Math 104 - Fall 2012

Set up the problem: draw a diagram if appropriate, name the variables you are using and use them to express the objective function and all relevant constraints. The idea today is to practice translating word problems into these statements, so stop before you take the derivative (unless you have time at the end).

1. Find the dimensions of the rectangle of maximum area that can be inscribed in a right triangle with sides of length 3 and 4 and hypothenuse of length 5, if two sides of the rectangle lie along the two sides of the triangle. Make sure you justify that your answer is a maximum. In solving this problem, follow these steps:
   - Draw a sketch and assign names to all relevant variables.
   - State what quantities are given.
   - Express in your own words what the problem is asking.
   - Identify the quantity that needs to be optimized.
   - Identify all the constraints given in the problem.
   - Express the quantity that needs to be optimized as a mathematical function. In other words, write down the "objective function".
   - Express in mathematical form all the constraints given in the problem.
   - Simplify the "objective function" as a function of only one variable.
   - Apply appropriate calculus techniques to find maxima/minima of the "objective function".
   - Discuss your results.
2. You are in a dune buggy at point P in the desert, 12 km due south of the nearest point A on a straight east-west road. You want to get to a town B on the road 18 km east of A. If your dune buggy can travel at a speed of 15 km/h through the desert, and 39 km/h along the road, toward what point on the road should you head to minimize your travel time from P to B? Apply the problem-solving strategy described above.
3. A piece of wire 10 cm long is cut into two peices. One piece is bent into a circle while the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is minimal? Maximal?
4. A right circular cone is turned upside-down (vertex is at the bottom); the height of cone is 10 cm and the diameter of the top is also 10 cm. The cone is filled half-way (by depth), and water is draining out. The depth of the water is dropping at a rate of 4 cm per second. How fast is the volume of the water changing at this time? You may use the fact that the volume of a cone with radius r and height h is $\pi r^2 h/3$.
5. A shoebox, complete with overlapping lid, has a base with length 5/3 times its width. The rim of the lid overlaps the side of the main box by 1/5 of the height of the box. Assuming the thickness of the box is negligible and of uniform density, and the total volume of the box is 3000 cubic centimetres, what dimensions will result in the minimum weight of the empty box? Make sure you follow the steps outlined in the problem-solving strategy above. Draw a picture of the box and remember that if your computations lead to large numbers you can always express such numbers as a product of their factors: this will help you to carry out algebraic calculations by hand.
6. The price p in \$ for a product and the quantity q of demand (that is, the quantity of units sold) for a product are related by the equation $p^2 + 2q^2 = 1100$. If the price is increasing at a rate of \$2 per month when the price is \$30, find the rate of change of the revenue R in dollars per month.