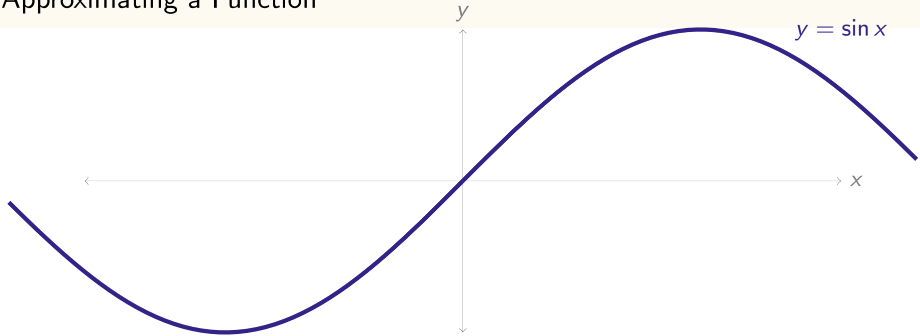
Approximating a Function



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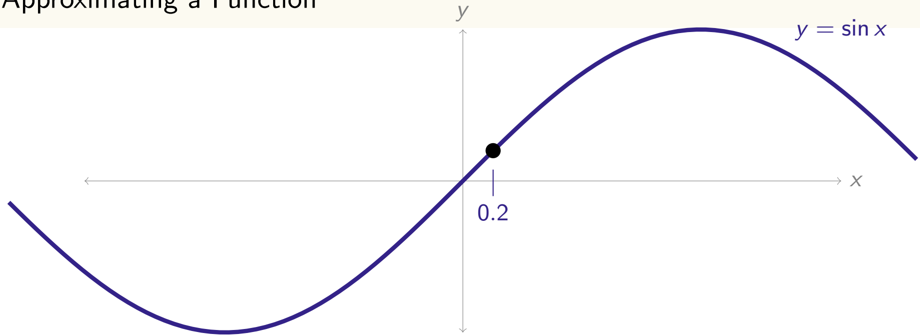

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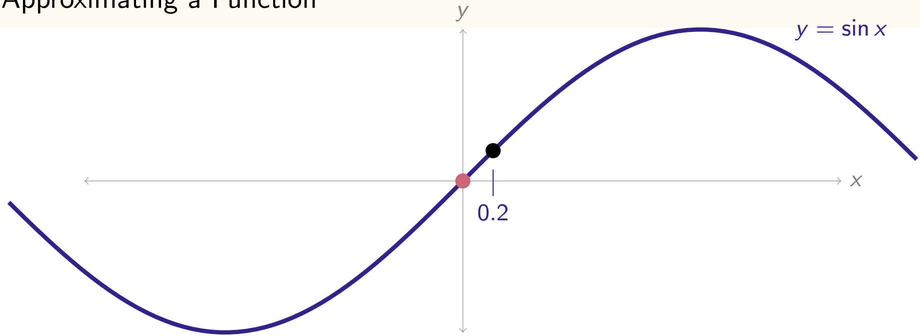

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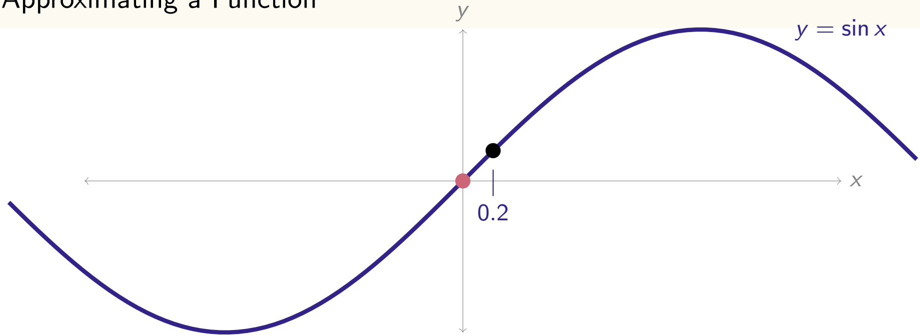

Constant Approximation

We can approximate $f(x)$ near a point a by

$$f(x) \approx f(a)$$

Constant approx: $\sin(0.2) \approx 0$;

