

MATH 210 – Introduction to Mathematical Computing
Learning Objectives - Draft 4

Course pre-requisites and co-requisites:

As a pre-requisite to this course students are required to have a reasonable mastery of differential and integral calculus. This includes being able to

- extensively understand algebraic and transcendental functions;
- understand and evaluate limits, derivatives and integrals of single-variable functions;
- understand series and determine their convergence using convergence tests;
- understand Taylor series and determine their interval of convergence.

Students are required to take the following courses in advance of or concurrently with this course. Topics relevant to this course are listed below.

- A course on differential equations (Math 215, 255, 256, 265 or equivalent)
 - basic solution methods for initial-value and boundary-value problems
 - direction fields and solution curves
 - phase portraits of systems of ordinary differential equations
 - Euler methods
- A course on linear algebra (Math 152, 221, 223 or equivalent)
 - basic operations with vectors and matrices
 - systems of linear equations
- A course involving mathematical reasoning (Math 220, 226 or equivalent)
 - logics: boolean expressions, conditional statements and quantifiers

Course-level learning goals:

In this course students will learn the basics of mathematical computing primarily on Maple and MATLAB. Basic computer algebra, graphing and programming techniques as well as some numerical methods will be introduced. In particular, students will learn to

- manipulate and operate numbers, matrices and symbolic expressions;
- create graphs or animations;
- write Maple and MATLAB programs involving loops, conditional statements and functions;
- perform floating point operations and control round-off errors;
- solve linear systems or nonlinear equations;

- symbolically and numerically differentiate and integrate functions;
- find or approximate solutions of differential equations symbolically, graphically or numerically.

Topic-level learning goals:

Here are the major learning goals of the course. Not all of these goals are of equal importance.

Basic calculations with Maple:

1. Input integers, fractions, algebraic numbers, floating point numbers and complex numbers.
2. Symbolically evaluate algebraic and transcendental numbers (involving arithmetics, powers, roots and elementary functions) and their numerical values.
3. Specify the number of significant digits used in floating point calculations.
4. Input expressions involving symbolic variables.

Elementary commands:

1. Assign/unassign a number or an expression to a variable or an indexed variable. List all assigned variables.
2. Define and operate on a mathematical function or a piecewise defined function.
3. Expand, factor and simplify polynomials, rational functions and other expressions.
4. Manipulate polynomials (e.g. sorting and collecting terms, finding coefficients, etc) and rational functions (e.g. finding quotients and remainders, partial fraction decomposition)
5. Plot a graph or an animation of a function.
6. Solve an algebraic equation or a system of linear equations symbolically and evaluate the numerical values of the roots.

Differential and integral calculus:

1. Evaluate the limit of a function at a specific point or at infinity.
2. Find first- or higher-order (partial) derivatives of an expression or a function symbolically.
3. Differentiate an implicitly defined function.
4. Find the anti-derivative of an expression or a function symbolically.
5. Evaluate a definite or an improper integral symbolically and determine its numerical value.

Solutions of ODEs:

1. Set up an ODE or a system of ODEs with initial or boundary conditions.
2. Find the analytic solution of an ODE or a system of ODEs.
3. Plot and interpret the direction field and solution curves of an ODE.
4. Plot and interpret phase portraits of systems of ODEs.

Basic calculations and programming with MATLAB:

1. Input and operate with floating point numbers (real and complex). Display floating point numbers in different formats.
2. Input and operate with vectors and matrices. Evaluate sums, products, determinants and inverses of matrices. Evaluate scalar multiples, dot products and componentwise products of vectors.
3. Solve a system of linear equations.
4. Understand the MATLAB programming syntax. Construct a simple executable m-file.
5. Create and understand the logic behind a function, a loop and a conditional statement.

Round off errors

1. Understand the concept of round-off errors.
2. Understand the limitations of floating point representation of numbers, and the significance of round-off errors in mathematical operations. Be aware of operations that result in a loss of significant number of digits.

Numerical solutions of nonlinear equations

1. Understand the bisection method, the Newton method and the concept of a fixed-point iteration. Understand the convergence behaviour of the Newton method.
2. Write MATLAB programs for the bisection method or the Newton method.

Numerical integration

1. Understand the basic quadrature rules (trapezoidal, midpoint, Simpson's rule) and their discretization errors. Write MATLAB programs involving these basic quadrature rules and their associated composite rules.
2. Understand and generate Newton-Cotes formulas. Write MATLAB programs involving Newton-Cotes formulas.
3. Improve the accuracy of numerical integration by using Richardson extrapolation (Romberg's method).

Euler methods for ODEs

1. Use forward, backward and central difference quotients to approximate derivatives of functions. Understand the errors associated with different difference quotients.
2. Understand the forward Euler, backward Euler and Crank-Nicholson methods for first-order initial value problems and their errors. Write MATLAB programs involving these methods.
3. Understand the absolute stability of Euler and Crank-Nicholson methods.
4. Understand the forward and backward Taylor method for first-order initial value problems. Write MATLAB programs involving these methods.