Science One Mathematics - Midterm Exam,
February 9th, 2016
*This midterm has **6 questions** on **11 pages**, for a total of 70 points.*

Duration: 80 minutes

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations; for changes of variables state how the variables are related. For integration by parts, state what the parts are. Answers without justifications will not be accepted.
- Continue on blank pages if you run out of space.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First name: _____ Last name: _____

Student #: _____ Bamfield #: _____

Signature: _____

Question:	1	2	3	4	5	6	Total
Points:	15	25	8	9	7	6	70
Score:							

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15 marks 1. Answer the questions below. Show all your work and provide justifications.

(a) If $f(2) = 12$, f' is continuous and $\int_2^4 f'(t)dt = 17$, find $f(4)$.

(b) A function f is defined by $f(x) = \int_0^x (1 - t^2)e^{t^2}dt$. On what interval(s) is f increasing?

(c) Find the limit as $n \to \infty$ of the sum $\sum_{i=1}^{n} \frac{2}{n} \cos(1 + i/n)$.

25 marks

2. Evaluate any **five** integrals of your choice from the list below. Remember to include integration constants whenever appropriate. Continue your work on the next page if you need more space.

(a) $\displaystyle\int \frac{x^2+1}{x^3-x}\,dx$

(b) $\displaystyle\int x\tan^2 x\,dx$

(c) $\displaystyle\int \ln\sqrt[3]{x}\,dx$

(d) $\displaystyle\int \arctan(1/x)\,dx$

(e) $\displaystyle\int_0^1 \frac{x^2}{(4-x^2)^{3/2}}\,dx$

(f) $\displaystyle\int_0^4 xe^{x^2}\,dx$

This page has been left blank for your workings and solutions.

8 marks

3. Find the area between the graphs of $\sin x$ and $\cos x$, between two consecutive points of intersection.

9 marks

4. Consider the following solid S. The base of S is the region in the x–y plane with boundary curve $x^2 + y^2 = 4$; Cross-sections perpendicular to the x-axis are squares, with lower edge in the base. Write down an integral that represents the volume of the solid S. You do not need to evaluate the integral.

7 marks 5. Find the value(s) of a for which the following improper integral converges:

$$\int_1^\infty \left(\frac{ax}{x^2+1} - \frac{1}{2x} \right) dx.$$

Evaluate the corresponding integral(s).

6 marks

6. If a, b, c are constants such that $\displaystyle\int \frac{ax^2 + bx + c}{x^2(x+1)^3}\,dx$ is a rational function, and $c = 1$, find the value of b.

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