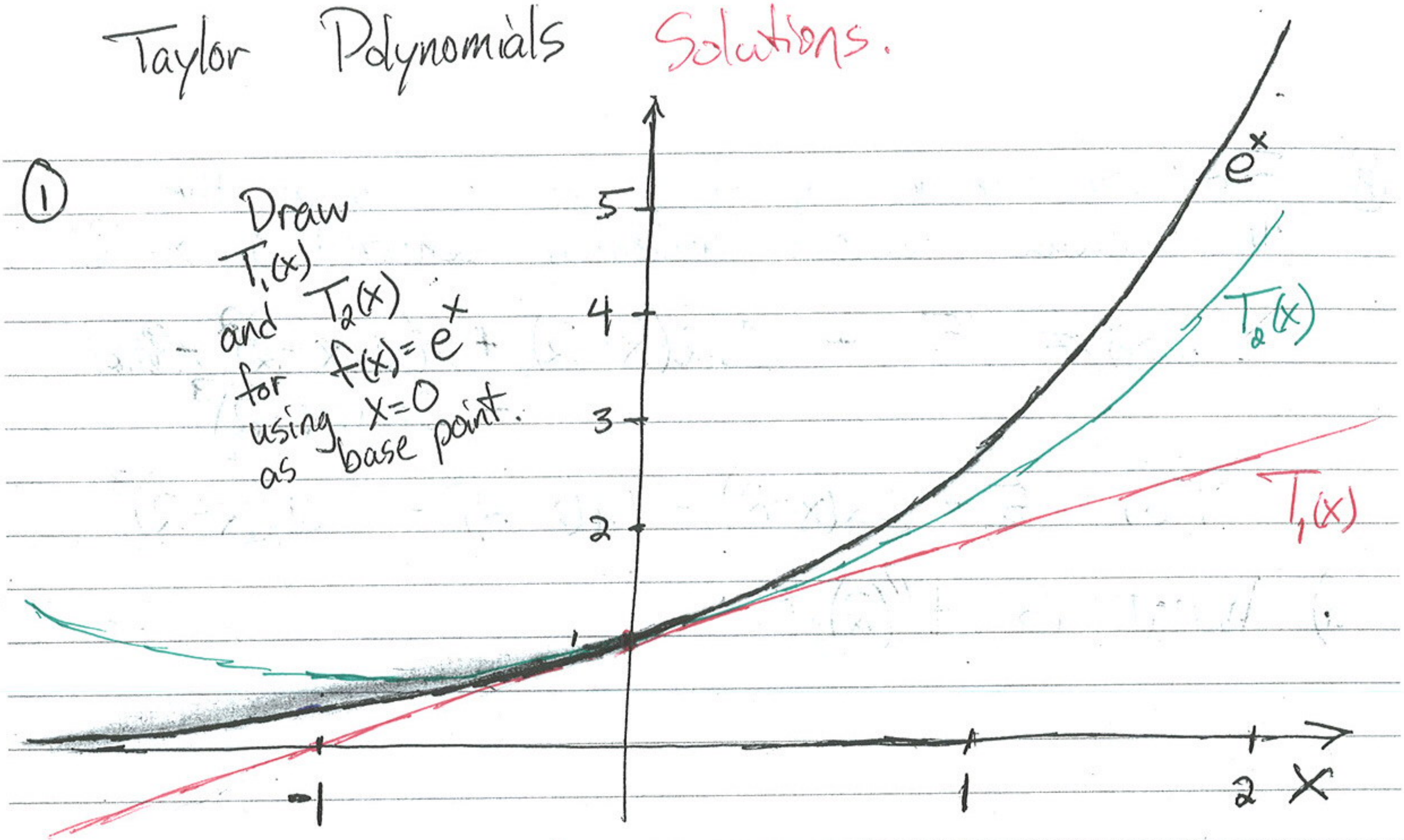
Taylor Polynomials Solutions.

