

Solutions to Extra Business Problems

5/28/11

1. (a). Taking the derivative of $C(q)$, we find that $MC(q) = 12q + 14$. Thus $MC(5) = 74$ - the marginal cost at a production level of 5 is 74 thousand dollars/unit.

(b). To estimate the cost, we use the formula Increase in C = Marginal Cost at q · Increase in Production. Note that we increase from 5 to 5.25, so we care about the marginal cost at $q = 5$, and that the increase is $5.25 - 5 = 0.25$ - we move up 0.25 units from 5 to 5.25. So we get the increase is $MC(5) \cdot 0.25 = 4.25$ thousands of dollars.

(c). The break even point occurs when $R = C$. Substituting in the equation formulas for R and C given in the problem, we obtain

$$6q^2 + 14q + 18 = -q^2 + 37q + 38$$

To solve for q , we collect terms all on one side and solve the resulting quadratic via the quadratic equation. It yields $q = -\frac{5}{7}, 4$. We disregard the negative solution (we will never produce a negative quantity), so the point where we break even is at a production level of 4.

(d). This reasoning here is crucial. At a break even point, $R = C$. But if $MR > MC$, then R is growing faster, and so if we move to the right, R will have moved up more than C - so we will increase profit by moving to the right, i.e., by increasing production. If $MC > MR$, then the rate of change of cost, C , is greater than that of revenue, so moving to the right will cause cost to move up higher than revenue,

while backwards will lower revenue less than it lowers cost, in which case we should *decrease* production. See this image for an illustration - the two break even points occur at 1 and 2; at point 2, tangent lines are drawn in black, and we can see that the red curve has a tangent line with greater slope than the blue curve - red's marginal is greater. Consequently, red is higher than blue to the right of the break even point, and less than blue to the left of the break even point.

With that explanation in hand, we find that $MC(4) = 12 \cdot 4 + 14 = 62$, while $MR(4) = -2 \cdot 4 + 37 = 35$. So $MC(4) > MR(4)$. In the picture we linked to, this corresponds to the point 2 (a break even point) and since $MC(4) > MR(4)$, we conclude that the red curve represents cost and the blue curve represents revenue. We should therefore move to the left (decreasing q , or production) in order for the revenue to be bigger than the cost.

2. (a). We assume that the demand relationship between p and q is linear - a line. The phrase "every \$1 increase in price ... the number of surge protectors ... decreases by 15" tells us that a move of one unit to the right results in a decrease of 15, or in other words that the slope is -15. Another way to see this is to note that the points $(50, 3000)$ and $(50 + 1, 3000 - 15)$ lie on the line, and use those points to compute the slope. Using the point slope form

$$y - y_1 = m(x - x_1)$$

and remembering that x stands for the independent variable, which we are treating as p , we have $q - 3000 = -15(p - 50)$, or $q = -15p + 3750$. So in the phrasing of this problem, $D(p) = -15p + 3750$.

(b) We are told that "fixed production costs" are \$50,000 and that "variable production costs" are \$30. This is jargon, and means that in order to produce anything at all, \$50,000 has to be spent and must still be spent regardless of how many things are produced, and that each unit produced costs an additional \$30. Adding this up, you get that to produce q units you must spend \$50,000 + $30q$, or $C(q) = 50000 + 30q$. To express this as a function of p , we replace q by $D(p)$ from part (a). We have an answer of $50,000 + 30(-15p + 3750)$.

(c). This is straight forward. Our general rule is that $R = pq$. The question wants us to give an answer both with only p and with only q on the right side - so we refer to our demand curve. It tells us $q = -15p + 3750$, or solving for p , that $p = -\frac{1}{15}q + 250$. Substituting either one of these into $R = pq$ gives either of the two responses requested in part (c) : $R = p(-15p + 3750)$ and $R = (-\frac{1}{15}q + 250)q$.

(d). Recall that “marginal” is just an econ. term for the derivative. So Marginal Profit MP is just P' . Profit of course is Revenue - Cost, or in our case, $P(p) = p(-15p + 3750) - [50,000 + 30(-15p + 3750)]$. Using the product and sum rules, we find $MP(p) = (-15p + 3750) + p(-15) - [30(-15)]$.

(e). The break even points are the points where $R = C$, or equivalently, where the profit is 0. So we wish to find p and q where $P = 0$. Since we know q if we know p (q is a function of p) we need only solve $P(p) = 0$ for p and then use our demand curve to find the q which corresponds to this p . So, we set $P(p) = p(-15p + 3750) - [50,000 + 30(-15p + 3750)] = 0$ and find p - blindly applying the quadratic equation requires the least thought, once we expand this expression out . It yields $p = \frac{10}{3}(42 \pm \sqrt{789})$ (so in fact, we couldn't have factored to find the roots had we tried).

Recalling that this question also asked for the quantity at a break even point, and from part (a) that $q = -15p + 3750$, we have that the break even points in terms of q are $q = -15[\frac{10}{3}(42 \pm \sqrt{789})] + 3750$.

(f). This is just like part (d) of question 1 - we compute the “marginals” of Cost and Revenue and see which one is larger (which is certainly less work than graphing these functions). We have $MC(q) = 30$, while $MR(q) = (-\frac{1}{15})q + (-\frac{1}{15}q + 250)$. At $q = -15[\frac{10}{3}(42 - \sqrt{789})] + 3750$, compute that MR is about -150 , so $MC > MR$ at the higher break even point - moving to the right will only make cost get bigger than revenue, so we should move to the left and decrease production. Decreasing production corresponds to increasing price. So price should be increased.

