Math 190 Integrals And Riemann Sum Worksheet

Questions:

1. Explain why the expression

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

should give exactly the area under the curve $f(x)$.

2. Explain why the expression

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

should give an approximation to the integral $\int_a^b f(x)dx$.

3. Explain why (possibly using a picture or possibly referencing the Fundamental Theorem of Calculus)

- $\displaystyle\int_a^a f(x)dx = 0$
- $\displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$
- $\displaystyle\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

are true.

4. Consider the function

$$f(x) = x^3$$

(a) Approximate the area under the curve $f(x)$ from $x = 0$ to $x = 3$ using Riemann Sums. Take $n = 3$ and use left endpoints. Is your approximation and underestimate an overestimate or exactly equal to the true value.

(b) Approximate again the area under the curve $f(x)$ from $x = 0$ to $x = 3$ using Riemann Sums. Take again $n = 3$ and this time use right endpoints. Is your approximation and underestimate an overestimate or exactly equal to the true value.

5. Consider the function

$$g(x) = -x^2 + 3x$$

(a) Approximate the integral

$$\int_{-1}^{2} g(x)dx$$

using Riemann Sums with $n = 3$ and left endpoints.

(b) Approximate the same integral but using $n = 4$ and again left endpoints.

(c) Approximate the same integral but using $n = 6$ and again left endpoints.

(d) Perform the integration and find the exact value. How do your approximations compare. Which is best? Why do you expect this?