Equations of Lines and Planes
Practice Problems

Find the parametric form $\vec{\mathbf{x}}(t) = \vec{\mathbf{p}} + t\vec{\mathbf{d}}$ of a line given the following information:

1. Two points $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ on the line, say $\vec{\mathbf{p}}_1 = [1, -1, 3]$ and $\vec{\mathbf{p}}_2 = [2, 3, -1]$:
2. One point $\vec{\mathbf{p}}$ on the line and a direction vector $\vec{\mathbf{d}}$, say $\vec{\mathbf{p}} = [1, 2, 3]$ and $\vec{\mathbf{d}} = [6, 5, 4]$.
3. One point $\vec{\mathbf{p}}$ on the line and parallel ($\|$) to another line $\vec{\mathbf{a}} + t\vec{\mathbf{b}}$, say $\vec{\mathbf{p}} = [1, 0, 2]$, $\vec{\mathbf{a}} = [2, 2, -2]$, and $\vec{\mathbf{b}} = [1, 4, 5]$.
4. One point $\vec{\mathbf{p}}$ on the line and $\perp$ to another line $\vec{\mathbf{a}} + t\vec{\mathbf{b}}$, say $\vec{\mathbf{p}} = [1, 0, 2]$, $\vec{\mathbf{a}} = [2, 2, -2]$, and $\vec{\mathbf{b}} = [1, 4, 5]$.
5. One point $\vec{\mathbf{p}}$ on the line and $\perp$ to a plane $ax + by + cz + d = 0$, say $\vec{\mathbf{p}} = [1, 0, 2]$, $a = 2$, $b = -4$, $c = 5$, and $d = 1$.
6. One point $\vec{\mathbf{p}}$ on the line and $\|$ to a plane $ax + by + cz + d = 0$, say $\vec{\mathbf{p}} = [1, 0, 2]$, $a = 2$, $b = -4$, $c = 5$, and $d = 1$.
7. The line is the intersection of the two planes: $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$, where $a_1 = 2$, $b_1 = -4$, $c_1 = 5$, $d_1 = 1$, $a_2 = 1$, $b_2 = -7$, $c_2 = 1$, and $d_2 = 4$.
8. One point $\vec{\mathbf{p}}$ on the line and $\perp$ to each of two given lines: $\vec{\mathbf{a}}_1 + t\vec{\mathbf{b}}_1$ and $\vec{\mathbf{a}}_2 + t\vec{\mathbf{b}}_2$, where $\vec{\mathbf{p}} = [1, 0, 2]$, $\vec{\mathbf{a}}_1 = [2, 2, -2]$, $\vec{\mathbf{b}}_1 = [1, 4, 5]$, $\vec{\mathbf{a}}_2 = [2, 4, -2]$, and $\vec{\mathbf{b}}_2 = [4, 2, -3]$.
9. One point $\vec{\mathbf{p}}$ on the line and $\|$ to each of two given planes: $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$, where $\vec{\mathbf{p}} = [1, 0, 2]$, $a_1 = 2$, $b_1 = -4$, $c_1 = 5$, $d_1 = 1$, $a_2 = 1$, $b_2 = -7$, $c_2 = 1$, and $d_2 = 4$.
10. Equidistant to two given planes (here we mean that each point on the line is equidistant to the two planes): $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$, where
    a) $a_1 = 2$, $b_1 = -4$, $c_1 = 5$, $d_1 = 1$, $a_2 = 1$, $b_2 = -7$, $c_2 = 1$, and $d_2 = 4$.
    b) $a_1 = -2$, $b_1 = 14$, $c_1 = -2$, $d_1 = 1$, $a_2 = 1$, $b_2 = -7$, $c_2 = 1$, and $d_2 = 4$.
11. Equidistant to two given lines (here we mean that each point on the line is equidistant to the two given lines): $\vec{\mathbf{a}}_1 + t\vec{\mathbf{b}}_1$ and $\vec{\mathbf{a}}_2 + s\vec{\mathbf{b}}_2$.
    a) $\vec{\mathbf{a}}_1 = [2, 2, -2]$, $\vec{\mathbf{b}}_1 = [1, 4, 5]$, $\vec{\mathbf{a}}_2 = [2, 2, -2]$, and $\vec{\mathbf{b}}_2 = [4, 2, -3]$.
    b) $\vec{\mathbf{a}}_1 = [2, 2, -2]$, $\vec{\mathbf{b}}_1 = [1, 4, 5]$, $\vec{\mathbf{a}}_2 = [2, 4, -2]$, and $\vec{\mathbf{b}}_2 = [3, 12, 15]$.
    c) $\vec{\mathbf{a}}_1 = [2, 2, -2]$, $\vec{\mathbf{b}}_1 = [1, 4, 5]$, $\vec{\mathbf{a}}_2 = [2, 4, -2]$, and $\vec{\mathbf{b}}_2 = [4, 2, -3]$.

