Science One Math

November 15, 2018

Today we will

- prove an important fact in Calculus (one that you have just used in the test):

If $\boldsymbol{f'(x) > 0}$ for all x in an interval, f is **increasing** on that interval.

If $\boldsymbol{f'(x) < 0}$ for all x in an interval, f is **decreasing** on that interval.

- Identify points where a function may attain local maximum (minimum) values

Defn: f is **increasing** on an interval $\mathcal{I}$ **if** for every $a < b$ in $\mathcal{I} \Rightarrow f(a) < f(b)$.

Theorem: If $f'(x) > 0$ for all x in $\mathcal{I}$, f is increasing on $\mathcal{I}$.

Proof: Consider $a < b$ for any a and b in $\mathcal{I}$. By MVT, there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} > 0$$

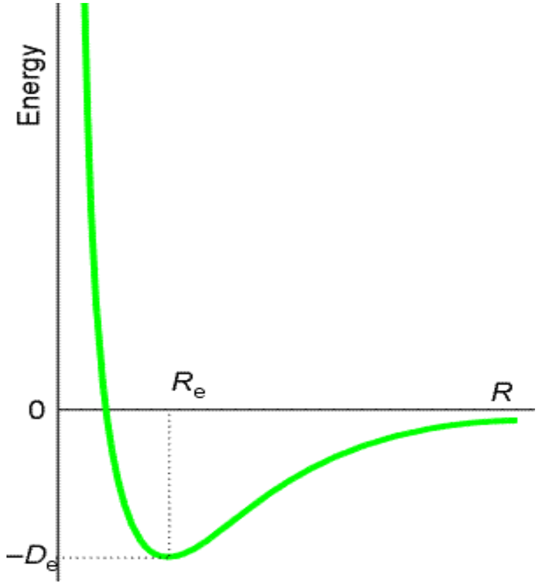
We chose $b > a$, so from above it follows $f(b) - f(a) > 0$, that is

$$f(b) > f(a) \quad \Rightarrow \quad f \text{ is increasing on } \mathcal{I}$$

Shape of a curve

What are the main features of this curve?

- Intervals of increase or decrease
- a local extreme value (minimum)
- asymptotic behaviour for $r \to \infty$



Lennard-Jones potential

Local Extreme Values (maxima and minima)

Defn: Let $f(x)$ be defined on $[a, b]$. Consider $a < c < b$, we say $f(x)$ has

- a local (or relative) **minimum** at $x = c$ if $f(x) \geq f(c)$ for all $a < x < b$,
- a local (or relative) **maximum** at $x = c$ if $f(x) \leq f(c)$ for all $a < x < b$.

A known fact (can be proved without calculus):

If f has a maximum (or minimum) at $x = c$ and $f'(c)$ exists, **then** $f'(c) = 0$.
(Fermat's Theorem)

Points where $f' = 0$ are good candidates for local extrema.

Are there other points where f could attain a local extremum?

Good candidates for extrema are points where $f' = 0$ and f' DNE.

Defn: A **critical number** of f is a number c (in the domain of f) such that either $f'(c) = 0$ or $f'(c)$ is undefined.

Does f always attain a local extremum at a critical number? Not always!

e.g. $f(x) = x^3$ no extremum at $x = 0$ even though $f'(0) = 0$.

We need a test to identify which critical numbers correspond to local extrema of f.

☞ first derivative test

☞ second derivative test (next week!)