Problem 1. Show that each positive integer can be written as a sum of integers of the form $2^a \cdot 3^b$ with the property that no integer from the chosen sum divides a different integer from the sum.

Problem 2. Let $n \in \mathbb{N}$ and let $P \in \mathbb{C}[z]$ be a polynomial of degree $2n$, all of whose roots have absolute value equal to 1. Let

$$g(z) := \frac{P(z)}{z^n}.$$ 

Prove that each solution for $g'(z) = 0$ (where $g'$ is the derivative of $g$) has absolute value equal to 1.

Problem 3. Let $A$ be an $N$-by-$N$ matrix with the property that each one of its entries is equal to 1 or $-1$ and also satisfying that $A \cdot A^t = N \cdot \text{id}_N$ (where $\text{id}_N$ is the $N$-by-$N$ identity matrix). Assume there exists an $a$-by-$b$ submatrix of $A$ whose entries are all equal to 1. Prove that $ab \leq N$.

Problem 4. Evaluate

$$\int_{0}^{1} \frac{\ln(x + 1)}{x^2 + 1} \, dx$$