

Solutions for “More Business Problems”

Math 184 - 14 May 2011

1. Initial information: when $p=4$, $q = 10000$, but when $p=4.4$, $q=8000$.

a. Since we want the *price* where the revenue is maximized, and

$$R = pq$$

it would be helpful to look at revenue, R , as a function of price, p . To do so, we want q in terms of p in the demand equation.

To determine the demand equation, we note that if it is linear, then it is determined by the two points of information we have been given. The slope, rise over run, would be

$$\frac{\text{change in } q}{\text{change in } p} = \frac{8000 - 10000}{4.4 - 4} = \frac{-2000}{0.4} = \frac{-20000}{4} = -5000$$

We can now write the demand equation as

$$q - q_0 = -5000(p - p_0)$$

(the point-slope formula where we know the slope is -5000). We can plug in either point of information we know, though $p=4$, $q=10000$ seems like it will be easier to work with. This gives $q - 10000 = -5000(p - 4)$, so $q = -5000p + 30000$. Back to the revenue, $R(p) = p(-5000p + 30000)$ and so the revenue is a downward-pointing parabola (the p^2 coefficient is -5000) and the maximum will occur halfway between the roots. The form $R(p) = p(-5000p + 30000)$ is already factored, and $p=0$ is one root. The other is where $(-5000p + 30000) = 0$, which is at $p=6$. So the roots are $p=0$ and $p=6$, and the halfway point is 3, so

The revenue is maximized when the price is \$3.

b. The cost function in terms of q is $C(q) = 1000 + 0.6q$ and to get it in terms of p (since we want the price for maximum profit), we have

$$C(p) = 1000 + 0.6(-5000p + 30000) = 1000 - 3000p + 18000 = 19000 - 3000p$$

and we can now calculate

$$P(p) = R(p) - C(p) = p(-5000p + 30000) - (19000 - 3000p)$$

$$P(p) = -5000p^2 + 30000p - 19000 + 3000p$$

$$P(p) = -5000p^2 + 33000p - 19000$$

This is a downward-pointing parabola, whose roots are the break-even points (where $R(p) = C(p)$ which means $P(p) = 0$), and the maximum will occur halfway between those roots. To calculate the roots, we use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, that means

$$p = \frac{-33000 \pm \sqrt{33000^2 - 4(-5000)(-19000)}}{2(-5000)}$$

but to simplify, we could divide everything by 1000 as a first step:

$P(p) = 1000(-5p^2 + 33p - 19)$ and then we have

$$p = \frac{-33 \pm \sqrt{33^2 - 4(-5)(-19)}}{2(-5)}$$

Without a calculator, we can remember that it is only the halfway point between the roots that we want. Since the roots are

$$p = \frac{-33 - \sqrt{33^2 - 4(-5)(-19)}}{2(-5)}, \quad \frac{-33 + \sqrt{33^2 - 4(-5)(-19)}}{2(-5)}$$

the halfway point is $\frac{-33}{2(-5)} = \frac{-33}{-10} = 3.3$ since the square root part is added and subtracted from this amount.

