

**MATH 110 Midterm 1**, October 26th, 2015**Duration: 90 minutes***This test has 7 questions on 10 pages, for a total of 60 points.*

- Read all the questions carefully before starting to work.
- All questions require a full solution; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First Name: \_\_\_\_\_ Last Name: \_\_\_\_\_

Student-No: \_\_\_\_\_ Section: \_\_\_\_\_

Signature: \_\_\_\_\_

Question:	1	2	3	4	5	6	7	Total
Points:	8	9	10	12	6	10	5	60
Score:								

**Student Conduct during Examinations**

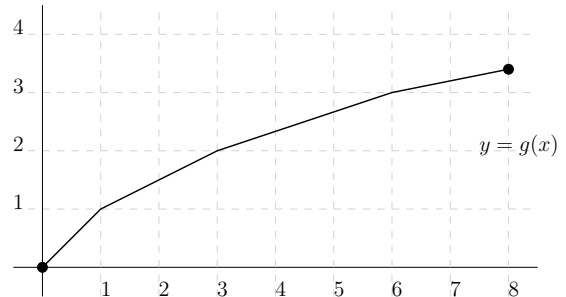
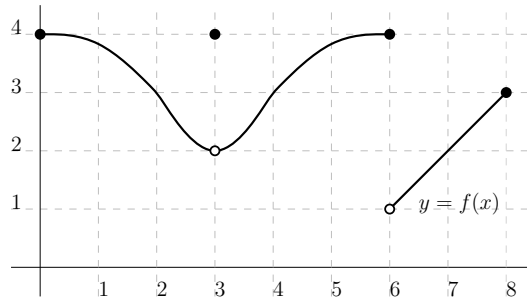
- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
  - purposely exposing written papers to the view of other examination candidates or imaging devices;
- Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
  - purposely viewing the written papers of other examination candidates;
- No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
  - using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  - using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
  - Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  - speaking or communicating with other examination candidates, unless otherwise authorized;
  - Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

**Full-Solution Problems.** In the following questions, justify your answers and **show all your work**. Unless otherwise indicated, **simplification of answers are required**.

1. This question has 3 different problems.

2 marks

(a) Consider the functions  $f(x)$  and  $g(x)$  whose values are below:



Compute the following values.

(i)  $f(f(2))$

(ii)  $g^{-1}(3)$ , where  $g^{-1}$  is the inverse of  $g$ .

(iii)  $g(f(1))$

3 marks

(b) (i) Find the domain of the following function

$$h(x) = \frac{\sqrt{3x-1}}{x^2 - 2x + 1}.$$

(ii) Find the range of the function  $f(g(x))$  where  $f(x) = \frac{2x}{x+1}$  and  $g(x) = 5$ .

3 marks

(c) Find the slope of the line which passes through the  $y$ -intercept and the positive  $x$ -intercept of the parabola  $y = x^2 - x - 2$ .

9 marks
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2. Evaluate the following limits, if they exist. If the limit does not exist, specify whether it is  $\infty$ ,  $-\infty$ , or neither. Show all your work.

(a)  $\lim_{x \rightarrow 1} (x + \sqrt{x})$ .

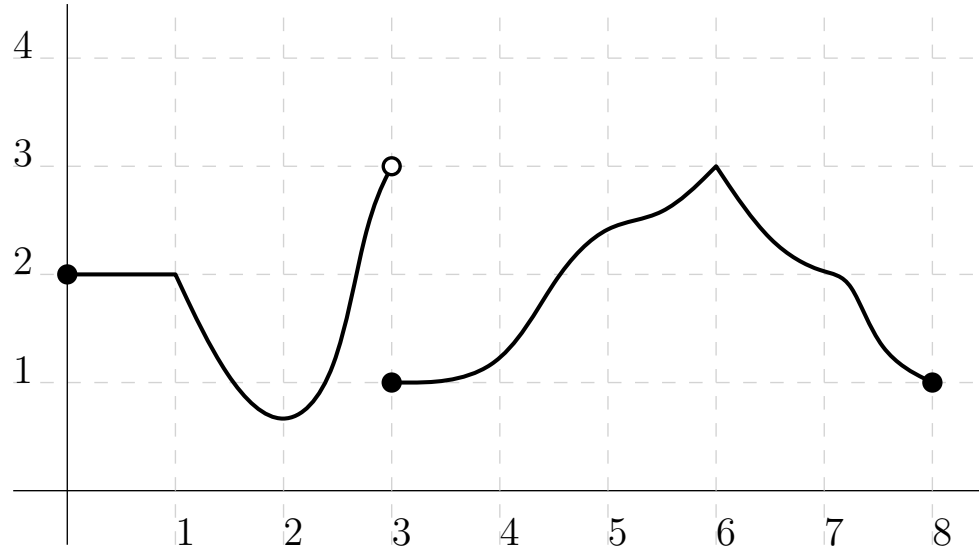
(b)  $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 2x}$ .

