Sept. 30

- HW #2 Solutions posted
- HW #3 Due Monday
- Quiz #2 Friday Oct. 9 (limits).
- Learning Objectives posted
  - will be updated each week.

Limits:

But how to compute limits given an equation?

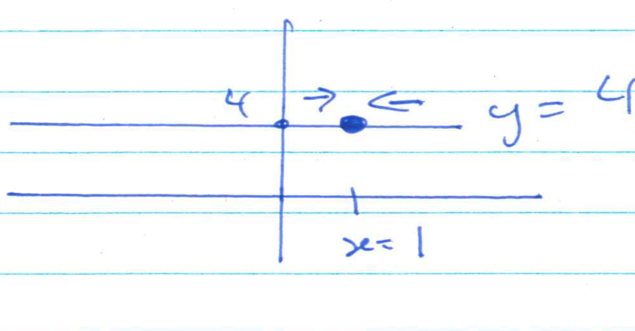
Last class we computed limits using graphs (§ 1.1).

Today equations (§ 1.2).

Clicker Q: What is $\lim_{x \to 1} 4$, $= 4$.

A) 0 C) 2 E) D.N.E.

B) 1 → D) 4

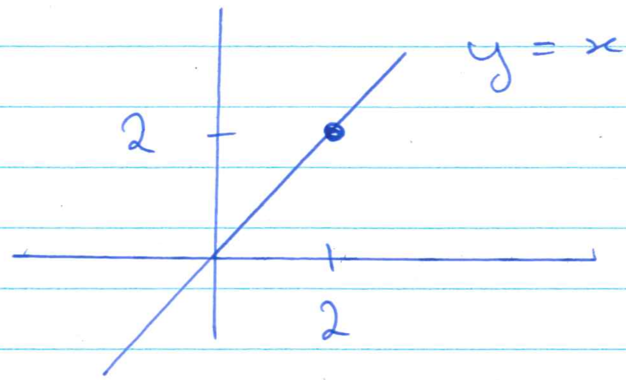


Sept. 30

Clicker Q: $\lim_{x \to 2} x$

Some options.

(E).



$\lim_{x \to 2} x = 2$.

Sept. 30.

Using these simple limits and the following limit laws we can compute limits of "nice" functions.

Limit Laws (Start of § 1.2)

If $\lim\limits_{x \to a} f(x) = L$ and $\lim\limits_{x \to a} g(x) = M$

then

a) $\lim\limits_{x \to a} \big(f(x) + g(x)\big) = \lim\limits_{x \to a} f(x) + \lim\limits_{x \to a} g(x) = M + L$

b) $\lim\limits_{x \to a} \big(f(x) - g(x)\big) = \lim\limits_{x \to a} f(x) - \lim\limits_{x \to a} g(x) = M - L$

c) $\lim\limits_{x \to a} k f(x) = k \lim\limits_{x \to a} f(x) = kL$

d) $\lim\limits_{x \to a} \{f(x) g(x)\} = \left[\lim\limits_{x \to a} f(x)\right]\left[\lim\limits_{x \to a} g(x)\right] = M \cdot L$

e) $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} = L/M \quad (M \neq 0)$

f) $\lim\limits_{x \to a} \big(f(x)\big)^n = \left(\lim\limits_{x \to a} f(x)\right)^n = L^n$.

Sept-30.

Example: Find $\lim_{x \to -2} \frac{x^2 - 3x + 2}{x^3 + 6}$.

$$= \frac{\lim_{x \to -2} (x^2 - 3x + 2)}{\lim_{x \to -2} (x^3 + 6)}.$$

(e).

(f). (c).

$$= \frac{\lim_{x \to -2} x^2 - \lim_{x \to -2} (3x) + \lim_{x \to -2} 2}{\lim_{x \to -2} (x^3) + \lim_{x \to -2} 6}$$

(a) (b) (e)

$$= \frac{\left(\lim_{x \to -2} x\right)^2 - 3 \lim_{x \to -2} x + \lim_{x \to -2} 2}{\left(\lim_{x \to -2} x\right)^3 + \lim_{x \to -2} 6}.$$

