A discussion about the Marginal Profit

What is the marginal profit exactly?

Estimate of the error of linear approximation

So far, we have studied how using some information around a point a we can construct a linear approximation to approximate nearby values of the function.

An interesting question is now to estimate how good a linear approximation is. There are several ways to do that. An effective estimate is possible if we are given some information about the second derivative. Consider an interval I around the point a. Then for any point x in that interval, the error (in absolute value) made when approximating $f[x]$ using $L[x]$ can be estimated as follow:

$$|error| \leq \frac{M}{2}(x-a)^2$$

Where M is a positive number giving us information about the second derivative of the function f. More precisely, M gives us an idea of how big the second derivative might be on the interval I that we are considering. That is, it tells us that the absolute value of the second derivative of any point in the interval is at most M. Mathematicians write:

$$|f''[x]| \leq M \qquad \text{for all values of } x \text{ in the interval } I$$

More than one value of M can be used, but clearly, the smaller it is, the smaller we can guarantee the error to be. Let us see this in action in a more familiar setting.

Error in approximating sine values

Let us see what we can say about the error made when using a linear approximation to approximate the value of $sin[0.2]$

Indeed, we can do this by using information at the point 0, so a=0 and x=0.2 and we obtain that

$$L_0[x] = sin[0] + cos[0]\cdot(x-0) = 0 + 1\cdot(x-0) = x$$

Hence we can say that

$$sin[0.2] \approx L_0[0.2] = 0.2$$

How big is our error?

First idea

Second idea

Conclusion