Math 102 Learning Goals

January 7, 2014

1 Course-level Learning Goals

Upon completing this course, students will be able to

1. demonstrate fluency in the basic techniques of differential calculus: differentiation, graphing of functions, optimization, analysis of rates of change, and solving basic differential equations.

2. use the basic techniques of differential calculus to model and solve problems in the life sciences.

2 Week and Topic-level Learning Goals

Week 1

Topics: Introduction to course; review of polynomials and power functions; introduction to Hill functions.

Learning Goals:

1. Explain that a power function with a greater power is greater than one with a lower power for sufficiently large values of the variable and vice versa for sufficiently small values of the variable.

    (a) Equivalently: Explain how the plot of the power function $y = x^n$ changes as n changes.

2. Approximate a polynomial or rational function (especially Hill functions) with simpler functions for sufficiently large/small values of the independent variable.

Week 2

Topics: Average and instantaneous rates of change; limits and continuity; definition of the derivative; asymptotes.

Learning Goals:

1. Explain the connection between rate of change of a function and the slope of a secant line.

2. Calculate the slope of a tangent line (when one exists) as a limit of the slopes of secant lines approaching the tangent line.

3. Define "limit of a function" unambiguously.

4. Define "continuous function" in terms of left and right-hand limits and use this definition to determine if a given function is or is not continuous.

5. Calculate finite and infinite limits of any power function and of functions comprised of the summation, difference, product, quotient, and composition of power functions.

6. Define "asymptote" as a line to which a function gets arbitrarily close in either a finite limit (for a vertical asymptote) or an infinite limit (for a horizontal asymptote).

7. Define "derivative of a function" in terms of secant and tangent lines.

Week 3

Topics: Properties of derivatives; tangent lines; calculating the derivative using the definition; displacement, velocity, and acceleration.

Learning Goals:

1. State the limit definition of the derivative.

2. Compute the derivative of a power function, polynomial or a function involving a square root using the definition of the derivative.

3. Compute the derivative of any polynomial using the power and sum rules.

4. Find the equation of a tangent line to any type of function discussed so far that passes through a point, either on or off the graph of the function.

5. Use given information about the tangent lines of a parametrized function (a function defined with unspecified constants) to identify the unknown constants.

6. Find the velocity and acceleration given the displacement function (graph or formula), and vice versa.

Week 4

Topics: Product and quotient rules; graphing; critical, extreme, and inflection points.

Learning Goals:

1. Calculate the derivative of products and quotients of power and rational functions.

2. Given the graph of a function, sketch the graph of the derivative function.

3. Given a function, calculate its intervals of increase/decrease, concavity, inflection points, and local and global maximum/minimum. Use this information to plot the function.

Week 5

Topics: Global extrema; introduction to optimization.

Learning Goals:

1. Recognize that a given word problem is an optimization problem.

2. Set up an optimization problem by finding the objective and constraint functions.

3. Find an optimal value and show that it is optimal using appropriate methods.

Week 6

Topics: Optimization; linear regression.

Learning Goals:

1. Find the slope of the line of best fit, that passes through the origin, to a set of data by minimizing the sum of the squared residuals.

2. Identify which line in a given set of lines best fits a given set of data.

Week 7

Topics: Chain rule; implicit differentiation.

Learning Goals:

1. Calculate the derivative of functions of the form p(q(x)) , where p(x) and q(x) are either polynomials or rational functions.

2. Calculate the derivative of polynomial functions defined implicitly.

3. Recognize that a given word problem is a related rates problem, translate the word problem into a mathematical problem, solve this mathematical problem and relate the answer back to the original word problem.

Week 8

Topics: The exponential and natural logarithm functions.

Learning Goals:

1. Use the properties of exponential and logarithmic functions to simplify, or rearrange expressions involving exponential and logarithmic functions.

   (a) In particular, transform an expression involving general exponential and logarithmic functions to one involving the natural exponential and logarithm, e and ln.

2. Calculate the derivative of y=af(x), where a is a constant and f(x) is a function of x.

3. Calculate the derivative of y=loga(f(x)), where a is a constant and f(x) is a function of x.

4. Convert (x,y) data to (ln(x),ln(y)) data.

5. Find a line of best fit to (ln(x),ln(y)) data and transform this line to a power function in the xy plane.

Week 9

Topics: Exponential growth and decay; introduction to differential equations.

Learning Goals:

1. Verify that a given function satisfies a given differential equation.

2. Find a function that satisfies a given elementary differential equation.

3. Find a function that describes the change in a quantity given information about that quantity's doubling or characteristic time.

4. Solve for the constants in a general solution to a differential equation using the information given in an initial value problem.

5. Solve for the doubling times and half lives in exponential growth/decay problems.

Week 10

Topics: Euler's method; trigonometric functions.

Learning Goals:

1. Use Euler's method to find approximate values of a solution to a given initial value problem.

2. Alter a trig function by introducing a change in amplitude or frequency, a phase shift, or a vertical shift.

3. Rewrite expressions involving trig functions using basic trig identities.

Week 11

Topics: Derivatives of trigonometric functions; related rates; second order differential equations; inverse trigonometric functions.

Learning Goals:

1. Calculate the derivative of a given trig function.

2. Evaluate an inverse trig function ($arcsin$, $arccos$, $arctan$) applied to a trig function (sin, cos, tan).

3. Calculate the derivative of a given inverse trig function.

4. Find a solution to $y + a^2 y = 0$.

Week 12

Topics: Linear approximation and Newton's method; qualitative methods for differential equations.

Learning Goals:

1. Find the linear approximation of a function at a point.

2. Use Newton's method to approximate the value of a root of a given function.

3. Given a differential equation, draw its slope field.

4. Find the steady states (equilibria) of a given differential equation.

5. Determine the stability of the equilibria of a given differential equation in two ways: by using the differential equation's

    (a) slope field, and,
    (b) phase line.

Week 13

Topics: Examples of differential equations.

Learning Goals:

1. Derive a differential equation from a given "word problem". Find the equilibria of this differential equation and classify them as stable, unstable, or semistable.