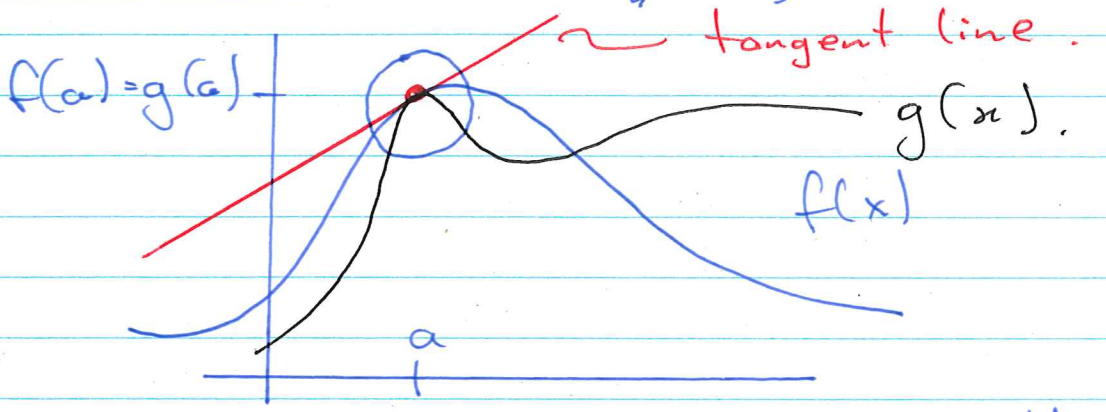
Sept. 28

- HW # 2 is Due
- HW # 3 due Monday.
- Quiz # 2. Friday Oct 9.



Our goal is to find the slope of the tangent line (speed).

In order to find the tangent line what do we need to know about $f(x)$?

We need information about $f(x)$ "close" to $x = a$.

We need limits.

Limits: (Section 1.1 / 1.2 in Contemporary Calculus).

The limit is an operation that we perform on a function.

We write $\lim_{x \to a} f(x) = L$.

to mean: the limit of $f(x)$ as x approaches a is L.

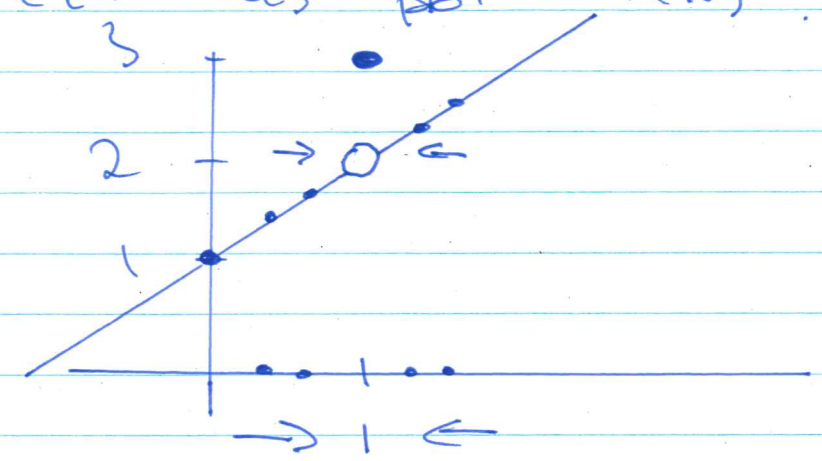
the values $y = f(x)$ "gets close" to the number L as x "gets close" to the number a.

Let's see an example.

Example: Find $\lim_{x \to 1} f(x)$ where

$$f(x) = \begin{cases} x+1, & x \neq 1 \\ 3, & x = 1 \end{cases}.$$

Let us plot $f(x)$.



To get an idea of what is going on we can compute some values of $f(x)$ when x is close to 1.

$f(1.1) = 2.1$ $\quad$ $f(0.9) = 1.9$

$f(1.01) = 2.01$ $\quad$ $f(0.99) = 1.99$

It looks like the values of $f(x)$ are approaching 2 (this guess is correct).