Google: $\sin(0.2) = 0.19866933079\ldots$

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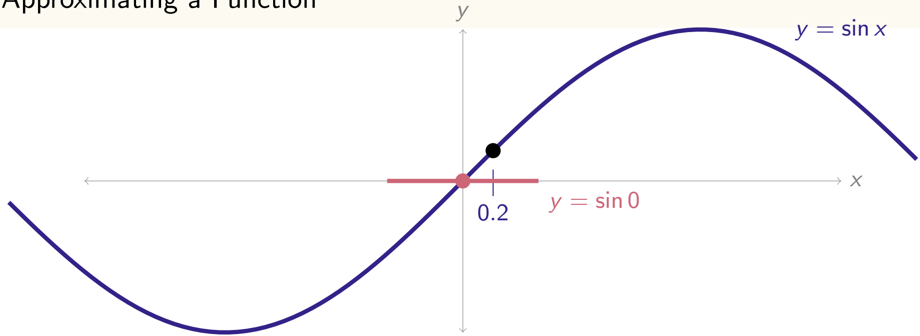

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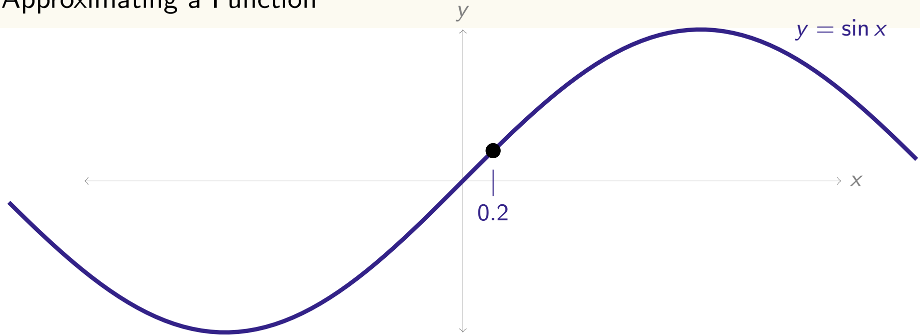

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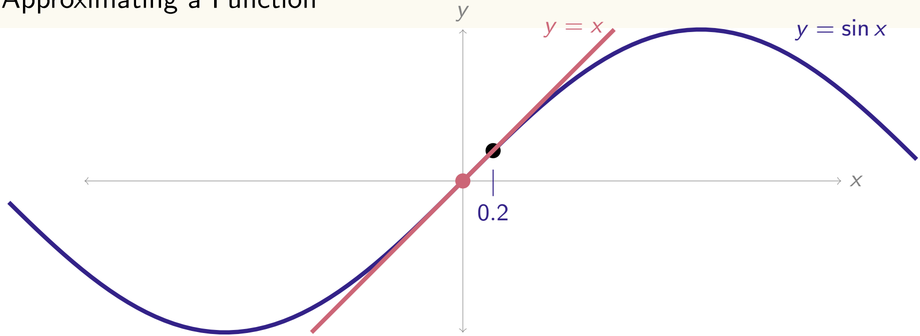

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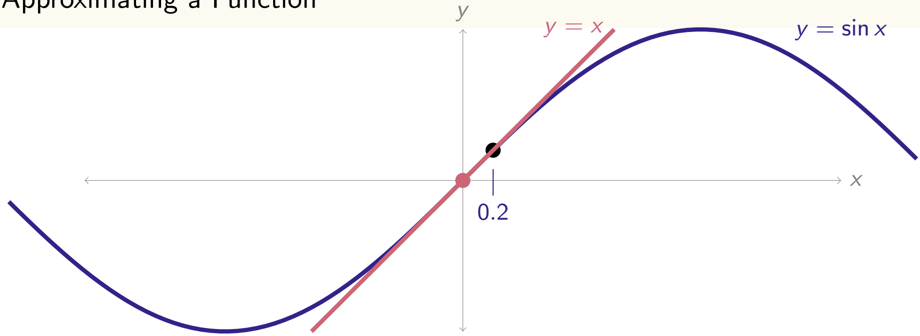

Linear Approximation (Linearization)

We can approximate $f(x)$ near a point a by the tangent line to $f(x)$ at a, namely

