Math 361 Winter 2001/2002
Assignment 6 (Quiz on Monday, November 5)

1. Consider the following differential equation model for the dynamics of a single population:

$$\frac{dN}{dt} = rN\left[1 - (N/K)^{\theta}\right],$$

where $\theta > 0$ is a parameter.

   (a) Graph the growth rate dN/dt as a function of N and do a graphical stability analysis.

   (b) Find the equilibria of the model analytically and determine their stability using linear stability analysis.

2. Consider the following differential equation model for the dynamics of a single population:

$$\frac{dN}{dt} = rN(N-a)\left(1 - N/K\right),$$

where $r, a, K > 0$ and where $a < K$.

   (a) Graph the growth rate dN/dt as a function of N and do a graphical stability analysis.

   (b) Find the equilibria of the model analytically and determine their stability using linear stability analysis.

   (c) Discuss the behavior of this model and contrast it with the behaviour of the simple logistic equation. (Note: the model given is analogous to the Allee effect discussed in an earlier homework for difference equations.)

3. Consider the following differential equation model for the dynamics of a single population in which individuals are harvested at a rate H:

$$\frac{dN}{dt} = rN\left[1 - N/K\right] - HN.$$

Here $H > 0$ is a parameter that represents the per capita harvesting rate.

   (a) Find the equilibria of the model analytically and determine their stability using linear stability analysis. What restriction on H is necessary for the existence of an equilibrium $N^* > 0$?

   (b) What happens to the population when H is larger than the value found in (a)?

4. Consider the following differential equation model for the dynamics of a single population with predation:

$$\frac{dN}{dt} = N\,[1 - N] - \frac{H \cdot N}{0.1 + N}.$$

Here the per capita harvesting rate $H/(0.1 + N)$ is a function of population size N. The dynamics of this system are determined by the parameter $H > 0$.

   (a) Plot the per capita harvesting rate a function of N.

   (b) Find the equilibria of the model analytically and determine their stability using linear stability analysis.

   (c) Compare this model to the model of question 3. Which one is more realistic?

5. Solve problem 2 on p. 152 in the textbook.

6. Solve problem 3 on p. 152 in the textbook.

7. Solve problem 4 on p. 152 in the textbook.

8. Solve problem 5 on p. 152 in the textbook.

9. Solve problem 13 on p. 154 in the textbook.

10. Solve problem 18 on p. 155 in the textbook.