Summary of Math 220 Basic Proof Diagnostic Skills Test
Validation Interviews

A total of 13 students were interviewed. 7 had completed MATH 101, 4 had completed MATH 103, and 1 had completed MATH 105 in the most recent term (Jan-Apr 2011). I do not know the course for the remaining student.

Each student completed the test one question at a time. After completing the question, they were asked to indicate if they found the question confusing, vague or ambiguous and also to explain how they came to the answer they chose.

Below are the comments for each question given by the students, as well as my conclusions about what action should be taken.

Question 1: Select the function that corresponds to the following graph
(a) 1   (d) 1
(b) 0   (e) 4
(c) 0   (f) 7

Overall, 7/13 correct

Clarity of Question:
No problems at all interpreting the question or possible answers.

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	(f) correct reasoning: acceptable	eliminated (a)-(d) because there is a horiz. Asymptote at y=0. Plugged in x=1 to pick between (e) and (f).
2	(e) incorrect	chose (e) because it is "the graph of 1/x shifted up by one". (the given graph is actually -1/x shifted up one unit)
3	(e) incorrect	Chose (e) because it is "1/x moved up one"
4	(f) correct reasoning: acceptable	Started by turning the page 90 degrees and saying the given graph was y=1/(x-1) and trying to solve for x to get the inverse. This didn't work, so then plugged in the point (1,0) and eliminated (a)-(e) and confirmed (f) with this.
5	(f) correct reasoning: acceptable	Eliminated (a)-(d) because these are undefined at x=1, whereas the graph shown is not. Then plugged in (1,0) to choose between (e) and (f).
6	(e) incorrect	Chose (e) because it is "1/x shifted up one"
7	(e) incorrect	Vertical asymptote at 0 means it must be (e) or (f). Then, since the graph is shifted up by one the constant should be positive (this is odd reasoning), therefore choose (f).
8	(d) incorrect	Eliminated (e) and (f) by putting in y=0 and claiming they give x=2 rather than x=1 when y=0. Eliminated (a) and (b) because they don't have an asymptote when x=0. Last, chose between (c) and (d) by

		looking at behaviour as x approaches 0 from the left.
9	(f) correct reasoning: acceptable	Eliminated (a) and (e) by plugging in (1,0). Eliminated (b) and (d) because thought the graph shown was like 1/x, not -1/x (also eliminated (f) temporarily this way). Then plugged in the point (1,0) into all and chose (f).
10	(f) correct reasoning: acceptable	Eliminated (a)-(e) by plugging in (1,0) and confirmed (f) this way also.
11	(f) correct reasoning: acceptable	Eliminated (a)-(e) by plugging in (1,0) and confirmed (f) this way also.
12	(f) correct reasoning: acceptable	Eliminated (a)-(e) by plugging in (-1,2) and confirmed (f) this way also.
13	(a) incorrect	Choose (a) because there is an asymptote at y=1. (incorrect reasoning). Did not consider other options.

Comments and Conclusions:
(1) Many students use the approach of plugging in the point (1,0) to arrive at the correct answer. We should change one of the distractors to also go through this point.
(2) In general, students arrive at wrong answers due to poor understanding of asymptotes, or misremembering what the graph of 1/x looks like (i.e. Confusing 1/x and -1/x).
(3) Overall a good question, and only needs the change mentioned in (1).

Question 2: Find the set of all values of k such that the given inequality is true.
(a) 1   (d) 0   (g) 1
(b) 0   (e) 0   (h) 1
(c) 9   (f) 0
spoiled: 1 (selected all of (a), (c), (e) and (f))

Overall, 9/13 correct

Clarity of Question:
Only one problem. One student interpreted the answers as (x,y) pairs rather than intervals and so chose several answers.

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	(c) correct reasoning: good	Solved algebraically to get k>6/13, then chose the correct interval.
2	(c) correct reasoning: poor	Eliminated all those with negative infinity because "we want greater than". Then eliminated (a) by plugging in k=0. Next, chose (c) because it is the "lowest possible one that works" (claimed to mentally plug in k=6/13).
3	(h) incorrect	Solved algebraically (incorrectly) to get -17k<6. Then chose (h) because it looked most similar to her solution.
4	(c) correct	Solved algebraically to get k>6/13, then chose the correct interval

		reasoning: good	
5	(c) correct reasoning: poor	Solved algebraically (incorrectly) to get k>4/9. Then chose (c) because "it is bigger than what I got"
6	(c) correct reasoning: good	Solved algebraically, initially with errors to get k>2/7, then tried to correct and eventually got k>6/13. Then, for some reason differentiated wrt k, realized it was not useful and chose the correct answer.
7	(c) correct reasoning: good	Solved algebraically to get k>6/13, then chose the correct interval.
8	(c) correct reasoning: good	Solved algebraically, initially with errors to get k>6/8, then tried to correct and got k>6/13 and chose the correct interval.
9	(a), (c), (e) and (g) incorrect	Interpreted the answer options as (k, y) pairs. Solved algebraically (mentioned "you could just plug in though"), drew the line y=13k-6 on a plot and shaded one side, then chose the (k,y) pairs in the unshaded region.
10	(g) incorrect	Plugged in k=0 (eliminated (a)), then k=1, then k=100. Concluded the right end point should be infinity so eliminated (b), (d), (f) and (h). Tried plugging in k=6/13, k=6/7 and k=8/17, generally had problems with calculations but chose (g).
11	(c) correct reasoning: good	Solved algebraically to get k>6/13, then chose the correct interval. Used the substitution A=k-1
12	(a) incorrect	Claims to have plugged in the endpoints (0 and infinity) and found that it works.
13	(c) correct reasoning: good	Solved algebraically to get k>6/13, then chose the correct interval.