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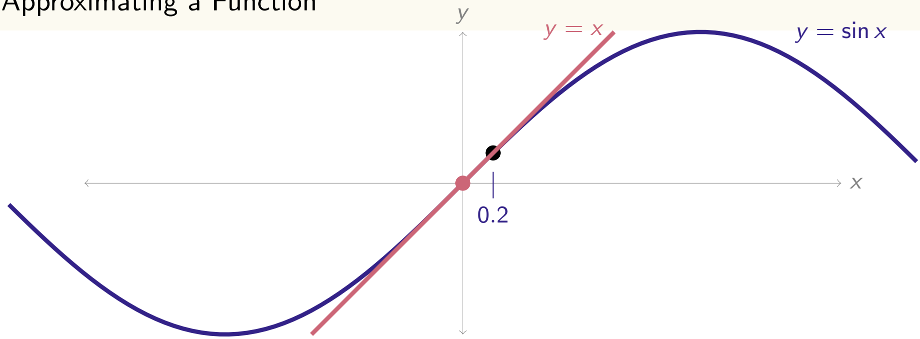

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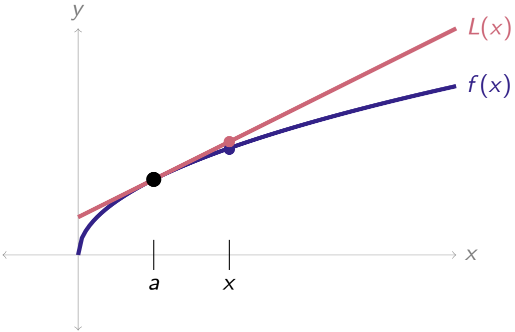

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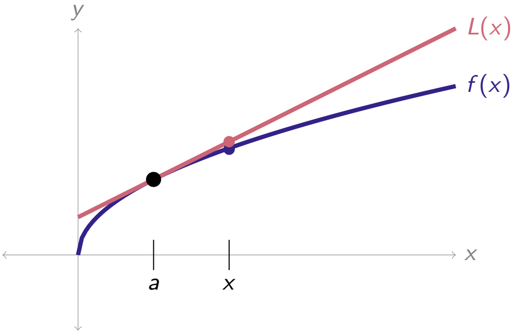

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Google: $\sqrt{8.9} = 2.98328677804...$

Characteristics of a Good Approximation

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Accurate

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Accurate

Possible to calculate (add, subtract, multiply, divide integers)

Can we Compute?

Suppose we want to approximate the value of $\cos(1.5)$. Which of the following linear approximations could we calculate by hand? (You can leave things in terms of π.)

A. tangent line to $f(x) = \cos x$ when $x = \pi/2$

B. tangent line to $f(x) = \cos x$ when $x = 3/2$

C. both

D. neither

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A. tangent line to $f(x) = \cos x$ when $x = \pi/2$

B. tangent line to $f(x) = \cos x$ when $x = \pi/4$

C. constant approximation: $\cos 1.5 \approx \cos \pi/2 = 0$

D. the linear approximations should be better than the constant approximation, but both linear approximations should have the same accuracy

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$\pi/2$ is very close to 1.5.

Linear Approximation

Example: Approx 2

Approximate sin(3) using a linear approximation. It is OK to use π in your answer.

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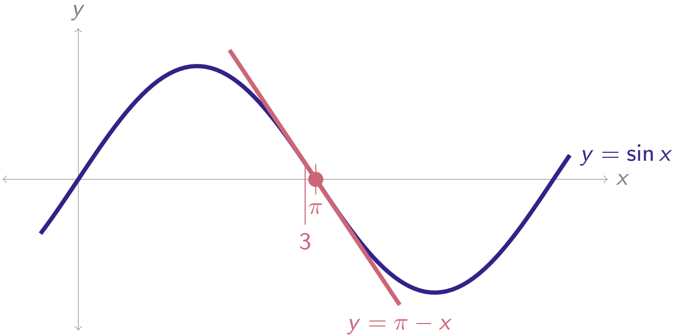
Approximate sin(3) using a linear approximation. It is OK to use π in your answer.
Let $f(x) = \sin x$ and $a = \pi$.
$f(3) \approx f(\pi) + f'(\pi)(3-\pi) = \sin(\pi) + \cos(\pi)(3-\pi) = \boxed{\pi - 3} \approx 0.14159$