(c)  $\lim_{x \rightarrow 3} f(x)$ , where  $f(x) = \begin{cases} 4 - x & \text{if } x \leq 3 \\ x^2 - 3x + 1 & \text{if } x > 3. \end{cases}$

(d)  $\lim_{x \rightarrow -1^-} \frac{x^2 + 4x + 3}{x^2 + 2x + 1}$ .

(e) Find the value of the constant  $a$  such that  $\lim_{x \rightarrow 0} (ae^{-x} + |x - 1|) = 2$ .

10 marks

3. (a) Consider the function  $f$  whose graph is below.

(i) At what point(s), if any, is  $f$  not continuous? Give the  $x$ -coordinate of each point and explain why  $f$  is not continuous at that point.

(ii) At what point(s), if any, is  $f$  not differentiable? Give the  $x$ -coordinate of each point and explain why  $f$  is not differentiable at that point.

(iii) Find the two intervals of  $x$ -values in which the derivative  $f'$  of  $f$  is negative.

(b) Let  $a, b$  be real numbers and consider the function

$$g(x) = \begin{cases} b5^x + a + 1 & \text{if } x \leq 0 \\ ax^2 + b(x + 3) & \text{if } 0 < x \leq 1 \\ -a\sqrt{x} + 7bx & \text{if } x > 1. \end{cases}$$

Find the values of  $a$  and  $b$  such that  $g(x)$  is continuous for all real  $x$ . Make sure you show all your work and justify your claims.

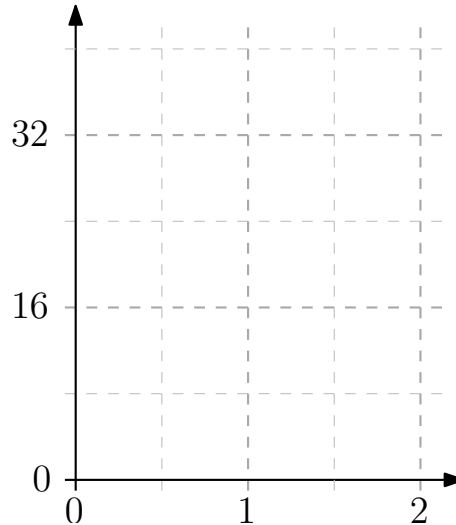
12 marks

4. A water balloon is tossed vertically in the air from a window. The balloon's height (in feet) at time  $t$  (in seconds) after being launched is given by

$$s(t) = -16t^2 + 16t + 32.$$

Use this function to respond to the following questions.

- (a) Sketch an accurate graph of  $s(t)$  on the diagram below.



- (b) Compute the average velocity of the balloon on the interval  $[1, 2]$ . Include units in your answer.
- (c) Use the limit definition to compute the instantaneous velocity of the balloon at the instant  $t = 1$ . Show your work using proper notation, include units. No marks will be given if the answer is obtained without using the limit definition of instantaneous velocity.

- (d) On your graph in part (a), sketch two lines: one whose slope represents the average velocity of the balloon on the interval  $[1, 2]$ , the other whose slope represents the instantaneous velocity at the instant  $t = 1$ . Label the first line  $L_1$ , and the second line  $L_2$ . Your sketch should be consistent with your calculations from part (b) and (c).
- (e) Using only one or two of the words *increasing*, *decreasing*, *positive*, *negative*, complete the following sentence:

The balloon's height at  $t = 1$  must be \_\_\_\_\_ because the instantaneous velocity at that instant is \_\_\_\_\_.

6 marks
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5. (a) Using the theorem(s) we discussed in class, carefully prove that the graph of the function  $f(x) = x^3 - 3x + 2$  crosses the line  $y = 3$  at least once in the interval between  $x = -2$  and  $x = 0$ . Make sure you justify your claim(s).

- (b) Explain why the graph of the function  $f(x)$  given in part (a) must cross the line  $y = 3$  at least twice in the interval between  $x = -2$  and  $x = 0$ .



10 marks
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6. Let  $f(x) = x^2 + 2x + 1$ . Use this function to answer the questions below.

(a) Find the equation of the tangent line to the graph of  $f(x)$  at the point where  $x = 2$ .

(b) Let

$$h(x) = \begin{cases} f(x) & \text{if } x \leq 2 \\ g(x) & \text{if } x > 2. \end{cases}$$

Find a function  $g(x) \neq x^2 + 2x + 1$  that makes  $h$  differentiable everywhere.

(c) Write down an expression for  $h'(x)$  and sketch its graph. Make sure your graph is clear and the axes are clearly labelled.

5 marks
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7. Find the equations of all lines that are tangent to the curve  $y = x^3$  and also pass through the point  $(1, 0)$ .