Comments and Conclusions:
(1) Most students attempt to solve this algebraically (which is what we want). Many make errors, and either rework their algebra to get the correct solution, or just pick something that looks similar or is somehow "close" or "bigger" than their answer.
(2) Incorrect answers are often chosen because students have used a "plug in endpoints" approach. This is a poor strategy and it's good that this question identifies this.
(3) Some students 2/13 arrived at the correct response with very poor reasoning.
(4) Overall, a good question and I doubt it could be improved. Recommend leaving it as is.

Question 3: Which of the following is a sketch of the function $g(x)=|x^2-4|$?.
(a) 1   (d) 0
(b) 0   (e) 1
(c) 10   (f) 1

Overall, 10/13 correct

Clarity of Question:
No problems with clarity of question. One student asked if the sides branches in option (c) were

straight or curved.

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	(c) correct reasoning: good	Graphed it. x^2-4 is a parabola, then reflected the negative part across y=0 and chose (c).
2	(e) incorrect	Chose (e) because it has the right slopes on the ends and past x=2 the absolute value doesn't matter.
3	(f) incorrect	Chose (f) because "it is the only one shifted right by 4"
4	(c) correct reasoning: good	Mentally reflected the negative part of x^2-4 across y=0.
5	(c) correct reasoning: good	Graphed x^2-4 and mentally reflected the negative part of x^2-4 across y=0.
6	(c) correct reasoning: good	Mentally reflected the negative part of x^2-4 across y=0.
7	(c) correct reasoning: acceptable	The inside is a quadratic, so eliminated (a), (d) and (f). Eliminated (b) because absolute values can't be negative. Then chose (c) because he's "pretty sure the shape should the mirror of a parabola".
8	(c) correct reasoning: good	Graphed x^2-4 and mentally reflected the negative part of x^2-4 across y=0.
9	(c) correct reasoning: good	Graphed it. x^2-4 is a parabola, then reflected the negative part across y=0 and chose (c). Note: this student asked if the side branches of (c) were straight or curved. Also, eliminated (a) because the sides had the wrong curvature.
10	(c) correct reasoning: good	Graphed it. x^2-4 is a parabola, then reflected the negative part across y=0 and chose (c). Also explicitly eliminated (a) because it "doesn't look right"
11	(c) correct reasoning: acceptable	First, checked the x-intercepts and eliminated (b) and (f). Then noted that it should look like the graph of x^2-4 on the outer edges (beyond x=2 and x=-2) so chose (c).
12	(a) incorrect	Checked the x-intercepts first, concluded it must be (a) or (c) because "it is an absolute value", then chose (a) because "it is a parabola with the negative part flipped"
13	(c) correct reasoning: good	Graphed it. x^2-4 is a parabola, then reflected the negative part across y=0 and chose (c).

Comments and Conclusions:
(1) Most correct answers were arrived at by very good reasoning (graphing the parabola and reflecting across y=0).
(2) Incorrect answers were chosen because of very poor reasoning and approaches, or difficulty mentally reflecting the graph.
(3) Some students 2/13 arrived at the correct response by imperfect, but acceptable, methods.

(4) Overall, a good question. Recommend leaving it as is.

Question 4: Select all statements below that are true (quantifiers).
(a) 11
(b) 6
(c) 11
(d) 0

Overall, for (a) 11/13 correct, for (b) 7/13 correct, for (c) 11/13 correct, for (d) 13/13 correct.

Clarity of Question:
No problems with clarity of question. Many students found the wording of the statements confusing or hard to interpret and indicated a general discomfort and difficulty parsing the statements. They are clearly not accustomed to translating such statements.

