Problem 1. What is the maximum number of points in the cartesian plane whose both coordinates are rational numbers, which lie on the same circle whose center is not a point whose both coordinates are rational numbers?

Problem 2. Let $F_0(x) = \log(x)$ and for each $n \geq 1$ and $x > 0$, we let

$$F_n(x) = \int_0^x F_{n-1}(t)dt.$$ 

Compute

$$\lim_{n \to \infty} n! \cdot \frac{F_n(1)}{\ln(n)}.$$ 

Problem 3. Let $p$ be a prime number and let $f \in \mathbb{Z}[x]$. Assume that the integers $f(k)$ for $0 \leq k \leq p^2 - 1$ are all distinct modulo $p^2$. Then prove that for each $n \in \mathbb{N}$, the integers $f(k)$ for $0 \leq k \leq p^n - 1$ are distinct modulo $p^n$.

Problem 4. Find all functions $f : \mathbb{R} \to \mathbb{R}$ whose derivative is continuous with the property that for each rational number $\frac{a}{b}$, written in lowest terms (i.e., $a, b \in \mathbb{Z}$ with $b \in \mathbb{N}$ and gcd$(a, b) = 1$), we have that also $f \left( \frac{a}{b} \right)$ is a rational number whose denominator, when we write $f(a/b)$ in lowest terms, is also equal to $b$. 