$$= \frac{(-2)^2 - 3(-2) + 2}{(-2)^3 + 6}.$$

$$= \frac{4 + 6 + 2}{-8 + 6} = \frac{12}{-2} = -6.$$

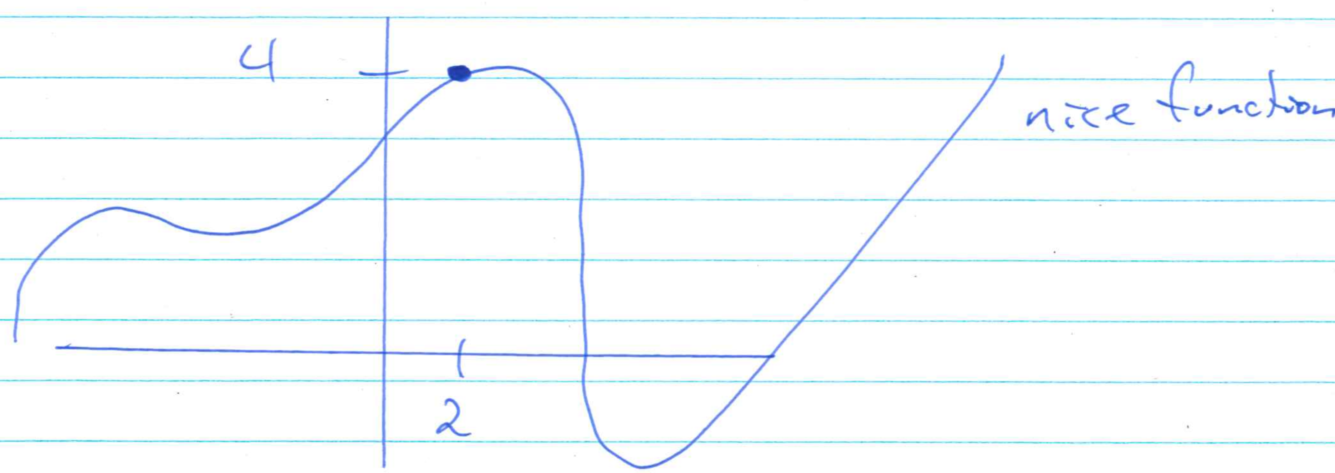
Sept. 30.

Notice that really all we did was plug in $x=-2$.
Observe,

$$\lim_{x\to -2} \frac{x^2-3x+2}{x^3+6} = \frac{(-2)^2-3(-2)+2}{(-2)^3+6}$$
$$= -6.$$

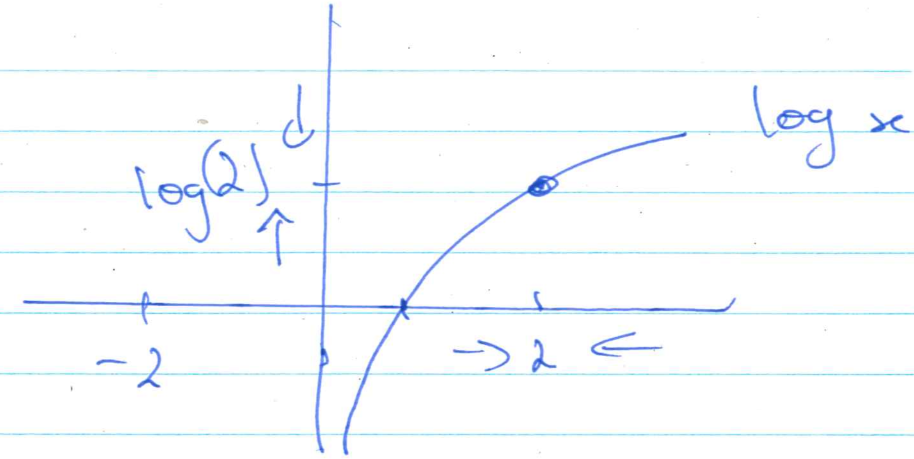
If you have a "nice." function to compute a limit you need only substitute.

Nice functions include
- polynomial.
- rational function of polynomials.
  - with no zero in the denominator.
- $\sin x$, $\cos x$, e^x
- $\sqrt{x}$ and $\log x$ (providing $x \geq 0$),



Example: $\lim_{x\to 2} \log(x) = \log(2)$.

Sept. 30.



Substitution won't always work.

Example: $$\lim_{x \to 3} \frac{\overbrace{x^2 - 2x - 3}^{f(x)}}{x - 3}.$$

If we try to plug in $x = 3$:

$$f(3) = \frac{3^2 - 2(3) - 3}{3 - 3} = \frac{0}{0}$$

not a number.

$f(3)$ does not exist
but the limit might still exist.

Let us factor.

~~lim~~

(7)

Sept. 30

$$\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{(x-3)}}$$

(we can cancel the $(x-3)$'s since x is just close to 3 and not equal to 3)