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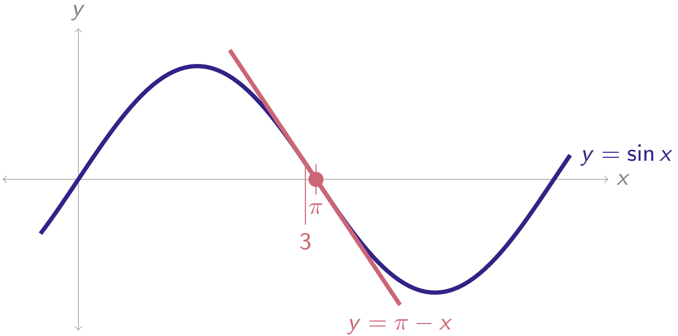

Google: $\sin(3) = 0.14112000806...$

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Example: Approx 3

Approximate $e^{1/10}$ using a linear approximation.

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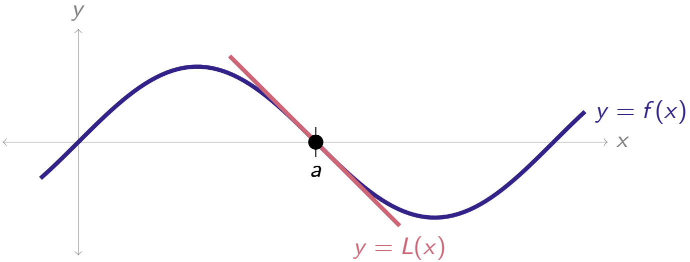
If $g(x) = x^{1/10}$:
$g'(x) = \frac{1}{10}x^{-9/10}$.
The closest number to e for which we can evaluate the tenth root is $a = 1$.
$g(e) \approx g(1) + g'(1)(e - 1) = 1 + \frac{1}{10}(-e - 1) = \frac{e+9}{10}$... but what's e?

Google: $e^{1/10} = 1.10517091808...$

Linear Approximation

Let $L(x) = f(a) + f'(a)(x - a)$, so $L(x)$ is the linear approximation (linearization) of $f(x)$ at a.



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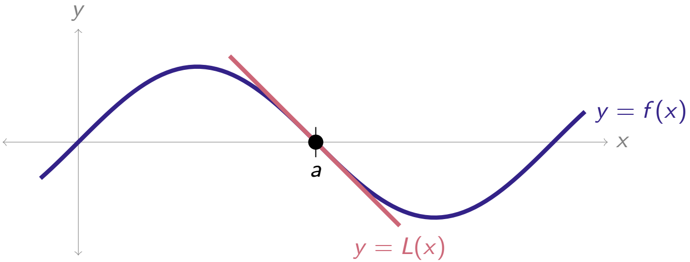

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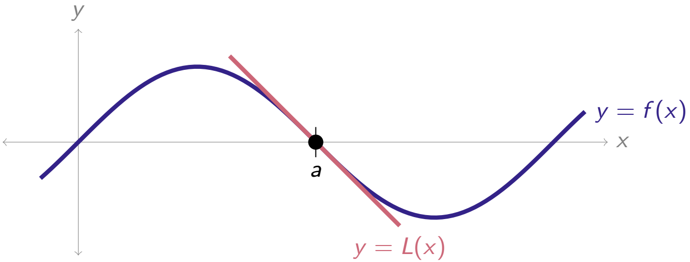

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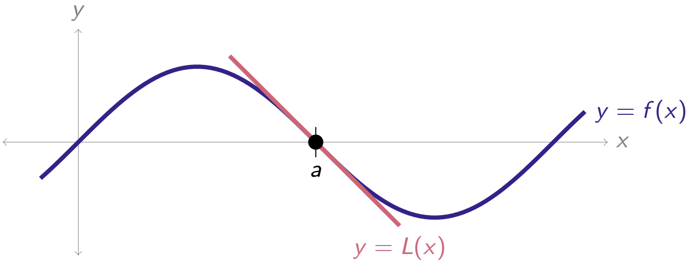

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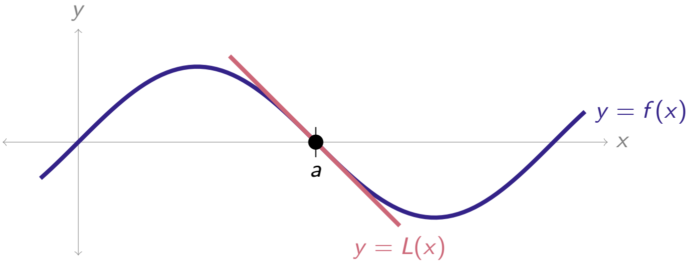

