Derivatives

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09:28

Find the derivative of $f(x)$.

1. $f(x) = x^2$

   $f'(x) = 2x$

2. $f(x) = 5x^2 + 1$

   $f'(x) = 5(2x) + 0$
   $= 10x$

3. $f(x) = 1 - x^2$

   $f'(x) = 0 - 2x$
   $= -2x$

4. $f(x) = x + 2$

   $f'(x) = 1 + 0$
   $= 1$

5. $f(x) = 3x^8 - 5x - 2$

   $f'(x) = 24x^7 - 5$

6. $f(x) = 5 - x$

   $f'(x) = -1$

(7) $f(x) = e^x$

$f'(x) = e^x$

(8) $f(x) = x^2 + e^2$

$f'(x) = 2x + 0$

$= 2x$

(9) $f(x) = 1 - e^x - 2x^7 + \pi$

$f'(x) = 0 - e^x - 14x^6 + 0$

$= -e^x - 14x^6$

(10) $f(x) = (x+1)(2x^2 - 3)$

$f'(x) = 1 \cdot (2x^2 - 3) + (x+1)(4x)$

$= 2x^2 - 3 + (x+1)4x$

(11) $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2} \cdot x^{-1/2} = \frac{1}{2\sqrt{x}}$

(12) $f(x) = \frac{3x^2+1}{\sqrt{x}} = 3x^{3/2} + x^{-1/2}$

$f'(x) = \frac{9}{2}x^{1/2} - \frac{1}{2}x^{-3/2}$

(13) $f(x) = \dfrac{\sqrt{x}}{3x^2+1}$

$f'(x) = \dfrac{\frac{1}{2}(x^{-1/2})(3x^2+1) - \sqrt{x}(6x)}{(3x^2+1)^2}$

(14) $f(x) = \dfrac{1+x}{x^{2/3}} = x^{-2/3} + x^{1/3}$

$f'(x) = -\dfrac{2}{3}x^{-5/3} + \dfrac{1}{3}x^{-2/3}$

(15) $f(x) = ax^2 + bx + c$
a, b, c are constants

$f'(x) = 2ax + b$

(16) $f(x) = 7x^2(x+1)e^x = (7x^3+7x^2)e^x$

$f'(x) = (21x^2+14x)e^x + (7x^3+7x^2)e^x$

(17) $f(x) = \pi^2 e^x + 1$

$f'(x) = \pi^2 e^x$

(18) $f(x) = \pi^2 e^{\pi x}$

$f'(x) = \pi^3 e^{\pi x}$

(19) $f(x) = x^2 e^{-3x}$

(19) $f(x) = x^2 e^{-3x}$

$f'(x) = 2xe^{-3x} + x^2(-3e^{-3x})$

$= 2xe^{-3x} - 3x^2 e^{-3x}$

(20) $f(x) = \sqrt[3]{x}\, e^{5x}$

$f'(x) = \left(\frac{1}{3}x^{-2/3}\right)e^{5x} + \sqrt[3]{x}\,(5e^{5x})$

(21) $f(x) = \cos(x)$

$f'(x) = -\sin(x)$

(22) $f(x) = e^x \sin(x)$

$f'(x) = e^x \sin(x) + e^x \cos(x)$

(23) $f(x) = \dfrac{1+x}{\sin(x)}$

$f'(x) = \dfrac{1 \cdot \sin(x) - (1+x)\cos(x)}{(\sin(x))^2}$

(24) $f(x) = x^8 e^{2x} \cos(x)$

$f'(x) = 8x^7 e^{2x}\cos(x) + x^8 \frac{d}{dx}\left(e^{2x}\cos(x)\right)$

$= 8x^7 e^{2x}\cos(x) + x^8\left(2e^{2x}\cos(x) - e^{2x}\sin(x)\right)$

(25) $f(x) = \dfrac{x-2}{x^2+3}$

$f'(x) = \dfrac{1\cdot(x^2+3) - (x-2)(2x)}{(x^2+3)^2}$

More Derivative Problems

(1) $f(x) = \dfrac{1}{\sqrt{x}} = x^{-1/2}$ $\qquad f(3) = \dfrac{1}{\sqrt{3}}$

$f'(x) = -\dfrac{1}{2}x^{-3/2}$ $\qquad f'(3) = -\dfrac{1}{2(3^{3/2})}$

equation of tangent to $f(x)$ at $x=a$:

$$y = f'(a)(x-a) + f(a)$$

here, $y = f'(3)(x-3) + f(3)$

so $\boxed{y = -\dfrac{1}{2(3^{3/2})}(x-3) + \dfrac{1}{\sqrt{3}}}$

(2) $g(x) = 5e^{2x}$

$g'(x) = 10e^{2x}$

$\boxed{g'(0) = 10e^0 = 10}$

(3) $h(x) = 2xe^{-x}$

$$h'(x) = 2e^{-x} + 2x(-e^{-x})$$
$$= 2e^{-x} - 2xe^{-x}$$

$$h''(x) = -2e^{-x} - \left(2e^{-x} + 2x(-e^{-x})\right)$$
$$= -2e^{-x} - 2e^{-x} + 2xe^{-x}$$

$$h''(1) = -2e^{-1} - 2e^{-1} + 2\cdot 1\cdot e^{-1}$$
$$h''(1) = -4e^{-1} + 2e^{-1}$$
$$\boxed{h''(1) = -2e^{-1}}$$

this line is okay.