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	(a), (b), (c)	Stated that (a) and (c) were "basically the same" and (b) and (d) were "very similar". Chose incorrect response (b) because "if there is a y then you can find an x"
2	(a), (c)	Expressed some confusion in statement (b) about whether x is "fixed" or not, and in the end decided "there exists" means "one fixed single number".
3	(a), (b), (c)	Chose the first 3 statements because "all 3 mean the same thing". Did not choose (d) because "it can't be true for every x **and** every y"
4	(c)	Did not choose (a) because "there are 2 dependent variables". Claimed the same reasoning for not choosing (d) .
5	(a), (b), (c)	Found all statements confusing and "can't tell the difference between them"
6	(a), (c)	Almost chose option (b) but then changed answer, because we can't find one single fixed x that works for all y.
7	(a)	Claimed (a) is true because we can change both both x and y. (b) is false because x can't change but y can. (c) is similar to (b) except here y is fixed (so also false). (d) can be disproved by picking x=-1 and y=-1.
8	(a), (b), (c)	Chose (a),(b) and (c) because "the function is continuous so there will be an x integer whenever there is a y integer". Did not choose (d) because since both are "for every" they won't necessarily add up to 3.
9	(a), (c)	Did not choose (b) because "if y is infinity there is no way you can get 3 by adding these two". Claimed (c) was similar to (a) except it must be true for every integer, so don't see why not. Did not pick (d) because it's "silly"
10	(a), (b), (c)	For (b), expressed some confusion about whether it means a single fixed value of x, or if it can vary.

11	(a)	Did not choose (b) because "if y can be every integer, the output can't be fixed". Did not choose (c) because "if x can be every integer, the output can't be fixed"
12	(a), (b), (c)	Chose (b) to be true because "it is the flip of (c)". Chose (c) because "we can always solve this"
13	(c)	Did not choose (a) because the wording was confusing. Only chose (c) because this student treated it as a single-answer multiple choice question.

Comments and Conclusions:
(1) Many students interpreted the various parts in terms of dependent vs. independent variables or free vs. Fixed quantities. Statement (b) was therefore often (6/13 times) selected as true because students read this as one free and one fixed variable (or one dependent and one independent), so the equation can be solved. In contrast, (d) was never chosen as true because it had "two fixed variables" or "two independent variables".
(2) (a) and (d) were generally interpreted correctly. (b) and (c) were often misinterpreted.
(3) Overall, a good question. Recommend leaving it as is but it would be interesting to do more student interviews on these types of problems.

Question 5: Do the following two statements mean the same thing? "If it is difficult , then it makes me stronger" and "If it is not difficult, then it does not make me stronger"
(a) Yes 2
(b) No 11

Overall, 11/13 correct.

Clarity of Question:
No problems with clarity of question.

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	Yes - incorrect	Claimed the second statement indirectly states the same thing as the first. In particular, "adding not to both sides doesn't change it"
2	No – correct reasoning: good	Because the first statement doesn't say that if it's easy it doesn't also make you stronger (i.e. "not difficult" and "it makes me stronger" is possible in the first statement)
3	No – correct reasoning: unclear	Because it does not exclude all other possible things that can make you stronger.
4	No – correct reasoning: good	Because it it's not difficult it could still make me stronger. (in first statement)
5	No – correct reasoning: unclear	Because there might be other possibilities
6	No – correct	Because it's possible that if it's not difficult it still makes you stronger

	reasoning: good	(in first statement)
7	No – correct reasoning: good	Because the first statement doesn't say ONLY difficult things make you stronger.
8	Yes – incorrect	Two negatives on both sides cancel out to give the first statement, like a mathematical equation.
9	No – correct reasoning: good	Because in the first case if it is not difficult it could also make me stronger.
10	No – correct reasoning: unclear	Because if it is not difficult it probably doesn't make me stronger, but it could.
11	No – correct reasoning: poor	Because if it is not difficult it could make me weaker or it could have no effect.
12	No – correct reasoning: poor	Can't explain why, just "remember" this from CPSC 121.
13	No – correct reasoning: unknown	I missed this student's explanation.

Comments and Conclusions:
(1) The two incorrect responses were arrived at by treating the statements as mathematical equations, and negating both sides doesn't "change" the meaning.
(2) Most of the correct answers were arrived at with good reasoning, although 2 students used poor reasoning to arrive at the correct answer.
(3) Overall, a good question. Recommend leaving it as is.

Question 6: Do the following two statements mean the same thing? "If I am healthy, then I will come to class" and "If I come to class, then I am healthy"
(a) Yes 3
(b) No 10

Overall, 10/13 correct.

Clarity of Question:
No problems with clarity of question.

Reasons for selecting/eliminating answers:
ID #	Answer Choice	Comments
1	Yes - incorrect	Because the first statement means that if I'm not healthy then I won't come to class.
2	No – correct reasoning: good	Because (in the first statement) you could be sick and come to class.
3	No – correct reasoning: good	Because (the first statement) doesn't say you won't go if you're sick.
4	Yes – incorrect	No reason given.
5	No – correct	Because when the order is different it means different things.

	reasoning: unclear	
6	No – correct reasoning: good	Because it (the first statement) does not exclude being unhealthy and going to class.
7	No – correct reasoning: poor	Because the cause and effect relationship differs. It doesn't say if they're healthy they can still choose to go to class or not.
8	Yes – incorrect	Because they are just the reverse order of each other (order doesn't matter, like in a math equation).
9	No – correct reasoning: ok	One this is a subresult of the other. If A then B is not enough info to know if B then A.
10	No – correct reasoning: good	Because the first statement doesn't exclude even if I'm not healthy I still come to class.
11	No – correct reasoning: unclear	Because there could be conditions other than being healthy that could make me come to class.
12	No – correct reasoning: poor	Because it is similar to the last one (#5).
13	No – correct reasoning: good	Because the 1st statement can be true but going to class doesn't mean you're healthy (i.e. Could be sick and go to class).

