Linear Approximation - Quiz Solutions

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15:22

1. $f(x) \approx L_a(x) = f(a) + f'(a)(x-a)$

$f(x) = e^x \qquad a = 0$

$f'(x) = e^x \qquad f(a) = e^0 = 1$

$f'(a) = e^0 = 1.$

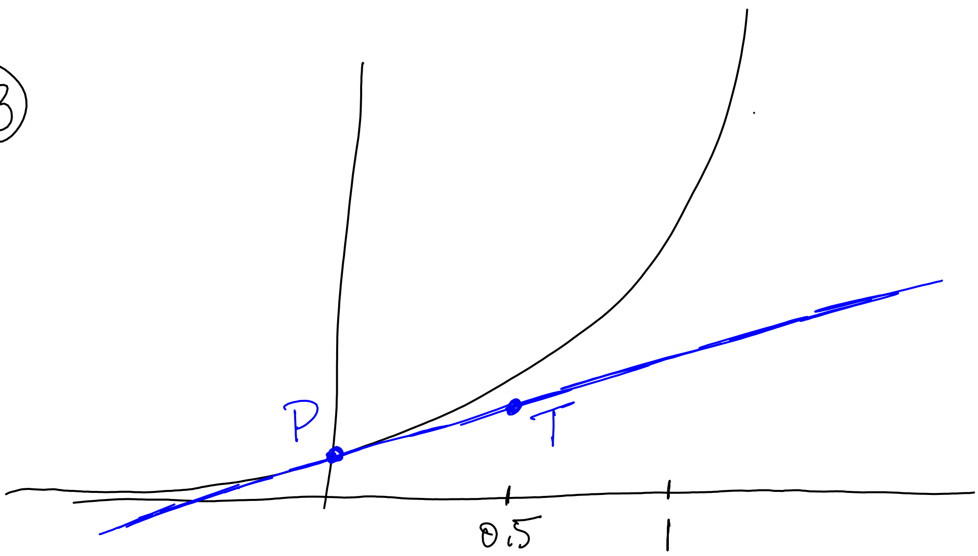
$L_0(x) = 1 + 1(x-0)$
$= 1 + x$

$f(0.5) = e^{0.5} \approx L_0(0.5) = 1 + 0.5$
$= 1.5$

2. b) underestimate.

The graph is concave up everywhere, in particular at $a=0$, So the tangent line will be below the graph at any point of approximation!

3.



4. $|\text{error}| \le \frac{M}{2}(x-a)^2$

we take $M = \max|f''|$ on an appropriate interval that contains both $a = 0$ and $x = 0.5$

$f'(x) = e^x \qquad f''(x) = e^x$

There are different M values that work. (we show a few possibilities here).

Attempt 1: Try interval $[0,1]$.

Since e^x is increasing its largest value on $[0,1]$ is $e^1 = e$

So $|f''(c)| \le e$ for all c in $[0,1]$

$$|\text{error}| \le \frac{e}{2}(0.5-0)^2 = \frac{e}{8}$$

also true and easier to interpret:

$$|\text{error}| \le \frac{e}{8} < \frac{3}{8} = 0.375$$

Attempt 2: Try interval $[0, 0.5]$.

max of f'' on $[0, 0.5]$ is $e^{0.5}$

$$|\text{error}| \le \frac{e^{0.5}}{2}(0.5-0)^2 = \frac{e^{0.5}}{8}$$

also true and easier to interpret:

$$|\text{error}| \le \frac{e^{0.5}}{8} < \frac{2}{8} = 0.25$$

Conclusion:

We can combine the error estimate from Attempt 2 with the fact that the approximation must be an **underestimate** to conclude that:

$$1.5 \leq e^{0.5} \leq 1.5 + 0.25$$

$$1.5 \leq e^{0.5} \leq 1.75$$