Find an equation of a plane (if possible) given the following information:

1. One point $\vec{\mathbf{p}}$ on the plane and a normal vector $\vec{\mathbf{b}}$ to the plane, say $\vec{\mathbf{p}} = [1,2,3]$ and $\vec{\mathbf{b}} = [6,5,4]$.
2. One point $\vec{\mathbf{p}}$ on the plane and $\perp$ to a line $\vec{\mathbf{a}} + t\vec{\mathbf{d}}$, say $\vec{\mathbf{p}} = [1,0,2]$, $\vec{\mathbf{a}} = [2,4,-2]$, and $\vec{\mathbf{d}} = [4,2,-3]$.
3. One point $\vec{\mathbf{p}}$ on the plane and $\|$ to another plane $ax+by+cz+d=0$, say $\vec{\mathbf{p}} = [1,0,2]$, $a=2$, $b=-4$, $c=5$, and $d=1$.
4. One point $\vec{\mathbf{p}}$ on the plane and $\perp$ to another plane $ax+by+cz+d=0$, say $\vec{\mathbf{p}} = [1,0,2]$, $a=2$, $b=-4$, $c=5$, and $d=1$.
5. One point $\vec{\mathbf{p}}$ on the plane and $\perp$ to two given planes: $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$, where
   a) $\vec{\mathbf{p}} = [1,0,2]$, $a_1=2$, $b_1=-4$, $c_1=5$, $d_1=1$, $a_2=1$, $b_2=-7$, $c_2=1$, and $d_2=4$.
   b) $\vec{\mathbf{p}} = [1,0,2]$, $a_1=-2$, $b_1=14$, $c_1=-2$, $d_1=1$, $a_2=1$, $b_2=-7$, $c_2=1$, and $d_2=4$.
6. Three points $\vec{\mathbf{p}}_1$, $\vec{\mathbf{p}}_2$ and $\vec{\mathbf{p}}_3$ on the plane, say $\vec{\mathbf{p}}_1 = [2,-1,3]$, $\vec{\mathbf{p}}_2 = [3,-1,3]$ and $\vec{\mathbf{p}}_3 = [2,-1,0]$.
7. One point $\vec{\mathbf{p}}$ on the plane and a line $\vec{\mathbf{a}} + t\vec{\mathbf{d}}$ on the plane, say $\vec{\mathbf{p}} = [1,0,2]$, $\vec{\mathbf{a}} = [2,4,-2]$, and $\vec{\mathbf{d}} = [4,2,-3]$.
8. Both of the lines: $\vec{\mathbf{a}}_1 + t\vec{\mathbf{b}}_1$ and $\vec{\mathbf{a}}_2 + t\vec{\mathbf{b}}_2$, are on the plane, where $\vec{\mathbf{a}}_1 = [2,2,-2]$, $\vec{\mathbf{b}}_1 = [1,4,5]$, $\vec{\mathbf{a}}_2 = [3,6,3]$, and $\vec{\mathbf{b}}_2 = [4,2,-3]$.
9. One point $\vec{\mathbf{p}}$ on the line and $\perp$ to each of two given lines: $\vec{\mathbf{a}}_1 + t\vec{\mathbf{d}}_1$ and $\vec{\mathbf{a}}_2 + t\vec{\mathbf{d}}_2$, where $\vec{\mathbf{p}} = [1,0,2]$, $\vec{\mathbf{a}}_1 = [2,2,-2]$, $\vec{\mathbf{d}}_1 = [1,4,5]$, $\vec{\mathbf{a}}_2 = [2,4,-2]$, and $\vec{\mathbf{d}}_2 = [4,2,-3]$.
10. Equidistant to two given planes (here we mean that each point on the plane is equidistant to the two given planes): $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$, where
    a) $a_1=2$, $b_1=-4$, $c_1=5$, $d_1=1$, $a_2=1$, $b_2=-7$, $c_2=1$, and $d_2=4$.
    b) $a_1=-2$, $b_1=14$, $c_1=-2$, $d_1=1$, $a_2=1$, $b_2=-7$, $c_2=1$, and $d_2=4$.
11. Equidistant to two given lines: $\vec{\mathbf{a}}_1 + t\vec{\mathbf{d}}_1$ and $\vec{\mathbf{a}}_2 + t\vec{\mathbf{d}}_2$ (here we mean that each point on the plane is equidistant to the two lines), where
    a) $\vec{\mathbf{a}}_1 = [2,2,-2]$, $\vec{\mathbf{d}}_1 = [1,4,5]$, $\vec{\mathbf{a}}_2 = [2,2,-2]$, and $\vec{\mathbf{d}}_2 = [4,2,-3]$.
    b) $\vec{\mathbf{a}}_1 = [2,2,-2]$, $\vec{\mathbf{d}}_1 = [1,4,5]$, $\vec{\mathbf{a}}_2 = [2,4,-2]$, and $\vec{\mathbf{d}}_2 = [3,12,15]$.
    c) $\vec{\mathbf{a}}_1 = [2,2,-2]$, $\vec{\mathbf{d}}_1 = [1,4,5]$, $\vec{\mathbf{a}}_2 = [2,4,-2]$, and $\vec{\mathbf{d}}_2 = [4,2,-3]$.