Comments and Conclusions:
(1) The incorrect responses were arrived at by treating the statements as mathematical equations or by incorrect logical reasoning.
(2) Most of the correct answers were arrived at with good reasoning, although 2 students used poor reasoning to arrive at the correct answer.
(3) Overall, a good question. Recommend leaving it as is.

Question 7: Do the following two statements mean the same thing? "If the hammer is in the library, then the butler is the murderer" and "If the butler is not the murderer, then the hammer is not in the library"
(a) Yes 9
(b) No 4

Overall, 9/13 correct.

Clarity of Question:
No problems with clarity of question.

Reasons for selecting/eliminating answers:
ID #	Answer Choice	Comments
1	No - incorrect	The hammer must be with the butler. If both are not in the library, then the butler could still be the murderer.
2	Yes – correct reasoning: unknown	Initially answered No, but switched to Yes on re-reading.

3	Yes – correct reasoning: poor	Because if it were the murderer then the hammer would be in the library.
4	Yes – correct reasoning: unknown	No reason given.
5	Yes – correct reasoning: unknown	No reason given.
6	Yes – correct reasoning: acceptable	Thought of the various possibilities.
7	Yes – correct reasoning: unknown	No reason given.
8	Yes – correct reasoning: poor	Because two negatives in a math equation cancel.
9	No – incorrect	Because one thing is a subresult of the other. If A then B is not enough info to know if B then A.
10	No – incorrect	The 1st is just one case to prove the butler is the murderer, but there could be other ways to prove the butler is the murderer.
11	Yes – correct reasoning: unknown	"Just seems true."
12	Yes – correct reasoning: poor	Went for "Yes" but "not too sure about it."
13	No – incorrect	Because the 1st statement could be false when the 2nd statement is true.

Comments and Conclusions:
(1) The incorrect responses were arrived at by treating the statements as mathematical equations or by incorrect logical reasoning.
(2) Most students with correct answers could not explain their reasoning. 3 correct responses were arrived at with poor reasoning.
(3) Overall, a good question. Recommend leaving it as is, but further interviews may be insightful.

Question 8: For real numbers x, $x^2 > x$
(a) always true 0
(b) sometimes true 13
(c) never true 0

Overall, 13/13 correct.

Clarity of Question:
No problems with clarity of question. However, several students (3) asked whether 0 was a real number (I did not tell them).

Reasons for selecting/eliminating answers:
ID #	Answer Choice	Comments
1	(b) – correct	Doesn't work for x=1, so can't always be true.

		reasoning: acceptable	
2	(b) – correct reasoning: good	Tested cases. Found one true (x=2) and one false (x=0.5)
3	(b) – correct reasoning: acceptable	Because it is not true for decimals, went with sometimes true.
4	(b) – correct reasoning: acceptable	Doesn't work for x=0, so can't always be true.
5	(b) – correct reasoning: acceptable	Asked if 0 was real. Assuming it is, went with sometimes true.
6	(b) – correct reasoning: acceptable	Not true for x=1, so went with sometimes true.
7	(b) – correct reasoning: good	Not true for x=1/2, true for x>1. Expressed confusion about what are real numbers... are decimals? 0?
8	(b) – correct reasoning: acceptable	Not true when x<1, so went with sometimes true.
9	(b) – correct reasoning: good	Graphed y= x^2 and y=x. Noted it was not true for x=1.
10	(b) – correct reasoning: poor	Not true for x=0, so $,\geq,$ would "probably work better"
11	(b) – correct reasoning: acceptable	Asked if 0 was a real number. Noted that if x=0 then $x^2=x$, so not always true.
12	(b) – correct reasoning: acceptable	Not true for x=1, so not >. So chose sometimes true.
13	(b) – correct reasoning: good	Not true for x=1, True for x=-1

Comments and Conclusions:
(1) Most students tested values of x, found one that did not work and so chose "sometimes true". It's unclear whether these students also had in mind cases that were true, or whether they have a bias toward "always true" and then just downgrade a step.
(2) Even though all students answered correctly, I think it is worth leaving on (and as is), as past test results suggest higher error rates than this (students may be thinking more carefully about this in person).

Question 9: For real numbers x and y, $(x+y)^2=x^2+y^2$
(a) always true 0
(b) sometimes true 9
(c) never true 4

Overall, 9/13 correct.

Clarity of Question:
No problems with clarity of question. However, one additional student (in addition to the 3 from the

previous question) asked whether 0 was a real number.