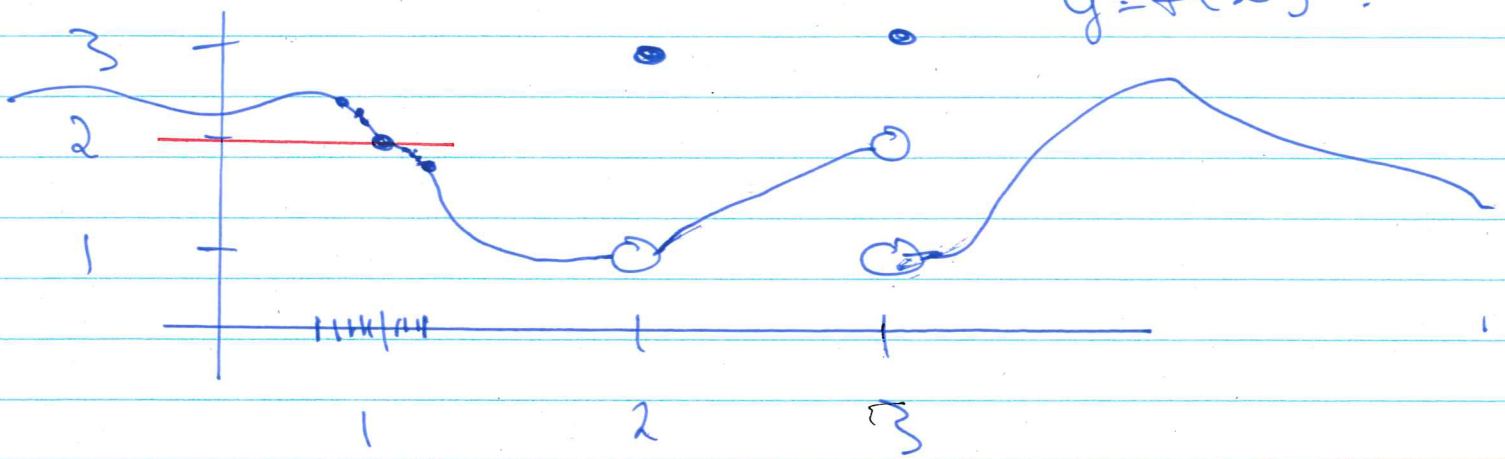
We write

$$\lim_{x \to 1} [f(x)] = 2.$$

Note however that $f(1) = 3 \neq 2$.

The limit doesn't care what happens at $x=1$ just near $x=1$.

Examples



Clicker Q: What is $\lim_{x \to 1} f(x)$?

A) 1

→ B) 2

C) 3

D) None of the above.

$\lim_{x \to 1} f(x) = 2$.

Clicker Q: What is $\lim_{x \to 2} f(x)$?

(A).

$\lim_{x \to 2} f(x) = 1$.

(5)

Clicked: $\lim_{x \to 3} f(x)$? (D).

This limit does not exist.
There is no one number that $f(x)$ approaches.

$\lim_{x \to 3} f(x)$ D.N.E.

This function does have one sided limits at $x = 3$.

$\lim_{x \to 3^-} f(x) = 2$, $\lim_{x \to 3^+} f(x) = 1$

$\lim_{x \to 3^-}$: one sided limit from the left/below.

$\lim_{x \to 3^+}$: one sided limit from the right/above.

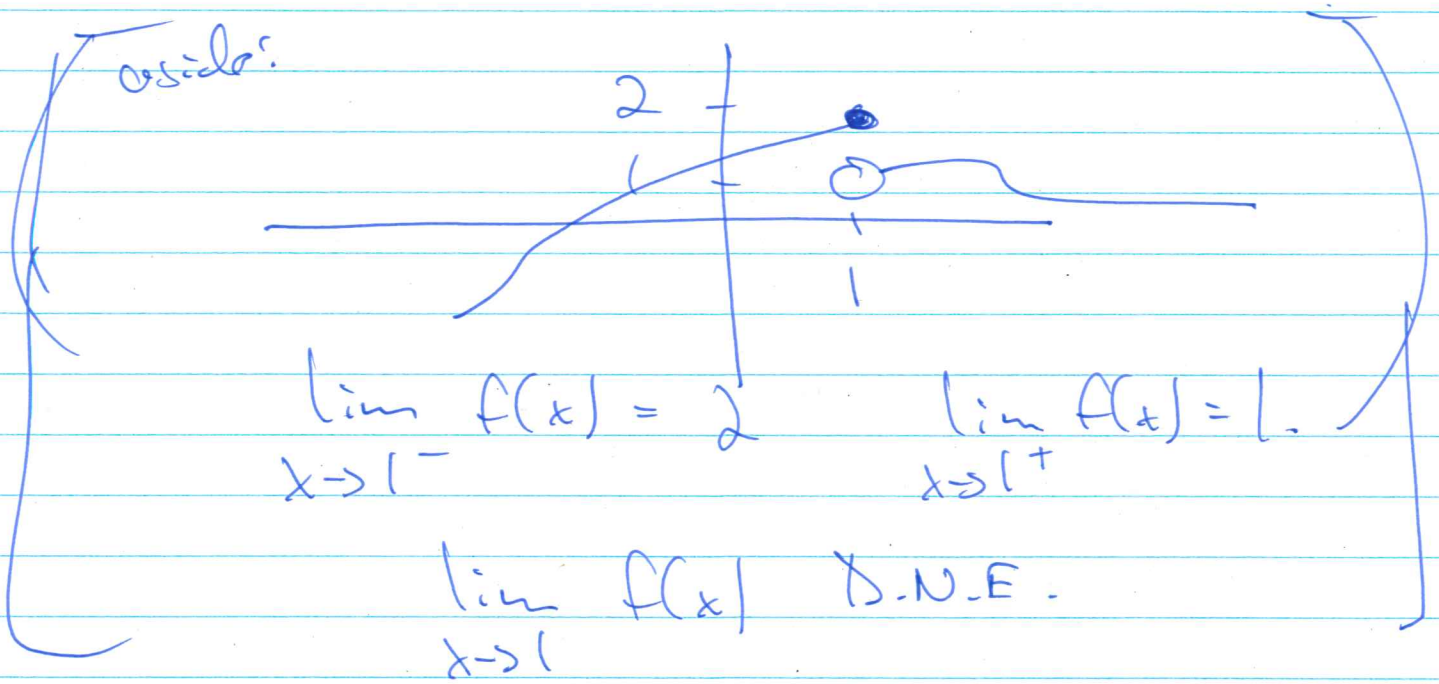
Let's find the one sided limits at $x = 2$.