HOWEVER, there is ambiguity in the term “higher” break even point

- does it mean the one corresponding to a higher price, or to a higher quantity? We have assumed that it refers to a higher quantity. Had you interpreted the question differently but arrived at the appropriate answer, you would receive full marks.

3. (a). This is easy. Set $R(q) = C(q)$ to get $800q - q^2 = 300q + 40000$, put it into a standard form, $q^2 - 500q + 40000 = 0$, and either factor or use the quadratic equation to find that $q = 100, 400$.

(b). You should be able to graph the profit function, which incidently is $P = R - C = -(q^2 - 500q + 40000)$. It is positive in the region between the two break-even points, i.e., for $100 < q < 400$.

(c). For this problem, it is easiest to consider marginal profit - the derivative of the profit function. If the profit P has a positive derivative at 300, then it is moving upwards - and increasing production q will increase profit. Conversely, if $MP(q)$ is negative for $q = 300$, then the profit P is moving downwards, and we do not want to increase production.

We compute $MP(q) = -(2q - 500)$, and so $MP(300) = -(600 - 500) = -100$. So we should not increase production beyond 300.

(d). We want to know the relationship between p and q . To do that you have to be a little clever - we are given revenue, $R(q) = 800q - q^2$. Since it is always true that $R(q) = pq$, we can set $800q - q^2 = pq$, and cancel the q 's so that $800 - q = p$. This is our demand function.

4. (a). The average cost represents the amount it costs, on average, to produce each unit - that is, $AC(q) := \frac{C(q)}{q}$, the total cost of producing q units divided by the number of units, q . When $q = 1$, $AC(q) = C(q)$. If $AC(q) = \alpha q^\beta$ as the problem suggests, then not only does $AC(1) = C(1)$, but $AC(1) = \alpha$. So α is the cost to produce 1 unit.

Learning by doing means that as q goes up, the average cost $AC(q)$ goes down. So αq^β should be getting smaller as q gets larger. If β is positive, then q^β just gets bigger - the square-root of a million is still bigger than the square-root of 10. But if β is negative, then $q^\beta = \frac{1}{q^{-\beta}}$,

where now $-\beta$ is positive, and so as q gets larger, $q^{-\beta}$ gets larger, so $q^\beta = \frac{1}{q^{-\beta}}$ gets smaller. Another way to see this is to take the derivative of $AC(q)$, obtaining $MAC(q) = \beta\alpha q^{\beta-1}$; if $AC(q)$ is getting smaller, it should have a negative derivative, so β should be negative.

If there is learning by doing, the average cost will go to 0 as q gets large - you can see this by drawing a picture, letting $\beta = -1$ and $\alpha = 1$.

5. What we have here is an example of a piecewise-defined function. If May works 8 or fewer hours, the labor costs \$10 for every hour she works - and incidentally, each hour corresponds to the production of 10 floral arrangements, so that the cost is \$1 per floral arrangement, or $C(q) = 1 \cdot q$. But once she works more than 8 hours (i.e., produces more than 80 floral arrangements), the cost of labor is the cost for the first 8 hours, which is \$80, plus the cost of the additional time, which now is \$15 per hour, or \$15 per every 10 arrangements produced - that is \$1.5 per floral arrangement. So the function becomes $80 + 1.5q$, for $q > 8$. So we write $C(q) = \begin{cases} q & \text{if } 0 \leq q \leq 80 \\ 80 + 1.5q & \text{if } q > 80 \end{cases}$.

(b). The average labor cost per q arrangements is the labor cost for q arrangements divided by q , or $AC(q) = \frac{C(q)}{q}$. Using part (a), this is $AC(q) = \begin{cases} q & \text{if } 0 \leq q \leq 80 \\ \frac{80+1.5q}{q} & \text{if } q > 80 \end{cases}$.

(c). To find marginal labor cost, we take the derivative. We get $MC(q) = \begin{cases} 1 & \text{if } 0 \leq q < 80 \\ 1.5 & \text{if } q > 80 \end{cases}$.

Note that the marginal labor cost does not really make sense at $q = 80$, since the function is not differentiable there. However, since in dealing with marginals the prime question is what happens when we increase the function slightly, we could take the derivative of the expression for $C(q)$ when q is just to the right of 80 - just be clear to state that that is what you are doing.

6. (a). This is just like 2(a). By the same procedure, we find that if we put p on the horizontal axis, then the slope is -1000 and the equation is $q = -1000p + 7000$, or $p = \frac{-1}{1000}q + 7$.

(b). We have $R(p) = p \cdot q = p \cdot (-1000p + 7000)$

(c). We look for the vertex of the parabola $R(p) = p \cdot (-1000p + 7000)$. At the vertex, the derivative must be 0 - we differentiate $p \cdot (-1000p + 7000)$ and set it equal to 0, then solve for p , that is $(-1000p + 7000) + p(-1000) = 0$, or $p = 3.5$.

Since $q = (-1000p + 7000)$, we have that the quantity should be $-1000 \cdot 3.5 + 7000 = 3500$ to maximize revenue (though why you would want to maximize revenue as opposed to profit is unclear ...)

(d). Recall that $R(p) = p \cdot (-1000p + 7000)$ The maximum monthly revenue is $R(3.5) = 3.5(3500)$, or \$12250.