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	(c) – incorrect	Because it's missing 2xy when expanded.
2	(c) – incorrect	It should be $x^2+2xy+y^2$ (we learned this).
3	(c) – incorrect	Because these two expressions don't mean the same thing.
4	(b) – correct reasoning: good	To be true we must have 2xy=0, so true if x=0 or y=0 (or both).
5	(b) – correct reasoning: good	Plugging in (1,2) and it's not true. Plugging in (0,0) and it is true.
6	(b) – correct reasoning: acceptable	Used x=1, y=0 as an example.
7	(b) – correct reasoning: good	True when x=0,y=0. Not true if x=2,y=3.
8	(b) – correct reasoning: good	It's only true for x=0 and y=0. Expanding out gives $x^2+2xy+y^2$, so not true for some.
9	(b) – correct reasoning: acceptable	True if they're both zero. Otherwise not true. Expanded out to give x^2+xy+y^2 .
10	(b) – correct reasoning: good	Should use FOIL, so not correct. Most numbers no, but for a few numbers (x=0,y=0) it could work.
11	(b) – correct reasoning: acceptable	Because if x and y are zero they are the same.
12	(b) – correct reasoning: poor	Because it is true only when x=1 and y=1, otherwise not true.
13	(c) – incorrect	This student asked if 0 was a real number. Assumed it was not. Tested x=1,y=1 and found it not true. Tested x=0,y=0 and found it was true, but assumed 0 was not a real number.

Comments and Conclusions:
(1) Most students who answered incorrectly failed to consider possible cases in (x,y), they just said it did not agree with the rule they knew (and chose never true). One student answered incorrectly because he assumed 0 was not a real number.
(2) Most students who answered correctly used valid reasoning.
(3) I think this question does a very good job at differentiating students who can deal with "open sentences" (statements where the true value depends on variable values) and so should be left on and as is.

Question 10: For real numbers x and y, $\sqrt{(x^2+y^2)}<x$
(a) always true 1
(b) sometimes true 6
(c) never true 6

Overall, 6/13 correct.

Clarity of Question:
One student asked whether x and y were required to be the same number, or whether they could be different (I did not say).

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	(b) – incorrect	Not always true because if y>0 it's not true.
2	(b) – incorrect	Because it's true if y=0.
3	(b) – incorrect	Depending on what the values are, x positive vs. x negative, it could be true or not.
4	(c) – correct reasoning: good	This student asked if x and y must be the same number, or if they can be different. Chose never true because it can only be = or >. If y=0, then you get equality, if y>0, then you get >x.
5	(b) – incorrect	Plugged in x=-2 and it didn't work, so chose sometimes true. After being prompted to explain her reasoning, this student changed her answer to never true.
6	(b) – incorrect	Because if x<0 then the LHS is positive, so it doesn't work.
7	(c) – correct reasoning: good	Because it is similar to the pythagorean theorem. The hypotenuse is the longest side of a triangle, which can never be shorter than either of the other two sides.
8	(c) – correct reasoning: good	This is simply pythagoras, $\sqrt{(x^2+y^2)}$ is hypotenuse, which is always the longest side on a triangle.
9	(c) – correct reasoning: acceptable	Noted that $\sqrt{(x^2)}=x$ but $\sqrt{(x^2+y^2)}$ is even bigger, so must be never true.
10	(b) – incorrect	This student oscillated between (b) and (c). Tried many cases, x=0, x=1, ... claimed that as x increases "it gets worse". In the end concluded that it works for negative numbers, so must be sometimes true.
11	(c) – correct reasoning: good	The smallest it could get is if y=0. Even then, it would have to be =, not <.
12	(c) – correct reasoning: acceptable	Because even with x=0 it is =. If x=1 it is >.
13	(a) – incorrect	This student initially chose (a), but then when prompted to explain switched to (c), with correct reasoning.

Comments and Conclusions:
(1) Students answered incorrectly due to flawed reasoning or misreading the question.
(2) All students who answered correctly did so after good or acceptable reasoning (either general reasoning about the inequality, or by using the pythagorean theorem).
(3) I think this question is good, and should be left as is.

Question 11: A polygon is regular when all of its sides are the same length and all its angles are the same. Circle all figures below that are regular polygons.

(a) 13/13 correct   (d) 13/13 correct   (g) 13/13 correct
(b) 12/13 correct   (e) 13/13 correct   (h) 13/13 correct
(c) 12/13 correct   (f) 13/13 correct

Overall, 12/13 students answered perfectly.

Clarity of Question:
No problems with clarity of question or answer options.

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	Incorrectly circled (b) and (c)	Did not choose (g) and (h) because they're symmetric, but have unequal angles.
2	All correct	Circled options because they seem to have the same side lengths and angles. Noted that the uncircled ones were "odd-looking"
3	All correct	Chose options based side lengths being the same.
4	All correct	Already knew what a regular polygon was and just picked them out.
5	All correct	Because the sides on those he/she circled all have the same lengths and angles.
6	All correct	The circled ones look like they have the same lengths and angles. The others didn't have the same lengths or angles.
7	All correct	The others all have sides with different lengths.
8	All correct	Circled those with equal angles and equal sides. This student had heard of regular polygons previously.
9	All correct	Eliminated (b) and (c) because side lengths differ. (no comment on (g) and (h)).
10	All correct	Eliminated options because angles or side lengths differed.
11	All correct	First looked for equal side lengths. All uncircled options had unequal sides.
12	All correct	Only checked if sides are the same length.
13	All correct	Eliminated those with different side lengths.

