Math 190 Quiz 5: Solutions

The quiz is 20 minutes long and has two questions. No calculators or other aids are permitted. Show all of your work for full credit.

Questions:

1. Find all values of b so that

$$\int_1^b (2x-5)\,dx = 0.$$

Solution1:
Let us compute the integral

$$0 = \int_1^b (2x-5)\,dx = x^2 - 5x\Big|_1^b = b^2 - 5b - \big(1^2 - 5(1)\big) = b^2 - 5b + 4.$$

So now we seek solutions, b, to

$$0 = b^2 - 5b + 4$$

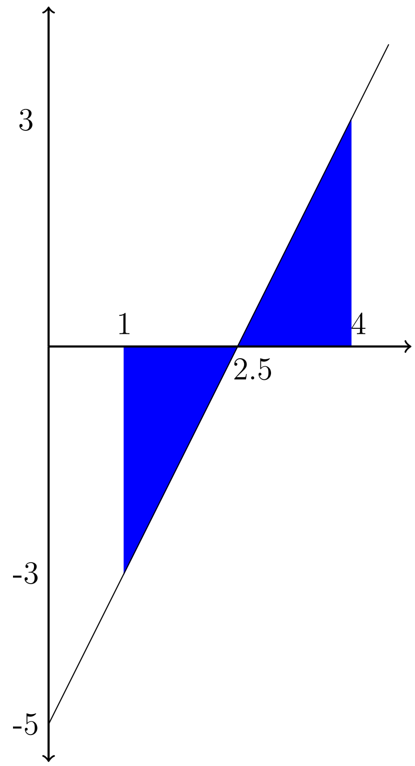
which we can find after factoring

$$0 = b^2 - 5b + 4 = (b-1)(b-4).$$

And so we have the two solutions $b = 1$ and $b = 4$.
Solution2:
Alternatively we can notice that

$$\int_1^1 (2x-5)\,dx = 0$$

and so $b = 1$ is a solution. To find the other we can graph the function and look for some symmetry. With a careful plot (see figure) of the graph we notice that if we take $b = 4$ then the total area under the curve will be zero (since the positive and negative area will cancel exactly).



2. Compute the following integral

$$\int \sin x \cos x dx.$$

Solution1: We can compute this integral by making the substitution $u = \sin x$. In this way $du = \cos x dx$. There follows

$$\int \sin x \cos x dx = \int u du = \frac{u^2}{2} + C = \frac{\sin^2 x}{2} + C.$$

Solution2: We can also compute this integral by making the substitution $u = \cos x$. In this way $du = -\sin x dx$. There follows

$$\int \sin x \cos x dx = -\int u du = -\frac{u^2}{2} + C = -\frac{\cos^2 x}{2} + C.$$

Note that while it may not look like it both of these functions are anti-derivatives to $f(x) = \sin x \cos x$. Observe that they differ only by a constant. We know that

$$\sin^2 x + \cos^2 x = 1$$

so

$$\sin^2 x = 1 - \cos^2 x$$
$$\frac{\sin^2 x}{2} = \frac{1}{2} - \frac{\cos^2 x}{2}.$$

Solution3: We can also solve this integral by doing an integration by parts. Let $u = \sin x$ and $dv = \cos x dx$. Then we have $du = \cos x dx$ and $v = \sin x$ so

$$\int \sin x \cos x dx = \sin^2 x - \int \sin x \cos x dx.$$

We can now rearrange the above as

$$\begin{aligned} \int \sin x \cos x dx + \int \sin x \cos x dx &= \sin^2 x \\ 2 \int \sin x \cos x dx &= \sin^2 x \\ \int \sin x \cos x dx &= \frac{\sin^2 x}{2} + C. \end{aligned}$$

Solution4: We could also use the trig identity $\sin x \cos x = \sin 2x/2$ to compute this integral

$$\int \sin x \cos x dx = \frac{1}{2} \int \sin 2x dx = -\frac{1}{2} \frac{\cos 2x}{2} + C = -\frac{1}{4} \cos 2x + C$$

Again note that this function differs from the other functions by a constant. For example if we employ the trig identity

$$\cos 2x = \cos^2 x - \sin^2 x$$

we see

$$\begin{aligned} -\frac{1}{4} \cos 2x &= -\frac{1}{4} \left(\cos^2 x - \sin^2 x\right) \\ &= \frac{1}{4} \left(\sin^2 x - \cos^2 x\right) \\ &= \frac{1}{4} \left(\sin^2 x - (1 - \sin^2 x)\right) \\ &= \frac{1}{4} \left(\sin^2 x - 1 + \sin^2 x\right) \\ &= \frac{1}{4} \left(2 \sin^2 x - 1\right) \\ &= \frac{\sin^2 x}{2} - \frac{1}{4}. \end{aligned}$$