

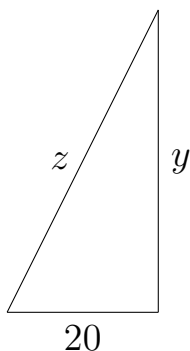
## Math 190 Quiz 4: Solutions

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*The quiz is 20 minutes long and has only one question. No calculators or other aids are permitted. Show all of your work for full credit. There is no need to simplify your final answer.*

1. Your trucker friend is 20 km West of base at a coffee shop enjoying a break. You are currently 80 km North of base travelling North (to pick up a shipment of lumber) at a speed of 60 km/h. How fast is the distance between you and your friend increasing at the present time?

**Solution:** We first consider the following triangle. Currently we have  $y = 80$  and  $z = \sqrt{20^2 + 80^2}$ .



The rate we are given is  $dy/dt$  and we require  $dz/dt$ . Note that the distance from your friend to the base is not changing in time. From our triangle we achieve the following equation (Pythagorean Theorem)

$$20^2 + y^2 = z^2.$$

Let us differentiate both sides in time. In this way we can relate the rates. So

$$\frac{d}{dt} (20^2 + y^2) = \frac{d}{dt} z^2$$

and since  $20^2$  is a constant we achieve

$$2y \frac{dy}{dt} = 2z \frac{dz}{dt}.$$

We can now solve for our required rate

$$\frac{dz}{dt} = \frac{y}{z} \frac{dy}{dt}$$

and substitute known values to achieve

$$\frac{dz}{dt} = \frac{80\text{km}}{\sqrt{20^2 + 80^2}\text{km}} \cdot 60\text{km/h}$$

or (simplification not necessary)

$$\frac{dz}{dt} = \frac{240}{\sqrt{17}}\text{km/h} \approx 58.2\text{km/h}.$$