Math 190 Quiz 3: Friday Oct 23

The quiz is 20 minutes long and has two questions. No calculators or other aids are permitted. Show all of your work for full credit.

Questions:

1. (a) Using the limit definition of the derivative (and not any other method) find the derivative of

$$f(x) = \frac{1}{3x}.$$

   (b) Now find the derivative of $f(x)$ again but using a different method of your choosing. State which method you are using.

Solution: (a) We first find the derivative using the limit definition. We compute

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{3(x+h)} - \frac{1}{3x}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{3x - 3(x+h)}{3(x+h)\cdot 3x}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{3x - 3x - 3h}{3(x+h)\cdot 3x}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{-3h}{3(x+h)\cdot 3x}\right) \\
&= \lim_{h\to 0} \frac{-1}{3(x+h)x} \\
&= -\frac{1}{3x^2}.
\end{aligned}$$

And so we have found the desired derivative.

(b) Let us now find the derivative using a different method. We will show two (only one was required). First we can use Power Rule by rewriting our function

$$f(x) = \frac{1}{3x} = \frac{1}{3}x^{-1}$$

and so

$$f'(x) = \left(\frac{1}{3}x^{-1}\right)' = \frac{1}{3}\left(x^{-1}\right)' = -\frac{1}{3}x^{-2} = -\frac{1}{3x^2}$$

as expected. We could have also used Quotient Rule to see

$$f'(x) = \frac{0 \cdot 3x - 1 \cdot 3}{(3x)^2} = -\frac{3}{3 \cdot 3x^2} = -\frac{1}{3x^2}$$

again, as expected.

2. Find the slope of the tangent line to

$$g(x) = \frac{1}{2}x^3 e^x$$

at the point $x = 2$.

Solution: To find the slope of the tangent line we first find the derivative. We employ product rule

$$\begin{aligned} g'(x) &= \left(\frac{1}{2}x^3 e^x\right)' \\ &= \frac{1}{2}\left(x^3 e^x\right)' \\ &= \frac{1}{2}\left(3x^2 e^x + x^3 e^x\right). \end{aligned}$$

Since the derivative gives the slope of the tangent line at a give point x to find the desired slope we compute

$$\begin{aligned} g'(2) &= \frac{1}{2}\left(3 \cdot 4e^2 + 8e^2\right) \\ &= 10e^2. \end{aligned}$$

And so the slope of the tangent line to $g(x)$ at $x = 2$ is $10e^2$.