A Business Problem
MATH 104 and MATH 184
Week 1: 13 September to 17 September 2010

Opple Inc. is the only manufacturer of the popular oPad. Opple estimates that when the price of the oPad is $200, then the weekly demand for it is 5000 units. For every $1 increase in the price, the weekly demand decreases by 50 units. Assume that the fixed costs of production on a weekly basis are $100 000, and the variable costs of production are $75 per unit.

(a) Find the linear demand equation for the oPad. Use the notation p for the unit price and q for the weekly demand.

(b) Find the weekly cost function, $C = C(q)$, for producing q oPads per week. Note that $C(q)$ is a linear function.

(c) Find the weekly revenue function, $R = R(q)$. Note that $R(q)$ is a quadratic function.

(d) The *break-even* points are where Cost equals Revenue; that is, where $C(q) = R(q)$. Find the break-even points for the oPad.

(e) On the same set of axes, sketch graphs of $C = C(q)$ and $R = R(q)$ and use these graphs to help you explain why there are two break-even points.

(f) *Profit* is defined as Revenue minus Cost: $P(q) = R(q) - C(q)$. Find the profit function $P(q)$. Note that it is a quadratic function.

(g) Graph $P = P(q)$ on the same axes as you sketched the graphs of $C(q)$ and $R(q)$. On this graph, indicate the regions of profit ($P(q) > 0$) and loss ($P(q) < 0$).

(h) How should Opple Inc. operate in order to maximize the weekly profit $P = P(q)$? Use mathematics in your explanation.