Science One

Mathematics

September 13 2018

Announcements

- First Math assignment on WebWork:

  MATH 1.1 Limits (for marks)

  Practice on Limits (optional)
- Quiz on Tuesday September 25, 25-30 minutes on limits

What does a bear dream about in hibernation?

What does a bear dream about in hibernation?

….about the limit of a function , of course!

The limit of a function

Last time:

- gave an informal definition of limit
- looked at a few examples of limits that do not exist
- Introduced one sided limits

Today:

- Discuss more examples of limits that do not exist
- Define vertical asymptotes
- Evaluate limits using limit laws and the squeeze theorem
- Give a mathematical definition of continuous function

Definition of a limit

Which one of the following statements explains the meaning of the notation

$$\lim_{x \to a} f(x) \text{ ?}$$

A. When $f(x)$ is close to L, x is close to a.

$B.$ $f(x)$ is close to L as x gets closer to a.

C. When x equals a, the function returns L.

Definition of a limit

Which one of the following statements explains the meaning of the notation

$$\lim_{x \to a} f(x) \text{ ?}$$

A. When $f(x)$ is close to L, x is close to a.

B. $\boldsymbol{f(x)}$ is close to $\boldsymbol{L}$ as $\boldsymbol{x}$ gets closer to $\boldsymbol{a}$.

C. When x equals a, the function returns L.

Definition of limit

$$\lim_{x \to a} f(x) = L$$

means

we can make $f(x)$ be as close as we like to the number L (i.e. we can make $f(x)$ be within any arbitrary number ε of L) by taking x-values to be sufficiently close to the number a (i.e. by taking x-values within a suitable interval of width δ).

If a function f is not defined at $x = a$

A. $\lim_{x \to a} f(x)$ does not exist.

B. $\lim_{x \to a} f(x)$ could be 0.

C. $\lim_{x \to a} f(x)$ must approach infinity.

D. None of the above.

If a function f is not defined at $x = a$

A. $\lim_{x \to a} f(x)$ does not exist.

B. $\boldsymbol{lim_{x \to a} f(x)}$ could be 0.

C. $\lim_{x \to a} f(x)$ must approach infinity.

D. None of the above.

Conditions for existence of a limit

- As $x \to a$ (on both sides), the function output must approach (get arbitrarily close to) a single, finite number.

- Left-hand limit must equal right-hand limit

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

$$\lim_{x \to (\pi/2)^+} \frac{1}{\cos(x)}$$

A. DNE because it diverges to $+\infty$.
B. DNE because it diverges to $-\infty$.
C. DNE because it oscillates.
D. equals 0.
E. equals 1.

$$\lim_{x \to (\pi/2)^+} \frac{1}{\cos(x)}$$

A. DNE because it diverges to $+\infty$.

B. DNE because it diverges to $-\infty$.

C. DNE because it oscillates.

D. equals 0.

E. equals 1.

Vertical Asymptotes

Defn. The line $x = a$ is a **vertical asymptote** to $y = f(x)$ if $\lim_{x \to a} f(x) = +\infty$ (or $-\infty$)

(or possibly if f approaches ∞ on one side)

Another example of a limit that does not exist

$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist because

A. $\frac{1}{x}$ is undefined at 0.

B. no matter how close *x* gets to 0, there are *x*-values near 0 for which $\sin(1/x) = 1$ and some for which $\sin(1/x) = -1$.

C. all of the above.

Another example of a limit that does not exist

$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist because

A. $\frac{1}{x}$ is undefined at 0.

B. no matter how close *x* gets to 0, there are *x*-values near 0 for which $sin(1/x) = 1$ and some for which $sin(1/x) = -1$.

C. all of the above.

Conditions for existence of a limit

- As $x \to a$ (on both sides), the function output must approach (get arbitrarily close to) a single, finite number.

- Left-hand limit must equal right-hand limit

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

Conditions for existence of a limit

- As $x \to a$ (on both sides), the function output must approach (get arbitrarily close to) a single, finite number.

  This condition is not satisfied if

  - $f(x)$ grows larger and larger (approaches infinity)
  - $f(x)$ may oscillate

- Left-hand limit must equal right-hand limit

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

  This condition is not satisfied by some piecewise functions

How to evaluate limits

Building blocks

$$\lim_{x \to a} c = c$$ where c is a constant

$$\lim_{x \to a} x = a$$

Then we use the limit laws

Limit Laws (p. 53, 57)

- $\lim_{x\to a} f(x) + g(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
- $\lim_{x\to a} f(x) - g(x) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
- $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$
- $\lim_{x\to a} f(x) \cdot g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
- $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$
- $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$,

then $\lim_{x \to a} \dfrac{f(x)}{g(x)}$

A. can not exist

B. must exist

C. not enough information to determine

If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$,

then $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)}$

A. can not exist

B. must exist

C. not enough information to determine

When the limit laws fail:

How to handle indeterminate form $\frac{0}{0}$

- Rewrite function in simplified form
  - Factor and cancel
  - Multiply by conjugate
  - Expand parenthesis and simplify
- Split the limit into right-hand and left-hand limits
- (later in the term we’ll apply l’Hopital’s rule)

$$\lim_{t\to 1^+} \frac{\sqrt{t+1}}{1-t+t^2}$$

$$\lim_{u\to 1^+} \frac{(1-u^2)(1+u)}{1-2u+u^2}$$

$$\lim_{y\to 3} \frac{y-3}{|y-3|}$$

$$\lim_{h\to 0} \frac{(2+h)^2-4}{h}$$

$$\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2}$$

$$\lim_{x\to -a} \frac{\left(x^3+a^3\right)^5+\left(x^3+a^3\right)^7}{(x^3+a^3)^5}$$