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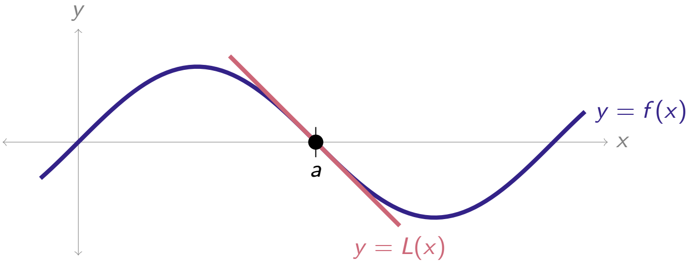

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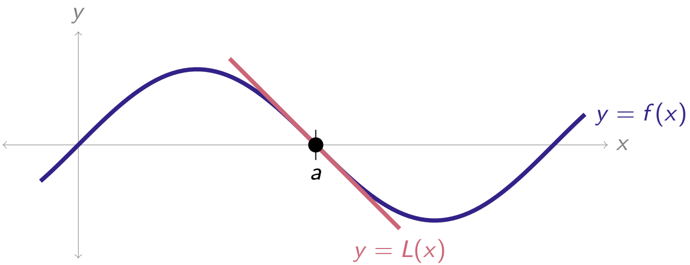

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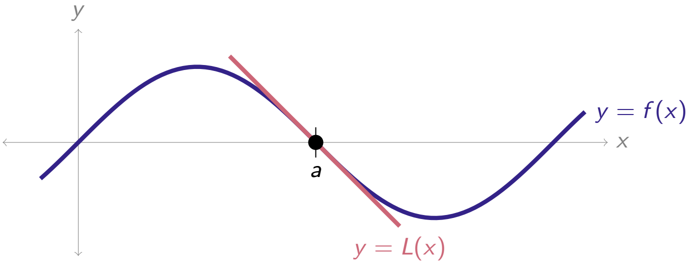
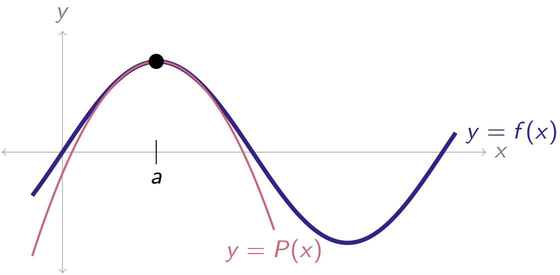


Recall: $L(x)$ is a *line*

Quadratic Approximation

Imagine we approximate $f(x)$ at a with a parabola $P(x)$.



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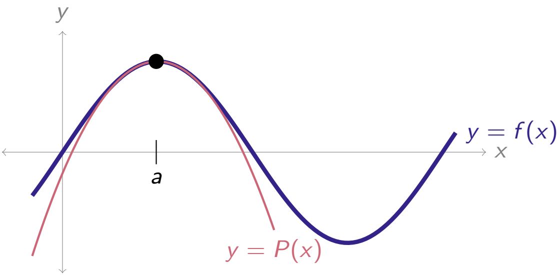

Then we can ensure:

$$P(a) = f(a)$$
$$P'(a) = f'(a), \text{ and}$$
$$P''(a) = f''(a).$$

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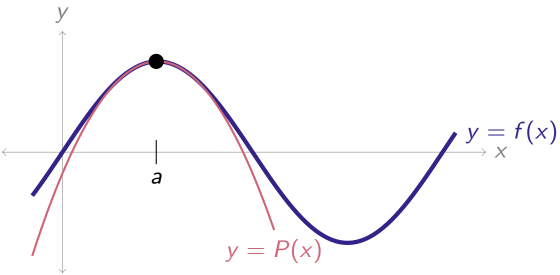

$P(x) = A + Bx + Cx^2$	$P(a) = A + Ba + Ca^2$	$f(a)$
$P'(x) = B + 2Cx$	$P'(a) = B + 2Ca$	$f'(a)$
$P''(x) = 2C$	$P''(a) = 2C$	$f''(a)$

