Profit function A company is interested in estimating its profit for various demand levels. For a demand of 1000 units, the company is making a profit of $4000 and has a marginal profit of -0.5 $/unit.	**General case** Given a differentiable function, we would like to estimate it at several points. For a given point *a* on the *x*-axis, the values of _____ and _____ are known.
If one makes the assumption that the marginal profit remains constant, then the profit function would be linear. The equation of this line is: for a demand *q* and a profit *P*. We call this the linearization of the profit for a demand of 1000 units.	If one makes the assumption that _____ _____ then the function would be linear. The equation of this line is:
For demand levels that are reasonably close to 1000 units, we can use the the above linearization to approximate the profit. For example, if the profit increases by 50 units, then the profit will approximately be: And for a decrease of 30 units, the profit will approximately be:	For _____ , we can use the line as an approximation for the _____. We write: The function _____ is called _____ _____ _____.

A picture is worth a thousand words...

Some practice

One common way to practice the art of linear approximations is to compute with explicit functions. On one hand it sounds silly since we actually know the function and could compute any value we would like, on the other it allows us to check what is going on and get some insight into the process.

A typical exam question

Using a linear approximation, estimate the value of $\sqrt{9.12}$

Note: during exams, you are not allowed calculators, so try to solve this problem without a calculator (you really won't need it).

Clicker question: What can we say about this estimated value?

Underestimates and overestimates

Estimating the error of linear approximations

Given the previous discussion, it should not come as too big a surprise that some information about the second derivative of the function will help us get an idea on the potential error our linear approximation is making.

A result

Given a differentiable function f for which we are approximating values around the point a on the x-axis using a linear approximation.

If the absolute value of the second derivative is at most a value M on some interval I around the point a, then the error made by using the linear approximation at a point x (that has to be in the interval I) is no more than

An illustration of this result