

Science One

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Local versus Global extrema

f attains

- a **local** maximum at $x = c$ if $f(c) \geq f(x)$ for all x **near c**
- a **absolute** maximum at $x = c$ if $f(c) \geq f(x)$ for **all x** in the domain of f

How to locate absolute extrema?

Goal: finding the global extrema of a function either over its entire domain or on some restricted portion.

If f is continuous on $[a, b]$, then

- A) There must be numbers m and M such that $m \leq f(x) \leq M$ for all x in $[a, b]$
- B) There must be local extreme values, but there may or may not be an absolute maximum or absolute minimum value for the function
- C) Any absolute max or min would be at either the endpoints of the interval, or at places in the domain where $f'(x) = 0$.

The Extreme Value Theorem

If f is a continuous function on a closed interval $[a, b]$, then f attains both an absolute minimum and absolute maximum on $[a, b]$.

The only possible locations for absolute extrema are

- at the **endpoints** of the interval or
- at a **critical point**.

Find the absolute extrema (minima and maxima) of the following functions

$$g(x) = xe^{-ax}, \quad a > 0$$

$$f(x) = xe^{-ax}, \quad \left[0, \frac{2}{a}\right] \text{ with } a > 0$$

How to locate absolute extrema

If f is defined and continuous on an **open** interval

👉 compare function values at critical points with asymptotic behaviour of f

If f is defined and continuous on a **closed** interval

👉 compare function values at critical points with values at endpoints.