Science One Mathematics - Midterm Exam,
February 14th, 2017
*This midterm has **6 questions** on **11 pages**, for a total of 54 points.*

Duration: 80 minutes

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations; for changes of variables state how the variables are related. For integration by parts, state what the parts are. Answers without justifications will not be accepted.
- Continue on blank pages if you run out of space.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First name: _____ Last name: _____

Student #: _____ Bamfield #: _____

Signature: _____

Question:	1	2	3	4	5	6	Total
Points:	8	6	20	6	8	6	54
Score:							

This page has been left blank for your workings and solutions.

8 marks

1. Determine whether each of the following statements is true or false. **Provide justification**.

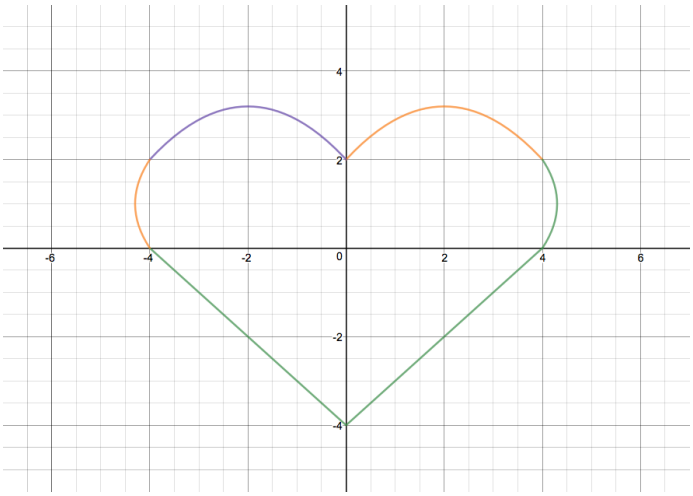
   (a) True/False. If f is continuous on $[a, b]$ then $\dfrac{d}{dx}\displaystyle\int_a^b f(t)dt = f(x)$.

   (b) True/False. Suppose $f(x)$ is a continuous function for all x and such that $f(0) = 0$. Let $g(x) = \int_{-1}^{x} f(t)dt$. Then the graph of g is tangent to the x-axis at $x = 0$.

   (c) True/False. If f has a discontinuity at $x = 0$, then $\int_{-1}^{1} f(x)dx$ does not exist.

   (d) True/False If f is continuous and $\int_0^9 f(x)dx = 4$, then $\int_0^3 xf(x^2)dx = 2$.

6 marks 2. Happy Valentine's Day.



Compute the area of the region enclosed by the heart-shaped curve shown above. The curve is described by the following equations,

$$\begin{aligned}
y &= |x| - 4 && \text{if } -4 \le x \le 4 \\
y &= 0.3x(4-x) + 2 && \text{if } 0 \le x \le 4 \\
y &= -0.3x(4+x) + 2 && \text{if } -4 \le x \le 0 \\
x &= -0.3y^2 + 0.6y + 4 && \text{if } 0 \le y \le 2 \\
x &= 0.3y^2 - 0.6y - 4 && \text{if } 0 \le y \le 2.
\end{aligned}$$

20 marks

3. Evaluate any **four** integrals of your choice from the list below. Remember to include integration constants whenever appropriate. Continue your work on the next page if you need more space.

(a) $\displaystyle\int \frac{1}{(x-1)(x^2+1)}\,dx$ (b) $\displaystyle\int \frac{\sin^3(x)}{\cos^2(x)}\,dx$ (c) $\displaystyle\int_0^3 (x+1)\sqrt{9-x^2}\,dx$

(d) $\displaystyle\int \frac{2x-1}{x^2-2x+5}\,dx$ (e) $\displaystyle\int_0^4 \log(1+x^2)\,dx$

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6 marks

4. Consider the solid whose base in the xy-plane is the region bounded by the curves $y = x^2$ and $y = 8 - x^2$, and whose cross-sections perpendicular to the x-axis are squares (with one side in the xy-plane). Find its volume.

8 marks

5. Determine whether each of the improper integrals is convergent or divergent, and if convergent find its value:

(a) $\int_0^\infty xe^{-x}dx$

(b) $\int_0^2 \frac{dx}{(x-1)^{2/3}}$

6 marks 6. (a) Find a Riemann Sum S_n that approximates the integral

$$I = \int_0^1 \sqrt{x} dx$$

and such that $S_n > I$. Construct S_n using n intervals, where n is a positive integer.

(b) Prove that the sum of the square roots of the first n positive integers can be approximated as

$$\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n} \approx \frac{2}{3} n^{3/2}$$

for large n.

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