# Critical exponents for the homology of Fortuin-Kasteleyn clusters on a torus

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Joint work with
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May 22, 2009

**Abstract**

A Fortuin-Kasteleyn cluster on a torus is said to be of type $\{a, b\}, a, b \in \mathbb{Z}$, if it possible to draw a curve belonging to the cluster that winds $a$ times around the first cycle of the torus as it winds $-b$ times around the second. Even though the Q-Potts models make sense only for Q integers, they can be included into a family of models parametrized by $\beta = \sqrt{Q}$ for which the Fortuin-Kasteleyn clusters can be defined for any real $\beta \in (0, 2]$. For this family, we study the probability $\pi(\{a, b\})$ of a given type of clusters as a function of the torus modular parameter $\tau = \tau_r + i\tau_i$. We compute the asymptotic behavior of some of these probabilities as the torus becomes infinitely thin. For example, the behavior of $\pi(\{1, 0\})$ is studied for $\tau_i \to \infty$. Exponents describing these behaviors are defined and related to weights $h_{r,s}$ of the extended Kac table for $r, s$ integers, but also half-integers. Numerical simulations are also presented.

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