① Draw $T_1(x)$ and $T_2(x)$ for $f(x) = e^x$ using $x=0$ as base point.



$$T_1(x) = L(x) = 1 + x$$

$$T_2(x) = 1 + x + \frac{x^2}{2}$$

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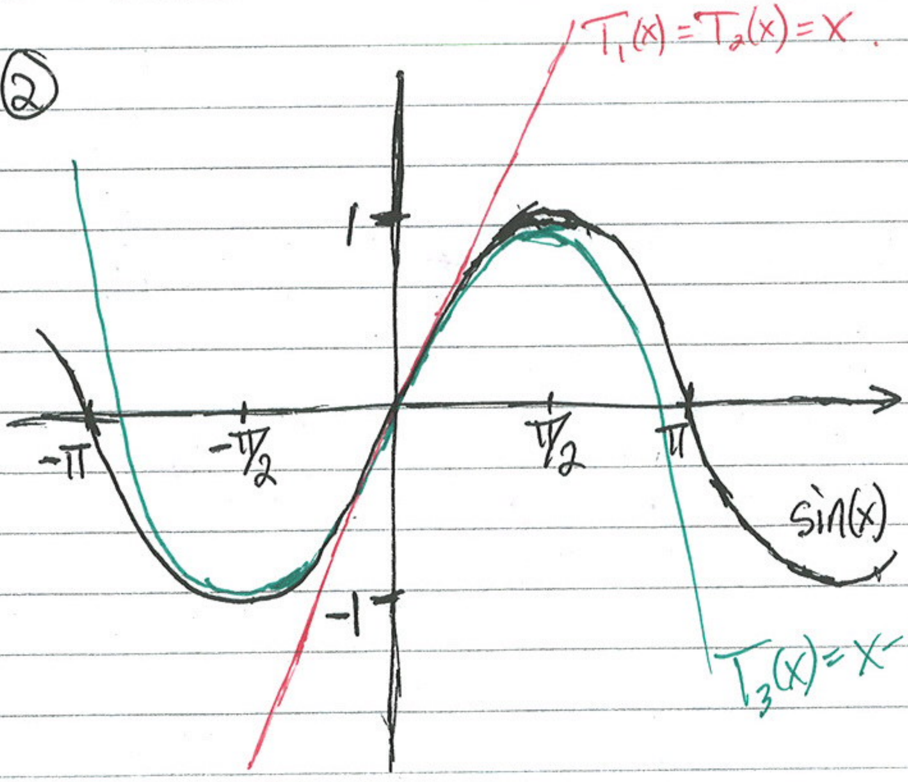
②



$$T_1(x) = x$$

$$T_2(x) = x$$ note! $f''(0) = 0$.

$$T_3(x) = x - \frac{x^3}{3!}$$

Draw these on the graph with sin(x), using $x=0$ as a base point.

③ For some model $y=f(x)$, we are given the third order Taylor polynomial centred at $x=2$:

$$\cancel{T_3(x) = 5 + 0.2(x-2) + 0.1(x-2)^2 + 0.01} \\ \cancel{- 0.01(x-2)^3}$$

on one line:

$$T_3(x) = 5 + 0.2(x-2) + 0.1(x-2)^2 - 0.01(x-2)^3$$

a) What is $f''(2)$?

$$0.1 = \frac{f''(2)}{2!}, \quad \text{so} \quad \boxed{f''(2) = 0.2}$$

b) What is $T_1(x)$ for this same function, also centred at $x=2$?

$$T_1(x) = 5 + 0.2(x-2)$$

Notice:

$$T_3(x) = T_1(x) + 0.1(x-2)^2 - 0.01(x-2)^3$$

from formula:

$$T_3(x) = \underbrace{\underbrace{f(a) + f'(a)(x-a)}_{T_1(x)} + \frac{f''(a)}{2}(x-a)^2}_{T_2(x)} + \frac{f'''(a)}{3!}(x-a)^3$$