Comments and Conclusions:
(1) The only errors observed were by one student who only checked for (the appearance of) equal angles. Almost all students answered all parts correctly, so this has very low utility. In addition, many students answered all parts correctly by only looking at the side lengths of the polygons.
(2) I recommend removing this question from the test and replacing it with a definition question that more effectively tests the need to satisfy multiple conditions.

Question 12: Below is a statement and 3 proofs. Select the proof of the statement that is correct and complete.
(a) 7
(b) 2
(c) 2
spoiled: 2

Overall, 2/13 students answered correctly.

Clarity of Question:
Two students misinterpreted the question and chose two proofs, (b) and (c).

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	(a) – incorrect	Chose (a) because "proofs usually start with what we're given, not what we end up with."
2	(a) - incorrect	(a) "seems like the best" and is "the most right." Did not like (b) because we don't know where the first line came from. Did not like (c) because it "seems true for some, but not true for others"
3	(a) - incorrect	Like (a) best because it starts with a familiar expression. Thought (c) had an error in it, and found that (b) starts with "something different" so prefer (a).
4	Spoiled (chose both (b) and (c))	Did not choose (a) because "the squaring step makes things positive and it is bad to do this." Noted that both (b) and (c) seem correct.
5	(c) – incorrect	(c) makes the most sense to me because there is a visual picture.
6	(a) - incorrect	Chose (a) because if we do it backwards, the final solution satisfies the statement. Commented that he/she wasn't sure what direction proofs should go in. Almost chose (c) because it "made sense to me"
7	(b) – correct reasoning: poor	This student had a hard time choosing. Said that "they all seem right" and spent most of his/her time analyzing the individual steps/calculations. Eliminated (c) because it doesn't explicity address "for positive a,b". Chose (b) because it has fewer steps (but noted that this was not a very good reason)
8	(a) - incorrect	Chose (a) because "this is the way I would have done it", i.e. "try to find a way to see that the assumption is true." Eliminated (b) because "I don't see why they started with this". Eliminated (c) because "maybe a different picture would make it wrong"
9	(b) – correct reasoning: good	(b) makes the most sense: start with something you know is true and rearrange to get what you want. Eliminated (a) because it is not what you should do to prove something ("seems risky"). Eliminated (c) because it "looks like the pythagorean theorem, which only shows true for some cases"
10	(c) - incorrect	Chose (c) because the picture really shows what's going on, whereas

		(a) and (b) seem like they could have errors. Noted that (b) was his/her least favourite because it is in the wrong order.
11	Spoiled (chose both (b) and (c))	Eliminated (a) because it didn't state "for positive numbers". If a=0, then the first line doesn't work. Chose both (b) and (c) because "I can't find any errors"
12	(a) - incorrect	Chose (a) because it looks most complete and easy to follow and the proof makes sense. Eliminated (c) because it has > rather than greater or equal and so is not complete.
13	(a) – incorrect	Noted that both (a) and (b) seemed right. Eliminated (b) because "when we divide we should the direction of the inequality". Eliminated (c) because it "doesn't go algebraically"

Comments and Conclusions:
(1) (a) was a common (incorrect) choice because it was familiar, or because students explicitly stated that this is the order they thought proofs should go in. (b) (correct) choice was often avoided because students couldn't tell where the first line came from.
(2) Often, students looked at the step-by-step computations and did not look at the overall structure of the proof in their assessment.
(3) Although there are some difficulties with the clarity of the question, I think it very clearly illuminates the tendency for students to think that proofs that start with the conclusion are correct, and their inability to look at the overall structure.
(4) I recommend leaving this question on, possibly with bolding of the word "the" or an additional (circle only one) statement to reduce the frequency of spoiled answers.

Question 13: Find the set of all values of x for which $|2-x^2|<2$ is true.

(a) 0 (d) 0 (g) 2
(b) 1 (e) 4 (h) 1
(c) 0 (f) 4
spoiled: 1

Overall, 4/13 students answered correctly.

Clarity of Question:
No problems with the question clarity, but for the answers one student interpreted the answer choices as (x,y) pairs. This student also asked what the union symbol was.

Reasons for selecting/eliminating answers:

ID #	Answer Choice	Comments
1	(e) – incorrect	Eliminated x=0 because of the strictly greater than. Chose the root 2 endpoints by solving $2-x^2>0$.
2	(e) - incorrect	Found the interval by solving for $|2-x^2|=0$, and noted that 0 and the endpoints can't be included because of the strictly less than.
3	(h) - incorrect	No attempt to solve algebraically. Plugged in endpoints. Almost chose (c), but then chose (h) because "they both give 2-2). Noted that it can't include 0 because then 2=2, but forgot this when choosing

