ANTIDERIVATIVES

Example: find a function $F(x)$ with $F'(x) = \sin(2x)$:

$$\frac{d}{dx}\left[-\frac{1}{2}\cos(2x)\right] = -\frac{1}{2}(-\sin(2x))(2) = \sin(2x)$$

An **antiderivative** of a function f is a function F satisfying $F'(x) = f(x)$.

Why might we need to find antiderivatives?

– to recover position from velocity, velocity from acceleration, etc.

– more generally, to solve ODEs (coming soon...)

– to compute definite integrals (Fundamental Theorem of Calculus)

ANTIDERIVATIVES

Example: Find *ALL* antiderivatives of $\sin(2x)$:

$$\frac{d}{dx}\left[-\frac{1}{2}\cos(2x) + C\right] = \sin(2x) \quad \text{for any constant } C$$

> Theorem: any two antiderivatives of a given function differ by a constant.

"Proof": If $F' = f$ and $G' = f$, then

$$\frac{d}{dx}[F(x) - G(x)] = F'(x) - G'(x) = f(x) - f(x) = 0,$$

and so $F(x) - G(x) = C$ for some constant C.

ANTIDERIVATIVES

The **indefinite integral** $\int f(x)dx$ of a function f is its general antiderivative:

$$\int f(x)dx = F(x) + C$$

where $F'(x) = f(x)$.

Example: $\int \sin(2y)dy = -\frac{1}{2}\cos(2y) + C$

Try these:

1. $\int \left(t^2 + t^3\right)\ dt$
2. $\int \left(\sqrt{s} + \frac{2}{s^2}\right)\ ds$
3. $\int e^{\cos(3x)} \sin(3x)\ dx$
4. $\int \cos(x^2)\ dx$

ANTIDERIVATIVES

1. $\int \left(t^2 + t^3\right)\ dt\ = \frac{1}{3}t^3 + \frac{1}{4}t^4 + C$

   (Why? $\frac{d}{dt}\left[\frac{1}{3}t^3 + \frac{1}{4}t^4 + C\right] = \frac{1}{3}\cdot 3t^2 + \frac{1}{4}\cdot 4t^3 + 0 = t^2 + t^3$)

2. $\int \left(\sqrt{s} + \frac{2}{s^2}\right)\ ds\ = \int \left(s^{\frac{1}{2}} + 2s^{-2}\right) ds = \frac{2}{3}s^{\frac{3}{2}} - 2s^{-1} + C$

   (Why?
   $\frac{d}{ds}\left[\frac{2}{3}s^{\frac{3}{2}} - 2s^{-1} + C\right] = \frac{2}{3}\cdot\frac{3}{2}s^{\frac{1}{2}} - 2\cdot(-1)s^{-2} + 0 = \sqrt{s} + \frac{2}{s^2}$)

3. $\int e^{\cos(3x)}\sin(3x)\ dx\ = -\frac{1}{3}e^{\cos(3x)} + C$

   (Why? $\frac{d}{dx}\left[-\frac{1}{3}e^{\cos(3x)} + C\right] =$
   $-\frac{1}{3}e^{\cos(3x)}\cdot(-\sin(3x))\cdot 3 + 0 = e^{\cos(3x)}\sin(3x)$)

4. $\int \cos(x^2)\ dx$ ...stay tuned...

ANTIDERIVATIVES

Example: Find the height as a function of time of an object shot straight up into the air with speed v_0 (no drag):

- acceleration: $a(t) = -g$ (constant)
- velocity: $v(t) = \int a(t)dt = \int(-g)dt = -gt + C_1$

  $v_0 = v(0) = C_1 \implies C_1 = v_0$
- height: $h(t) = \int v(t)dt = \int(-gt + v_0)dt = -\frac{1}{2}gt^2 + v_0 t + C_2$

  $0 = h(0) = C_2 \implies \boxed{h(t) = -\frac{1}{2}gt^2 + v_0 t}$

  (until it hits the ground!)