$\lim_{x \to 2^-} f(x) = 1$, $\lim_{x \to 2^+} f(x) = 1$.

Note: If the one sided limits exist and are the same number then the full limit exists and is the same number.

ie. $\lim_{x \to 2^-} f(x) = 1$ and $\lim_{x \to 2^+} f(x) = 1$

$\Longrightarrow \lim_{x \to 2} f(x) = 1$.



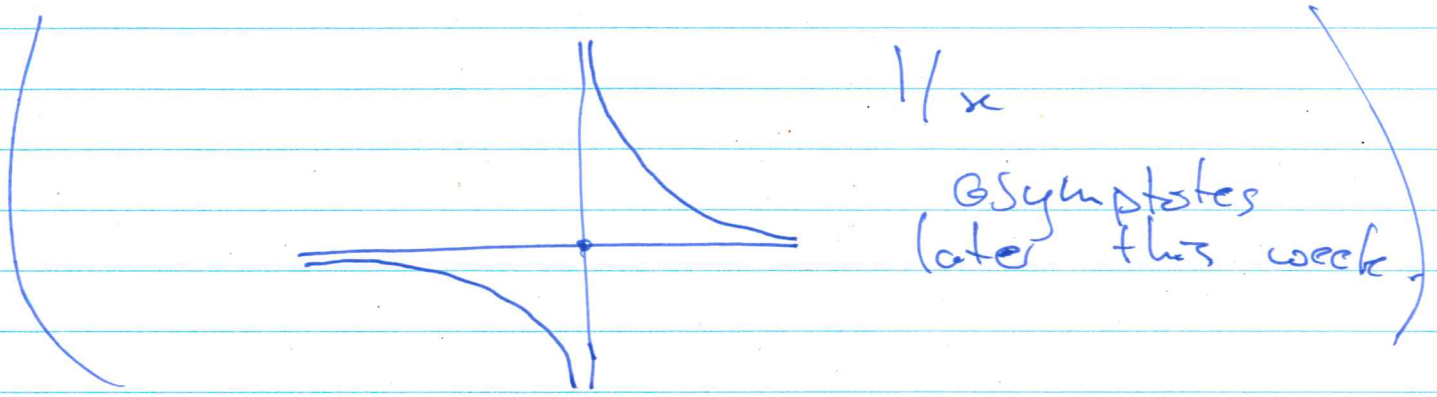
$\lim_{x \to 1^-} f(x) = 2$ $\lim_{x \to 1^+} f(x) = 1$.

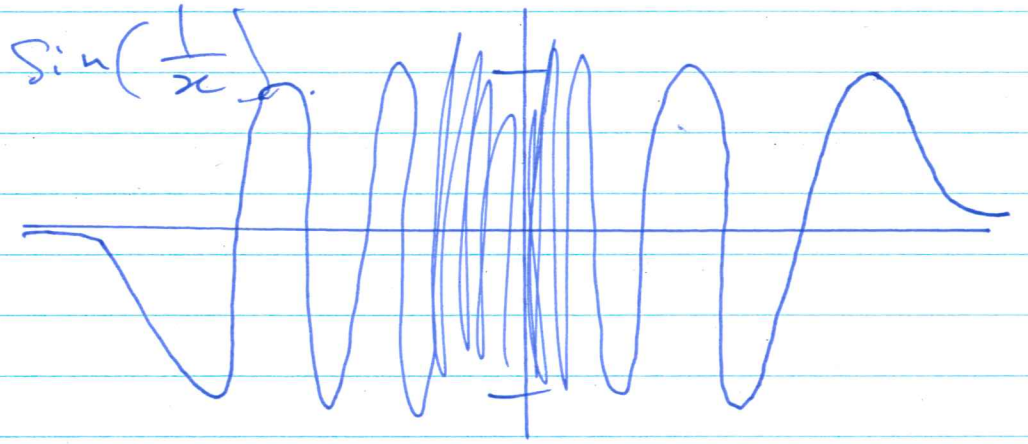
$\lim_{x \to 1} f(x)$ D.N.E.

If the two one sided limits are different then the full limit D.N.E.

(7)

Can anyone think of a function where the one sided limits do not exist?





$\lim_{x \to 0^+} \sin\left(\frac{1}{x}\right)$ does not exist.

So there is no one number. the y values approach so the limit D.N.E.

Also $\lim_{x \to 0^-} \sin\left(\frac{1}{x}\right)$ D.N.E.

and

$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ D.N.E.