Intermediate Value Theorem

Solutions

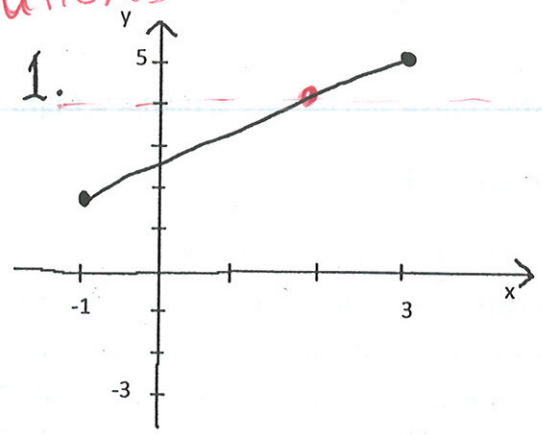
Thursday, September 20, 2012

Here are the graphs of five functions defined on the interval [-1,3].
For each case, decide:

- Does the function take the value of 4 on the interval [-1,3]?
- Does the function have a root (i.e. take the value zero) on the interval [-1,3]?
- For each of these values, does the Intermediate Value Theorem (IVT) apply?

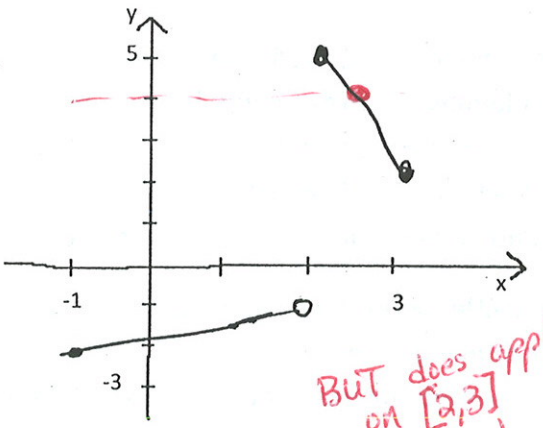
IVT: If a function is defined and continuous on the interval [a,b], then it must take all intermediate values between f(a) and f(b) at least once; in other words, for any intermediate value L between f(a) and f(b), there must be at least one input value c such that f(c) = L.

1.



Does f(c)=4 for any c in [-1,3]?	Y	Does IVT apply?	Y
Does f have a root on [-1,3]?	N	Does IVT apply?	N ← 0 not an intermediate val.

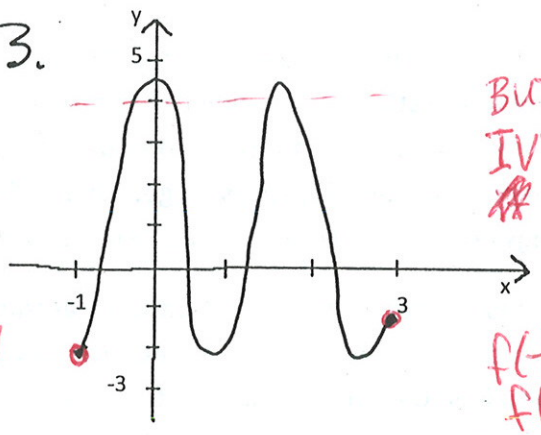
2.



BUT does apply on [2,3]

Does f(c)=4 for any c in [-1,3]?	Y	Does IVT apply?	N
Does f have a root on [-1,3]?	N	Does IVT apply?	N

3.

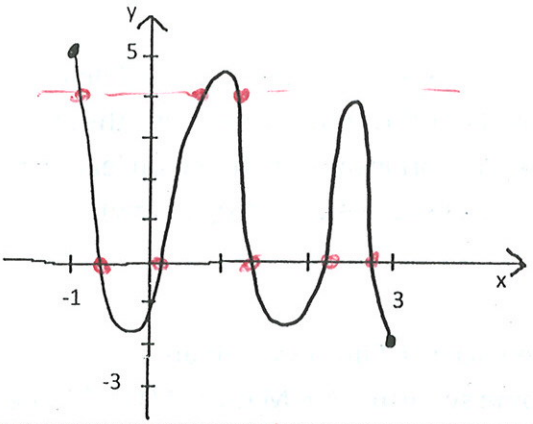


BUT, could apply IVT on [-1,0] using f(0)=4.5

f(-1) ≈ -2.2
f(3) ≈ -1.2

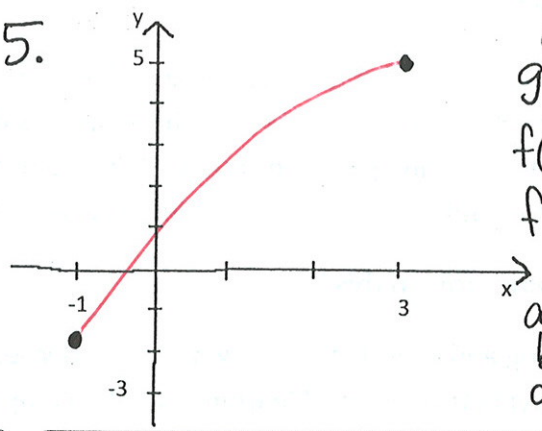
Does f(c)=4 for any c in [-1,3]?	Y	Does IVT apply?	N ← not actually intermediate
Does f have a root on [-1,3]?	Y	Does IVT apply?	N

4.



Does f(c)=4 for any c in [-1,3]?	Y	Does IVT apply?	Y
Does f have a root on [-1,3]?	Y	Does IVT apply?	Y

5.



draw graph with
f(-1) = -2
f(3) = 5
and answers below are true

Does f(c)=4 for any c in [-1,3]?	Y	Does IVT apply?	Y
Does f have a root on [-1,3]?	Y	Does IVT apply?	Y

What does the IVT tell you about function values on the interval that are not intermediate values?	NOTHING
What does the IVT tell you about functions that are not continuous on the interval?	NOTHING
What does the IVT tell you about where the c occurs in [a,b]?	NOTHING
What does the IVT tell you about how many such c values there might be?	At least one, no info about how many more