Exponential Growth and Decay *Math 104 Section 106 - Oct 2012*

Have you seen or heard the phrase "growing exponentially" in a news story?

What does it mean when something is "growing exponentially"? The quantity is . . .
 A. Growing very rapidly.
 B. Growing, maybe quickly, maybe slowly.
 C. Doubling every time period for a certain time period (every year, every day, etc.)
 D. Growing by a fixed % every time period for a certain time period (every year, every day, etc.).
 E. Doing something else not described by A-D.

Explain . . .

Technically, to say that a function models **exponential growth**, it must have the form:

(and it models **exponential decay** if   ). We often write the formula as:

Are there any other functions that represent exponential growth?

Pond Scum
If an initial population of surface algae in a pond at midnight (12:00am) is doubling every 10 minutes such that the pond is completely covered at noon, at what time of day was the pond half-covered?
   A. 12:10am
   B. 4:00am
   C. 6:00am
   D. 11:00am
   E. 11:50am

Pay Raise
Suppose you are negotiating a contract for a new job with a starting salary of $50,000. Which of the following raise schedules would you prefer?
   A. Raise of $1000 per year.
   B. Raise of 2% per year.
   C. It doesn't matter; these are the same.
   D. Need more information.
   E. Confused.

How is this related to exponential growth?

Radioactive decay
A radioactive material has a half-life of 1200 years. Suppose we call the amount of radioactive material left at time t to be A(t).
    a) How might we express the half-life in terms of A and t?
    b) What percentage of the original radioactivity of a sample is left after 10 years?
    c) How many years are required to reduce the radioactivity by 10%?

If you are ahead, here is a sad story about rabbits (you will need a separate page for your work):
Suppose that 1,000 rabbits are introduced onto an island where they have no natural predators. During the next five years the rabbit population grows exponentially. After the first two years the population grew to 3,500 rabbits. After the first five years a rabbit virus is sprayed on the island and after that the rabbit population decays exponentially. Two years after the virus was introduced the rabbit population dropped to 3,000 rabbits. How many rabbits will there be on the island 10 years after they were introduced?

Compound Interest
If you start with principal amount P, with a nominal interest rate r compounded n times per period, the formula for computing the amount at time t is:

For example, interest rates are often quoted at nominal rate meaning annual, though the compounding periods may be annual, monthly, daily, semi-annually, etc.

Loan Shark
Suppose a loan shark offers to loan you $1000 at an annual rate of 100% interest that you will pay back at the end of one year (so if it were not compounded, you would owe a total of $2000). You have the option to either pay $3000 at the end of the year OR let the loan shark choose the compounding rate to apply for the year. Which should you choose?

    A. Pay $3000.
    B. Let the loan shark choose the compounding rate.
    C. The results are about the same.
    D. There is not enough information to solve the problem.
    E. Confused.