Math 361 Winter 2001/2002
Assignment 4 (Quiz on Friday, October 5)

1. Solve Problem 9 on p. 63 in the textbook.

2. Consider the following model for a population that experiences intraspecific competition:

$$x(t+1) = \frac{\lambda \cdot x(t)}{[1+x(t)]^b},$$

where $x(t)$ and $x(t+1)$ are population sizes in subsequent year, λ is the maximal number of offspring per individual, i.e. the number of offspring per individual when there is no competition, and b is an additional parameter. (Note that this model differs from the one I used in class!)

   (a) Plot the number of offspring *per individual* as a function of population size for different values of λ and b.

   (b) Plot $x(t+1)$ as a function of $x(t)$ for different values of λ and b.

   (c) Find the carrying capacity of this model.

   (d) Determine the stability conditions for this equilibrium in terms of the parameters λ and b.

   (e) Use the graphical method of cobwebbing to illustrate the dynamics of this population for different starting population sizes and for different values of λ and b.

3. The following problem can be best done using a spreadsheet, but it can also be done using only a pocket calculator. Consider again the model used in problem 1. Fix $b = 10$. For the following parameter values of λ, start with a population size of $x(0) = 0.5$ in year 0 and plot the population sizes $x(t)$ in the next 10 years, i.e. for $t = 1, ...10$. Comment.

   (a) $\lambda = 2$

   (b) $\lambda = 5$

   (c) $\lambda = 10$

   (d) $\lambda = 20$

   (e) $\lambda = 40$

4. The model in problem 1 is usually interpreted in the following way: each year, each individual of the starting population $x(t)$ survives competition with a probability of $1/[1+x(t)]^b$, so that there are $x_s(t) = x(t)/[1+x(t)]^b$ survivors in year t. Each survivor then has λ offspring on average and dies after reproduction. The offspring form the starting population in the next year, so that $x(t+1) = \lambda \cdot x(t)/[1+x(t)]^b$. This allows

one to actually measure the parameter b in the field, as follows. Consider the logarithm of the ratio between initial population size and survivors of competition:

$$\log\left(\frac{x(t)}{x_s(t)}\right).$$

Express this as a function with $\log(x(t))$ as the independent variable, and conclude that for large $x(t)$, $\log\left(\frac{x(t)}{x_s(t)}\right)$ is a linear function of $\log(x(t))$ with slope b. Based on this finding, describe how you would go about measuring b in the field or in the lab.

5. Based on the method of problem 3 and on measuring maximal reproductive rates λ, Hassell et al. (*J. Anim. Ecol.* 45, 471-486, 1976) give the following estimates for the parameters λ and b in the model of problem 1 for several insect populations:

	λ	b
Moth: *Zeiraphera diniana*	1.3	0.1
Bug: *Leptoterna dolobrata*	2.2	2.1
Mosquito: *Aedes aegypti*	10.6	1.9
Potato Beetle: *Lepinotarsa decemlineata*	75.0	3.4
Parasitoid Wasp: *Bracon hebetor*	54.0	0.9

(a) Plot these values on a λ, b-parameter plane. (Recommendation: use a log scale for the λ-axis.)

(b) Use your results from problem 1 to determine which of these species will have a stable equilibrium, and which ones won't.

6. Consider the following graph of $x(t+1)$ as a function of $x(t)$:

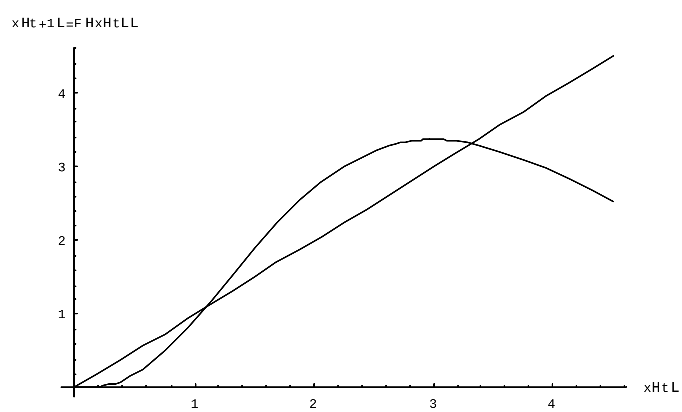

Figure 1: Allee Effect

This population is said to exhibit an *Allee effect*: For very small population sizes, the graph of F lies below the diagonal, which means that at very small population sizes the population is actually decreasing, e.g. because individuals have to spend too much time (and hence too many resources) looking for mates. This is in contrast to the models discussed in class, where populations are always growing exponentially when population sizes are low.

(a) How many non-zero equilibria are there for the dynamics of the population described by the graph in Figure 1?

(b) Which ones of those equilibria are stable?

(c) Suppose the population is at the lower of the equilibria found in (a), and is perturbed away from this equilibrium. What happens? (Hint: use cobwebbing; there are two cases to consider.)

7. Solve problem 13 on p. 64 in the textbook.

8. Solve problem 7 on p. 62 in the textbook.

9. Consider the population genetic model for the frequency dynamics of two genetic types A and B in a clonally (i.e. asexually) reproducing population (cf. class notes):

$$p(t+1) = \frac{p(t) \cdot w_A}{p(t) \cdot w_A + (1 - p(t)) \cdot w_B} = p(t) \cdot \frac{w_A}{\overline{w}(t)},$$

where $p(t)$ is the frequency of type A at time t, $1 - p(t)$ is the frequency of type B at time t, w_A and w_B are the fitnesses of the two types, and $\overline{w}(t) = p(t) \cdot w_A + (1 - p(t)) \cdot w_B$ is the mean fitness at time t. w_A and w_B are assumed to be constant over time.

(a) Show that this model has 2 equilibrium points provided that $w_A \neq w_B$, and that one of them is locally stable, while the other one is not.

(b) Classify the long term dynamic behaviour of this model (i.e., make a list of all possible outcomes).

10. Refresh your knowledge about Taylor series and partial derivatives by reading the Appendix to Chapter 2 in the textbook.