(4) $C(x) = x\sin(x)$

$$C(\pi) = \pi\sin(\pi)$$
$$= \pi\cdot 0$$
$$= 0$$

$$C'(x) = 1\cdot\sin(x) + x\cos(x)$$
$$= \sin(x) + x\cos(x)$$

$$C'(\pi) = \sin(\pi) + \pi\cos(\pi)$$
$$= 0 + \pi\cdot(-1)$$
$$= -\pi$$

equation of tangent line:

$$y = C'(\pi)(x - \pi) + C(\pi)$$

$$\boxed{y = -\pi(x - \pi) + 0}$$

(5) $f(x) = x^2+1$ $\qquad$ $f(a) = a^2+1$

$f'(x) = 2x \qquad f'(a) = 2a$

tangent line to $f(x)$ at $x=a$ is

$$y = f'(a)(x-a) + f(a).$$

so

$$y = 2a(x-a) + a^2+1$$

$$y = 2ax - 2a^2 + a^2 + 1$$

$$y = 2ax - a^2 + 1$$

given information: line passes through the point $(-1,-2)$.

plug into line equation:

$$-2 = 2a(-1) - a^2 + 1$$

$$0 = -2a - a^2 + 1 + 2$$

$$a^2 + 2a - 3 = 0$$

$$(a+3)(a-1) = 0$$

solutions. $a=-3,\ a=1$

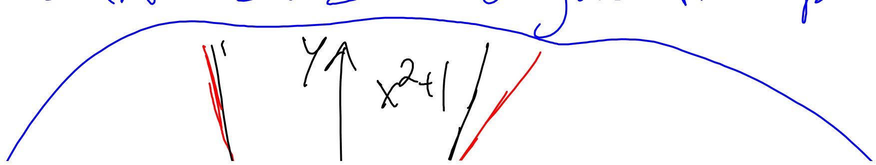
so there are two such lines

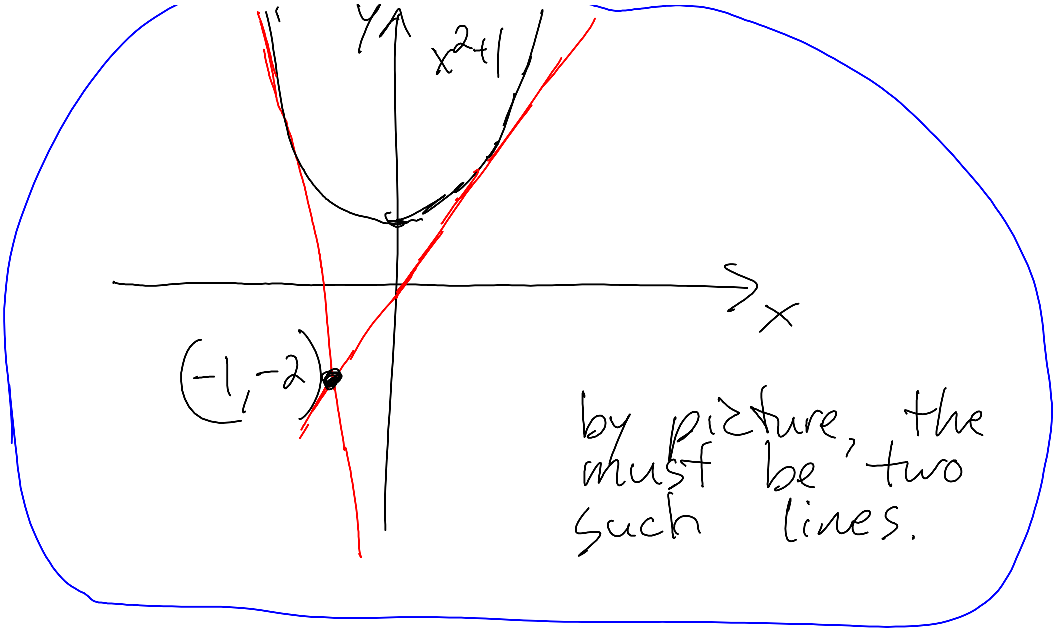
(their equations are

$$y = 2(-3)x - (-3)^2 + 1$$

and $y = 2\cdot1\cdot x - 1^2 + 1$)

ALTERNATIVE: argue from picture.





(6) $f(x) = 3x^2 - \pi x + e^{2x} - \cos(x)$

$f'(x) = 6x - \pi + 2e^{2x} + \sin(x)$

$f''(x) = 6 - 0 + 4e^{2x} + \cos(x)$

$f'''(x) = 0 - 0 + 8e^{2x} - \sin(x)$

$$\boxed{f'''(x) = 8e^{2x} - \sin(x)}$$

(7) $f(x) = \sqrt{x-5}$

$f(x+h) = \sqrt{x+h-5}$

limit definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \quad \frac{0}{0}$$

$h \to 0 \qquad h$ form 0

$$= \lim_{h \to 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \left(\frac{\sqrt{x+h-5} + \sqrt{x-5}}{\sqrt{x+h-5} + \sqrt{x-5}} \right)$$

conjugate

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h-5}\right)^2 - \left(\sqrt{x-5}\right)^2}{h\left(\sqrt{x+h-5} + \sqrt{x-5}\right)}$$

$$= \lim_{h \to 0} \frac{(x+h-5) - (x-5)}{h\left(\sqrt{x+h-5} + \sqrt{x-5}\right)}$$

add terms

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h-5} + \sqrt{x-5}\right)}$$

cancel h

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h-5} + \sqrt{x-5}}$$

can now evaluate the limit.

$$= \frac{1}{\sqrt{x+0-5} + \sqrt{x-5}}$$

$$= \frac{1}{2\sqrt{x-5}}$$