Definition of the derivative

Tuesday, September 25, 2012

Names and student numbers for group (minimum of 2):

1. ______________________

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2. ______________________

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3. ______________________

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Solutions.

① Use $\boxed{f(x) = \sqrt{x} + 1}$ in computing the derivative below.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{(\sqrt{x} + 1) - (\sqrt{a} + 1)}{x - a}$$

$$= \lim_{x \to a} \frac{\sqrt{x} \cancel{+1} - \sqrt{a} \cancel{-1}}{x - a}$$

$$= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} \left(\frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} \right)$$

$$= \lim_{x \to a} \frac{\cancel{x - a}}{\cancel{x - a}} \frac{1}{\sqrt{x} + \sqrt{a}}$$

$$= \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}} = \frac{1}{\sqrt{a} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} + 1) - (\sqrt{x} + 1)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \lim_{h \to 0} \frac{(\cancel{x} + h) - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{a+h} + 1) - (\sqrt{a} + 1)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{a+h} - \sqrt{a}}{h} \left(\frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} \right)$$

$$= \lim_{h \to 0} \frac{(\cancel{a} + h) - \cancel{a}}{h(\sqrt{a+h} + \sqrt{a})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{a+h} + \sqrt{a})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{a+h} + \sqrt{a}} = \frac{1}{\sqrt{a+0} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$$

Write two different limit expressions for $f'(1)$. Which would you prefer to use to compute $f'(1)$?

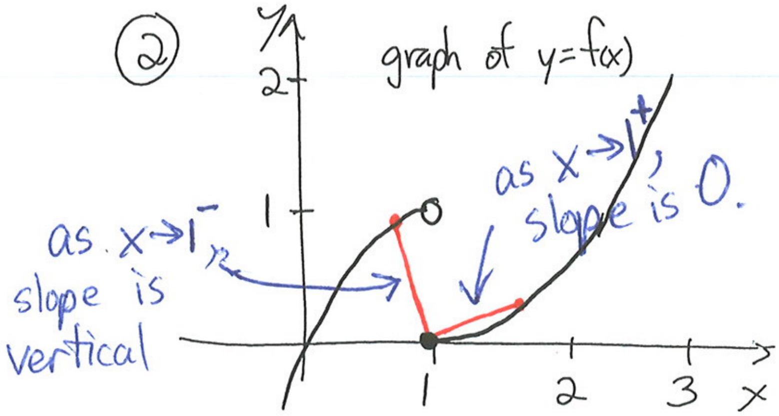
$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(\sqrt{1+h} + 1) - (\sqrt{1} + 1)}{h}$$

goal: manipulate to cancel h.

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{(\sqrt{x} + 1) - (\sqrt{1} + 1)}{x - 1}$$

goal: manipulate to cancel $x-1$

②



a) Draw the secant from $(1, f(1))$ to $(0.8, f(0.8))$ and the secant from $(1, f(1))$ to $(1.5, f(1.5))$.

Compute based on picture

b) On the left we have

$$\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{(2x-x^2)-0}{x-1} \quad \text{DNE}$$

and on the right:

$$\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{(x-1)^2-0}{x-1} = 0$$

$$f(x) = \begin{cases} 2x-x^2 & \text{if } x<1 \\ (x-1)^2 & \text{if } x\geq 1 \end{cases}$$

so $f(1)=0$

We can conclude that $f'(1)$ DNE because $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$ DNE.

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③ Which of these derivatives are you able to compute using derivative rules?

a) $f'(x)$ for $f(x) = 3x^2 - x + 5$

$$f'(x) = 6x - 1 + 0 = 6x - 1$$

b) $g'(x)$ for $g(x) = e^{3x}$

$$g'(x) = 3e^{3x}$$

c) $h'(t)$ for $h(t) = t\,\ln(t)$ note: $t>0$ must be true for this to make sense.

$$h'(t) = 1\cdot \ln(t) + t\cdot\frac{1}{t} = \ln(t) + 1$$

d) $R'(q)$ for $R(q) = p(q)\cdot q$, where p is some function of q.

$$R'(q) = p'(q)\cdot q + p(q)\cdot 1$$

e) $P'(q)$ for $P'(q) = \dfrac{q^2+q-1}{q\sin(q)}$ oops. assume $P(q) = \dfrac{q^2+q-1}{q\sin(q)}$.

$$P'(q) = \frac{(2q+1)q\sin(q) - (q^2+q-1)(\sin(q) + q\cos(q))}{(q\sin(q))^2}$$

it is correct, even though not simplified.