		answer.
4	(f) – correct reasoning: good	Drew a graph, y=2 and $y=2-x^2$ and used correct reasoning to choose (f).
5	(e) – incorrect	Attempted algebraic solution initially (but used very ineffective methods), then tried graphically. Eventually tried plugging in endpoint values and chose (e).
6	(g) - incorrect	Solved by drawing a graph and finding the intercepts, but neglected to remove x=0.
7	(f) – correct reasoning: acceptable	No attempt to solve the inequality. Noted that x cannot be 0 or 2 or -2 because of the strict inequality. Chose (f) because it satisfied these requirements.
8	(b) - incorrect	Solved algebraically, looking at the two cases. Got x>0 and x<2 (missed x>-2) and so chose (b).
9	Spoiled (chose (a), (c), (d) and (h))	Drew graphs of $2-x^2$ and y=2. This student interpreted the answer options as (x,y) pairs an circles those with a y-value less than 2.
10	(f) – correct reasoning: acceptable	Plugged in various x-values (x=0, x=1, x=2, x=3). Says it does work for negatives. Eliminated all but (e) and (f), then chose (f) by plugging in x=4/5 (later realized x=9/5 would be better choice).
11	(g) - incorrect	Looked at the two cases. Got -2<x<2, but neglected to remove x=0.
12	(e) – incorrect	Noted that 0 can not be included because of strict inequality. Noted that we "can't include beyond root 2" so chose (e).
13	(f) – correct reasoning: good	Plotted a bunch of points to sketch a graph of $y=2-x^2$, then also drew y=2. Found like answer should be (e), (f) or (g). Eliminated (e) and (g) by plugging in points and noting 0 cannot be included.

Comments and Conclusions:
(1) Many students try to just plug in endpoints, which usually led to incorrect answers. Often students are misled to solutions that ignore the 2 on the right side of the inequality (i.e. Only check the expression inside the absolute value is positive).
(2) Correct answers were for good or acceptable reasoning.
(3) Recommend leaving this question as is.

Question 14: A prime power is an integer of the form p^n where p is prime and n is a positive integer greater than 1. Select all numbers below that are prime powers.

(a) 6/13 correct   (d) 9/13 correct   (g) 1/13 correct
(b) 12/13 correct   (e) 10/13 correct   (h) 11/13 correct
(c) 11/13 correct   (f) 10/13 correct

Clarity of Question:
No problems with the question clarity.

Reasons for selecting/eliminating answers:

ID #	Answer Choices	Comments
1	(b), (e)	Said 2 is not prime and chose answers correspondingly. On review, changed her mind.
2	(b), (d), (e)	Selected those answers that were written in the form p^n with the given conditions on p and n. (so missed (a) and (g)). Eliminated incorrect options for valid reasons.
3	(a), (b), (d), (e)	Incorrectly eliminated (g) because "4 is not prime". All other eliminated answers were for valid reasons.
4	(a), (b), (d), (e), (g) (all correct) reasoning: good	Eliminated all incorrect answers for valid reasons and selected correct ones for good reasons.
5	(b), (c), (d), (e)	Incorrectly chose (c) because it "looks like 1^n". Missed (a) and (g) because the base was "not prime". Mentioned that he/she couldn't remember if 2 was a prime number.
6	(a), (b), (d), (e), (f), (h)	Looked for a prime number and a positive integer as the power. (Hence, incorrectly chose (h) and (f))
7	(a), (b), (d), (e)	Missed (g) because "4 is not prime". All other answers eliminated or chosen for good reasons.
8	(b), (d), (e), (f)	Circled these because they "were obviously in the form p^n". Eliminated (a) and (g) because they are not in this form. This student expressed some discomfort with this and asked if the MUST be in this form, or just COULD be.
9	(b), (d), (e), (f)	Did not choose (a) or (g) because the base was not prime. Not sure why (f) was chosen. All others chosen or eliminated for correct reasons.
10	(b), (e)	Eliminated (a), (c) and (h) because n=1. Eliminated (d), (f) and (g) because the base is not prime.
11	(a), (b)	Eliminated (g) because 4 is not prime. Eliminated (f) because -2 is not prime. Eliminated (c) because 1 is not prime.
12	(a), (b), (c), (d), (e), (h)	Only chose correct form answers, did not eliminate any. For (c), the student stated that 1 is prime. For (h), the student did not check the power.
13	(a), (d), (g)	This student didn't know what a prime number was. In the end, he/she decided they were even numbers and so chose (a), (d) and (g) based on this.

Comments and Conclusions:
(1) Errors by students can be attributed to (a) not knowing the definition of a prime number, (b) forgetting to check one of the conditions of the definition and (c) only considering the form the integers are expressed in, rather than how they COULD be expressed (so misinterpreting the definition).
(2) Correct answers were for good or acceptable reasoning.
(3) Recommend leaving this question as is.

Question 15: Please read the following passage and answer the question: "Did the South African Bushmen have an independent name for the number six?
(a) Yes 1
(b) No 12

Overall, 12/13 correct.

Clarity of Question:
No problems with the question clarity.

