Math 190 Quiz 2: Friday Oct 9

The quiz is 20 minutes long and has two questions. No calculators or other aids are permitted. Show all of your work for full credit. When asked to compute a limit: If the limit exists find its value. If the limit does not exist but equals $\pm\infty$ say so. Otherwise explain why the limit does not exist.

Questions:

1. Compute the following limits

   (a) $\lim_{x\to 4} 2x^2 - 7x - 4$

   (b) $\lim_{x\to 4} \dfrac{2x^2 - 7x - 4}{x - 4}$

   Solution:

   (a) We can compute this limit by direct substitution

$$\begin{aligned}\lim_{x\to 4} 2x^2 - 7x - 4 &= 2(4)^2 - 7(4) - 4 \\ &= 32 - 28 - 4 \\ &= 0\end{aligned}$$

   (b) For the following limit we factor the numerator and cancel the troublesome term

$$\begin{aligned}\lim_{x\to 4} \frac{2x^2 - 7x - 4}{x - 4} &= \lim_{x\to 4} \frac{(2x+1)(x-4)}{x-4} \\ &= \lim_{x\to 4} 2x + 1 \\ &= 2(4) + 1 \\ &= 9\end{aligned}$$

2. Compute $\lim_{x\to 3} f(x)$ where

$$f(x) = \begin{cases} -x^2 + 4, & x \leq 3 \\ \sqrt{x-3}, & x > 3 \end{cases}.$$

Solution: To understand this limit let us consider the one sided limits. The limit from the right we can compute by direct substitution

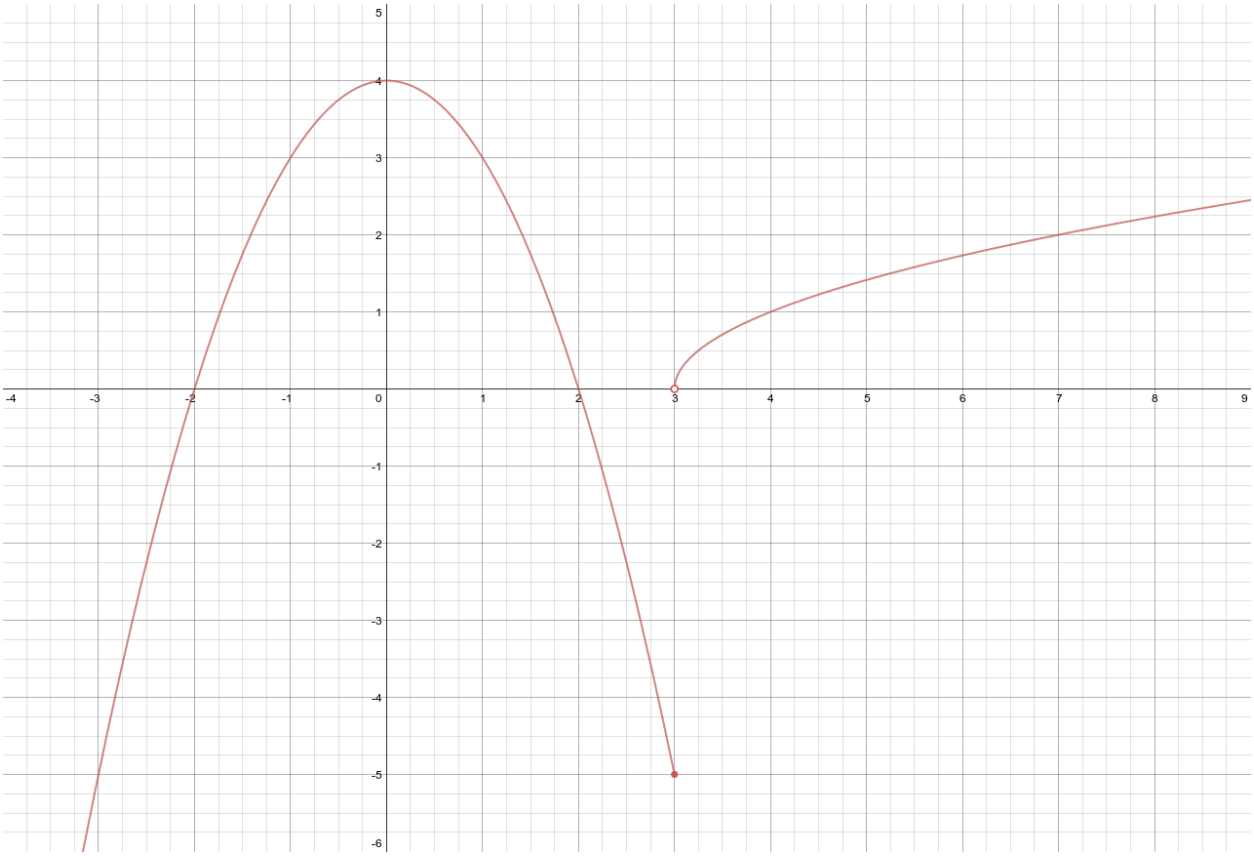
$$\begin{aligned} \lim_{x\to 3^+} f(x) &= \lim_{x\to 3^+} \sqrt{x-3} \\ &= \sqrt{3-3} \\ &= \sqrt{0} \\ &= 0. \end{aligned}$$

Note that since $x \to 3^+$ we have that $x > 3$ so we don't need to worry about taking the square root of a negative number. Similarly from the left we see

$$\begin{aligned} \lim_{x\to 3^-} f(x) &= \lim_{x\to 3^-} -x^2 + 4 \\ &= -(3)^2 + 4 \\ &= -5. \end{aligned}$$

And so, since the two one sided limits are not equal we conclude that $\lim_{x\to 3} f(x)$ does not exist.

Alternatively we can plot the graph of $f(x)$ (next page). From this graph we can see directly that the limit does not exist since the two one sided limits are not the same.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