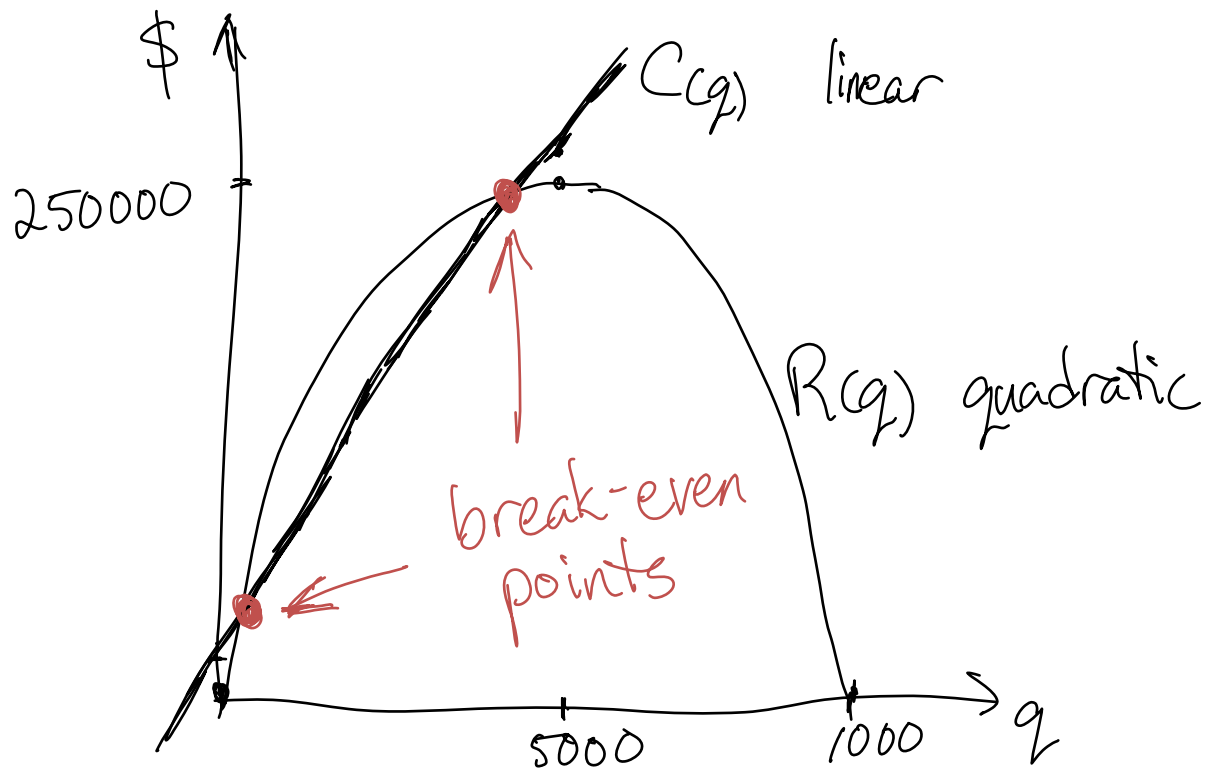
The maximum profit occurs at a price of \$3.30 per hamburger.

Note: As an alternative, you could solve both parts a and b by finding R, C and P as functions of q, find the q where the maximum occurs, and convert to p at the end using the demand equation. This gives the same values.

- Here, we already have the revenue as a function of q, and may proceed directly to finding the maximum. The graph of $R(q) = q(400 - q)$ is a downward-pointing parabola with roots at $q=0$ and $q=400$. So the maximum revenue occurs when $q=200$, but the question asks for the "largest possible revenue", which means we have to calculate $R(200) = 200(400-200) = 200*200 = 40000$.

The largest possible revenue is \$40,000.

3. We are given the demand equation as $p = 100 - 0.01q$ and, since we want to work in terms of q according to the problem (it says to plot R and C "as a function of quantity") we may substitute in $R = pq$ to obtain $R(q) = (100 - 0.01q)q$. This is a downward-pointing parabola with one root at $q=0$ and the other where $100 - 0.01q = 0$, which is: $100 - 0.01q = 0$ so $100 = 0.01q$ and hence $10000 = q$. With roots at $q=0$ and $q=10000$, the revenue will be a maximum halfway between at $q=5000$ and thus have a maximum value of $R(5000) = (100 - 0.01 \cdot 5000)5000 = 250000$. Thus the parabola representing $R(q)$ will have roots at $q=0$ and $q=10000$ and a maximum at $R(5000) = 250000$, while the line for $C(q)$ has slope 50, y-intercept 10000, and thus at $q=5000$ is above the parabola; $C(5000) = 50 \cdot 5000 + 10000 = 250000 + 10000 = 260000$. The break-even points are where $R(q)=C(q)$.



4. We are already given the demand equation, and here we want both the q and the p where R is maximized. $R = pq$ and we already have p in terms of q , so we can substitute in for p to obtain $R(q) = (2 - 0.01q)q$ whose graph is a downward-pointing parabola with roots at $q=0$ and q where $2 - 0.01q = 0$, which is where $2 = 0.01q$ and hence $200 = q$ (multiplied both sides by 100). So halfway between the roots of $q=0$ and $q=200$, we have the maximum revenue occurring at $q=100$. Then putting this back into the demand equation, we learn that when $q=100$, we have $p = 2 - 0.01(100) = 2 - 1 = 1$.
The maximum revenue occurs when $q=100$ which has a corresponding price of $p=1$.