Reasons for selecting/eliminating answers:

ID #	Answer Choices	Comments
1	(b) No – correct reasoning: good	Because it says no independent number names for groups of 3, 4, 5, or 6.
2	(a) Yes - incorrect	Because they counted up to 10, so must have a word for it.
3	(b) No – correct reasoning: good	Because they only have 2 words, 6 doesn't have its own name.
4	(b) No – correct reasoning: good	Because it is 2+2+2.
5	(b) No – correct reasoning: good	Because it says they have no independent name for 3, 4, 5, or 6.
6	(b) No – correct reasoning: good	Because it looks like they just use the two words to form 6.
7	(b) No – correct reasoning: good	Because they use two words to make it up.
8	(b) No – correct reasoning: good	According to the passage they only used 2 words and only counted in 2's.
9	(b) No – correct reasoning: good	It would be 2+2+2.
10	(b) No – correct reasoning: good	Reiterated the relevant part of the text to me.
11	(b) No – correct reasoning: poor	Used odd reasoning, said they would call it five-two (?)
12	(b) No – correct reasoning: acceptable	Because the passage says they needed two words to count to ten.
13	(b) No – correct reasoning: good	Because they used two-two-two.

Comments and Conclusions:
(1) All students but one got this question correct, and nearly all for good reasoning. Most students seem to have no difficulty reading and interpreting the passage.
(2) Correct answers (except one) were for good or acceptable reasoning.

(3) Recommend removing this question, as it is saturated and (now that we know students do not have reading comprehension difficulty) does not provide us with useful information.

Question 16: Please read the following passage and answer the question: "How would the South American Indians mentioned above refer to the number five?"
(a) 1   (d) 12
(b) 0   (e) 0
(c) 0   (f) 0

Overall, 12/13 correct.

Clarity of Question:
No problems with the question clarity.

Reasons for selecting/eliminating answers:

ID #	Answer Choices	Comments
1	(d) – correct reasoning: good	Eliminated other choices for good reasons. Noted that (d) was a better choice than (c) because it was shorter.
2	(d) – correct reasoning: good	Chose (d) because "this is the pattern shown. Eliminated some of the other choices for valid reasons.
3	(d) – correct reasoning: good	Because it matches the example two-two that they gave.
4	(d) – correct reasoning: good	Because they can't go over 2.
5	(d) – correct reasoning: good	Because it is more simple than (c) and matches the pattern in the text.
6	(d) – correct reasoning: good	Because they don't have 3 or 5 and (c) would be redundant with the one-one-one part. Also, (e) is different from the example.
7	(d) – correct reasoning: good	Because it matches the pattern. Also, eliminated all the other choices for valid reasons.
8	(d) – correct reasoning: good	Because it follows the pattern from two-one, two-two, ... and they can only use one and two.
9	(a) – incorrect	Because they "counted to six".
10	(d) – correct reasoning: good	Because it matches the pattern in the text.
11	(d) – correct reasoning: good	Follows pattern in the text.
12	(d) – correct reasoning: good	Because it matches the pattern.
13	(d) – correct reasoning: good	Because they could count up to 6 and this matches the pattern.

Comments and Conclusions:
(1) All students but one got this question correct, and nearly all for good reasoning. Most students seem to have no difficulty reading and interpreting the passage.
(2) Correct answers were for good reasoning. The only incorrect answer was due to poor reading comprehension.
(3) Recommend removing this question, as it is saturated and (now that we know students do not have much reading comprehension difficulty) does not provide us with useful information.

Question 17: Please read the following passage and answer the question: "How would the South African Bushmen exchange three cows for six pigs?"
(a) 0   (d) 0
(b) 12 (e) 0
(c) 0
no correct answer: 1

Overall, 12/12 correct. Note, there was a typo in the correct answer option that made it incorrect, which one student noticed. The typo was fixed for all subsequent validation interviews.

Clarity of Question:
No problems with the question clarity.

Reasons for selecting/eliminating answers:

ID #	Answer Choices	Comments
1	(b) – correct reasoning: good	Eliminated all other answers for valid reasons.
2	Noticed typo in the correct answer (b) – no correct answers	In the end, chose (e) because "none of the other answers work". Really, though, he noted there was no correct answer.
3	(b) – correct reasoning: good	Because that's the same as they did for 2 cows and 4 pigs.
4	(b) – correct reasoning: good	Because of the last line of the text. It says basically the same thing.
5	(b) – correct reasoning: good	Because that's what they did for the example.
6	(b) – correct reasoning: good	Because it matches the example given.
7	(b) – correct reasoning: good	Like in the example.
8	(b) – correct reasoning: good	Follows the pattern in the example.
9	(b) – correct reasoning: good	Reasoned correctly from example.
10	(b) – correct reasoning: good	Noted they wouldn't do (a), then chose (b) but I did not get his/her reasoning.

11	(b) – correct reasoning: good	Because it follows the pattern in the last line of the text.
12	(b) – correct reasoning: good	Because to do 2 for 4 they did it twice (matches pattern).
13	(b) – correct reasoning: good	Because it is mentioned at the end with the same pattern.

Comments and Conclusions:

(1) All students got this question correct, and all for good reasoning. Most students seem to have no difficulty reading and interpreting the passage.

(2) Recommend removing this question, as it is saturated and (now that we know students do not have much reading comprehension difficulty) does not provide us with useful information.