

Infinite-Dimensional Diffusions in the Kingman Simplex

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Abstract.

We construct a two-parameter family of infinite-dimensional diffusion processes $X^{\alpha,\theta}$ related to Pitman's two-parameter Poisson-Dirichlet distributions $P^{\alpha,\theta}$. The state space for $X^{\alpha,\theta}$ is an infinite-dimensional simplex called the Kingman simplex. The Poisson-Dirichlet distribution $P^{\alpha,\theta}$ is a unique stationary distribution for the corresponding process $X^{\alpha,\theta}$; the process is ergodic and reversible.

The diffusions $X^{\alpha,\theta}$ are obtained in a scaling limit transition from certain finite Markov chains on partitions of natural numbers. These chains have a natural relation with the Pitman's partition structure, this relation is explained.

In the special case when $\alpha = 0$, our finite Markov chains are similar to Moran-type model in population genetics, and our diffusion processes reduce to the infinitely-many-neutral-alleles diffusion model studied by Ethier and Kurtz (1981). Our main results extend those of Ethier and Kurtz, but our approach differs in some aspects.