

Group Names and Student Numbers (minimum of two names required for participation to be recorded):

1. _____ 2. Solutions 3. _____

Wacky Incorporated has an unusual widget that satisfies the demand relationship

$$q = 100p^2e^{-p},$$

where q is the number of widgets demanded (in thousands) when the price is p dollars per unit. Recall that the price elasticity of demand is given by $E(p) = \frac{p}{q} \frac{dq}{dp}$.

(a) Find the price elasticity of demand $E(p)$ explicitly as a function of p .

$$\begin{aligned} \frac{dq}{dp} &= 100(2pe^{-p} - p^2e^{-p}) \\ &= 100pe^{-p}(2-p). \end{aligned}$$

$$\begin{aligned} E(p) &= \frac{p}{q} \frac{dq}{dp} \\ &= \frac{p}{100p^2e^{-p}} \cdot 100pe^{-p}(2-p) \end{aligned}$$

$$E(p) = 2 - p$$

*** Note: this model is sketchy for $p \leq 2$ (gives $E(p) > 0$; linear case always has $E(p) < 0$).

$$R'(p) = q(1 + \epsilon)$$

(b) For which values of price p will decreasing the price slightly result in an increase in revenue for the company?

$$E(p) = 2 - p$$

$p \downarrow$ $R \uparrow$ when price is elastic.

Unit elasticity $|E| = 1$, ^{need} $E(p) = -1$

$$2 - p = -1$$

$$p = 3$$

elastic if $E < -1$ ($|E| > 1$).

$$p < 3: 2 - p > -1$$

(test $p = 2$)

$$p > 3: 2 - p < -1$$

(test $p = 4$)

elastic when $p > 3$

Answer:

$$p > 3$$

(c) If the price is currently at \$10 and is dropping at a rate of 5% per year, at what percent rate per year is demand changing; in other words, what is the relative rate of change of demand under these conditions?

$$\% \text{ change in price} = 5\%$$

$$E(p) = \frac{\% \text{ change in quantity}}{\% \text{ change in demand}}$$

$$E(10) = \frac{\% \text{ change in quantity}}{-0.05}$$

$$E(p) = 2 - p$$

$$E(10) = -8$$

so

$$\% \text{ change in quantity} = -0.05 \cdot E(10)$$

$$= -(0.05)(-8)$$

$$= +0.4$$

*** Time is not really passing here; we consider just this one moment with "these conditions."

Answer:

demand growing at 40% per year.