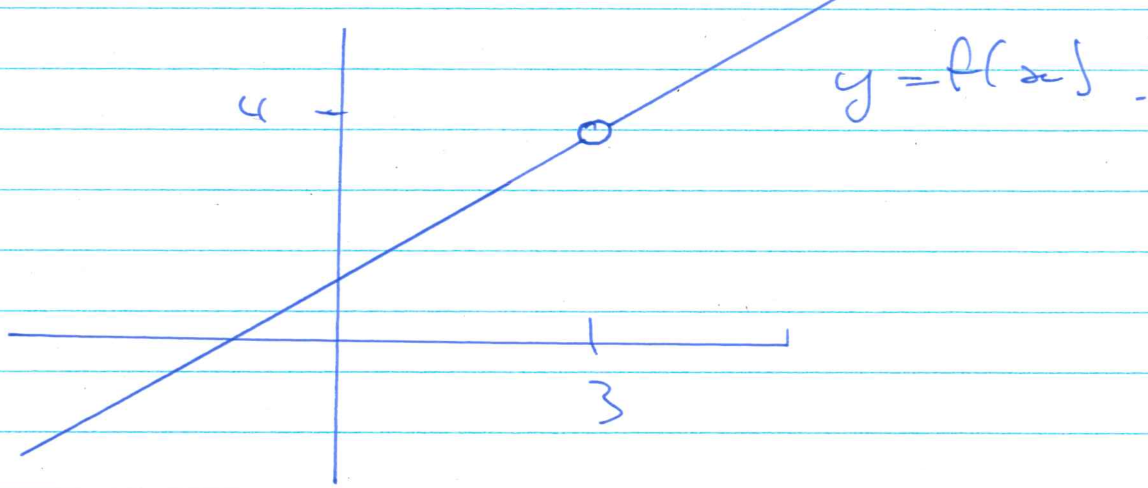
$$= \lim_{x \to 3} (x+1) \ \cancel{\approx} \ 4$$

← substitution.

$= 3 + 1$

$= 4.$

we stop writing limit after we substitute.



Sept. 30

same.

$$f(x) = \begin{cases} x+1 & x \neq 3 \\ \text{undefined}, & x = 3 \end{cases}$$

$$= \frac{x^2 - 2x - 3}{x - 3}.$$

Example: Find $\lim_{x \to 2^-} f(x)$, $\lim_{x \to 2^+} f(x)$

and $\lim_{x \to 2} f(x)$.

where
$$f(x) = \begin{cases} x+3, & x \geq 2 \\ -x^2 - 1, & x < 2 \end{cases}.$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} -x^2 - 1$$

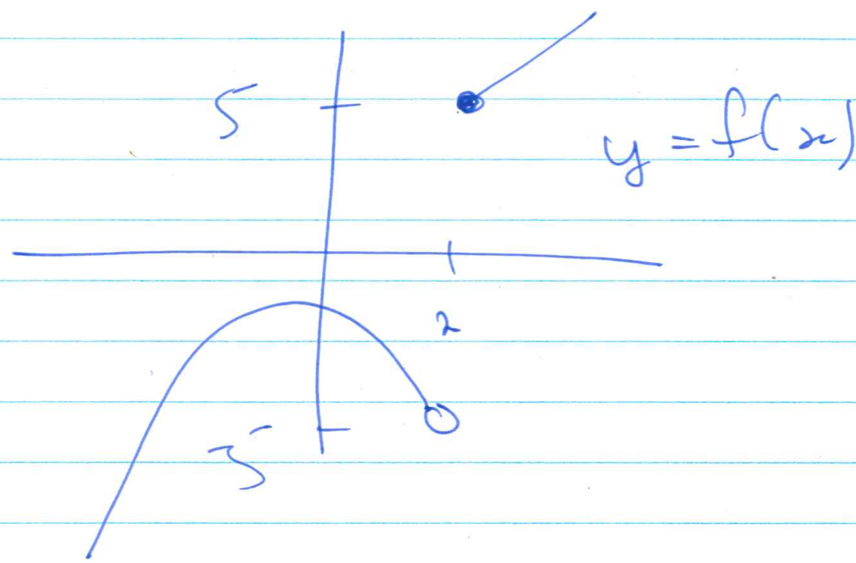
$$= -(2)^2 - 1 = -4 - 1 = -5.$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x + 3 = 2 + 3 = 5.$$

$\lim_{x \to 2} f(x) =$ D.N.E. since the one-sided limits are not equal.

(9) Sept. 30

It is true that $f(2) = 5$.



Example: $\displaystyle\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$.

If we substitute we get "$\frac{0}{0}$".

When you get "$\frac{0}{0}$" you need to do something to cancel the 0 in the bottom.

$$\lim_{x\to 4} \frac{(\sqrt{x}-2)}{x-4} \cdot \frac{(\sqrt{x}+2)}{\sqrt{x}+2}$$

multiply top/bottom by conjugate.

$$= \lim_{x\to 4} \frac{x + \cancel{2\sqrt{x}} - \cancel{2\sqrt{x}} - 4}{(x-4)(\sqrt{x}+2)}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2)} = \lim_{x\to 4} \frac{1}{\sqrt{x}+2}.$$

(18)

$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$ Sept. 30.

$= \frac{1}{\sqrt{4} + 2} = \frac{1}{2+2} = \frac{1}{4}$

For "$\frac{0}{0}$" algebra

$\Downarrow$

Cancel

$\Downarrow$

Substitute