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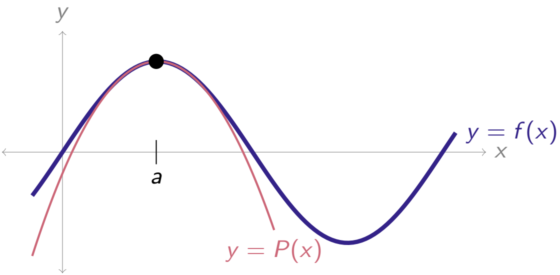

$$P(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$

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Quadratic Approximation

Constant:	$f(x) \approx f(a)$
Linear:	$f(x) \approx f(a) + f'(a)(x-a)$
Quadratic:	$f(x) \approx f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$

Quadratic Approximation

$$P(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$

Example: Approx 4

Approximate $\ln(1.1)$ using a quadratic approximation.

Quadratic Approximation

$$P(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$

Example: Approx 4

Approximate $\ln(1.1)$ using a quadratic approximation.

We use $f(x) = \ln x$ and $a = 1$. Then $f'(x) = x^{-1}$ and $f''(x) = -x^{-2}$, so $f(a) = 0$, $f'(a) = 1$, and $f''(a) = -1$. Now:

$$\begin{aligned} f(1.1) &\approx f(a) + f'(a)(1.1-a) + \frac{1}{2}f''(a)(1.1-a)^2 \\ &= 0 + 1(1.1-1) + \frac{1}{2}(-1)(1.1-1)^2 \\ &= 0.1 - \frac{1}{200} = \frac{20}{200} - \frac{1}{200} = \frac{19}{200} = \frac{9.5}{100} = 0.095 \end{aligned}$$

Google: $\ln(1.1) = 0.0953101798...$

Quadratic Approximation

$$P(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$

Example: Approx 5

Approximate $\sqrt[3]{28}$ using a quadratic approximation. You may leave your answer unsimplified, as long as it is an expression you could figure out using only plus, minus, times, and divide.

Quadratic Approximation

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We use $f(x) = x^{1/3}$ and $a = 27$. Then $f'(x) = \frac{1}{3}x^{-2/3}$ and $f''(x) = \frac{-2}{9}x^{-5/3}$. So, $f(a) = 3$, $f'(a) = \frac{1}{3^3}$, and $f''(a) = \frac{-2}{3^7}$.

$$\begin{aligned} f(28) &\approx f(27) + f'(27)(28-27) + \frac{1}{2}f''(27)(28-27)^2 \\ &= 3 + \frac{1}{3^3}(1) + \frac{-1}{3^7}(1^2) \\ &= 3 + \frac{1}{3^3} - \frac{1}{3^7} \\ &= 3.03657978967\ldots \\ \textit{Google}: \sqrt[3]{28} &= 3.03658897188\ldots \end{aligned}$$

Example: Approx 6

Determine what $f(x)$ and a should be so that you can approximate the following using a quadratic approximation.

$\ln(.9)$

$e^{-1/30}$

$\sqrt[5]{30}$

$(2.01)^6$

Example: Approx 6

Determine what $f(x)$ and a should be so that you can approximate the following using a quadratic approximation.

$\ln(.9)$
$f(x) = \ln(x)$, $a = 1$

$e^{-1/30}$
$f(x) = e^x$, $a = 0$

$\sqrt[5]{30}$
$f(x) = \sqrt[5]{x}$, $a = 32 = 2^5$

$(2.01)^6$
$f(x) = x^6$, $a = 2$. It is possible to compute this without an approximation, but an approximation in this case might save time, while being sufficiently accurate for your purposes.

Sum Notation

$$\sum_{n=17}^{20} g(n) = g(17) + g(18) + g(19) + g(20)$$

We let n take every *integer* value from 17 to 20 (including 17 and 20), and *sum* the values of $g(n)$.

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$$= \overbrace{a(-1) + 5}^{n=-1} + \overbrace{a(0) + 5}^{n=0} + \overbrace{a(1) + 5}^{n=1} + \overbrace{a(2) + 5}^{n=2} + \overbrace{a(3) + 5}^{n=3}$$

$$5(5) + (-a) + 0 + a + 2a + 3a = \boxed{25 + 5a}$$

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The answer is zero!

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