Derivatives of Products

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True or False:

$$\begin{aligned}\frac{d}{dx}\{2x\} &= \frac{d}{dx}\{x+x\}\\ &= [1]+[1]\\ &= 2\end{aligned}$$

True or False:

$$\begin{aligned}\frac{d}{dx}\left\{x^2\right\} &= \frac{d}{dx}\{x\cdot x\}\\ &= [1]\cdot[1]\\ &= 1\end{aligned}$$

What to do with Products?

Suppose $f(x)$ and $g(x)$ are differentiable functions of x.

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$$= f(x)g'(x) + g(x)f'(x)$$

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$$\frac{d}{dx}\left[x^2\right] = \frac{d}{dx}[x \cdot x] = x(1) + x(1) = 2x$$

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Example: suppose $f(x) = 3x^2$, $f'(x) = 6x$, $g(x) = \sin(x)$, $g'(x) = \cos(x)$.

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$$\frac{d}{dx}\left[3x^2\sin(x)\right] = 3x^2\cos(x) + \sin(x)6x.$$

Now You

$$f(x) = (2x+5)\ln(x^2)$$

Where

- $\frac{d}{dx}[2x+5] = 2$,
- $\frac{d}{dx}\left[\ln(x^2)\right] = \frac{2}{x}$, and
- $\frac{d}{dx}\left[x^2\right] = 2x$.

A. $f'(x) = (2)\left(\frac{2}{x}\right)(2x)$

B. $f'(x) = 2(2x) + 2x(2)$

C. $f'(x) = (2x+5)(2) + \ln(x^2)\left(\frac{2}{x}\right)$

D. $f'(x) = (2x+5)\left(\frac{2}{x}\right) + \ln(x^2)(2)$

E. none of the above

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Now You (Again)

$$f(x) = a(x) \cdot b(x) \cdot c(x)$$

What is $f'(x)$?

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$$f(x) = a(x) \cdot b(x) \cdot c(x)$$

What is $f'(x)$?

$$\begin{aligned} f(x) &= [a(x)b(x)]\, c(x) \\ f'(x) &= [a(x)b(x)]\, c'(x) + c(x)\frac{d}{dx}\{a(x)b(x)\} \\ &= a(x)b(x)c'(x) + c(x)\left[a(x)b'(x) + a'(x)b(x)\right] \\ &= a(x)b(x)c'(x) + a(x)b'(x)c(x) + a(x)b(x)c'(x) \end{aligned}$$

Derivatives of Ratios

Quotient Rule

Let $f(x)$ and $g(x)$ be differentiable and $g(x) \neq 0$. Then:

$$\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$$

Mnemonic: Low D'high minus high D'low over lowlow.

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Example: $\frac{d}{dx}\left\{\frac{2x+5}{3x-6}\right\} =$

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Now you: $\frac{d}{dx}\left\{\frac{5x}{\sqrt{x}-1}\right\} =$

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Now you: $\frac{d}{dx}\left\{\frac{5x}{\sqrt{x}-1}\right\} = \frac{(\sqrt{x}-1)(5)-(5x)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}-1)^2} = \frac{\frac{5}{2}\sqrt{x}-5}{(\sqrt{x}-1)^2}$

Rules

Product: $\frac{d}{dx}\{f(x)g(x)\} = f(x)g'(x) + g(x)f'(x)$

Quotient: $\dfrac{d}{dx}\left\{\dfrac{f(x)}{g(x)}\right\} = \dfrac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$

Practice! Differentiate the following.

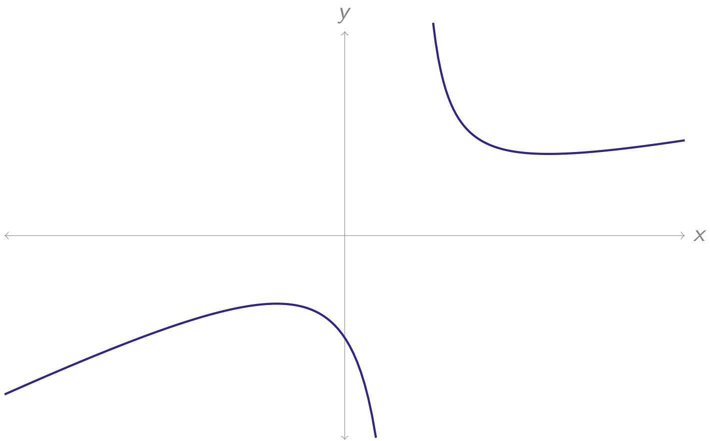
$f(x) = 2x + 5$

$g(x) = (2x + 5)(3x - 7)$

$h(x) = (2x + 5)(3x - 7) + 25$

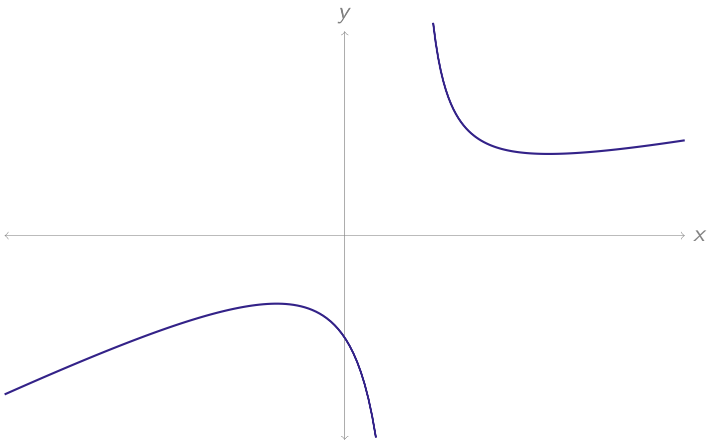
$j(x) = \dfrac{2x + 5}{8x - 2}$

$k(x) = \left(\dfrac{2x + 5}{8x - 2}\right)^2$



Above is a sketch of the function $f(x) = \dfrac{x^2+3}{x-1}$.

For which values of x is the tangent line horizontal?



Above is a sketch of the function $f(x) = \dfrac{x^2+3}{x-1}$.
For which values of x is the tangent line horizontal?

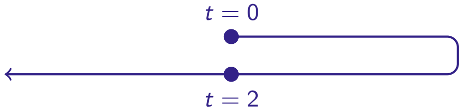
$$f'(x) = \frac{(x-1)(2x)-(x^2+3)(1)}{(x-1)^2} = \frac{(x-3)(x+1)}{(x-1)^2}$$

$x = -1$, $x = 3$

The position of an object moving left and right at time t, $t \geq 0$, is given by

$$s(t) = -t^2(t-2)$$

where a positive position means it is to the right of its starting position, and a negative position means it is to the left. At time $t = 0$, the object it at its starting position. First it moves to the right, then it moves to the left forever.



What is the farthest point to the right that the object reaches?

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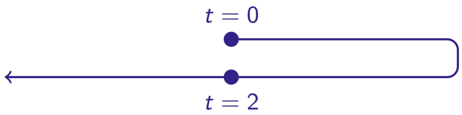

What is the farthest point to the right that the object reaches?

When the object turns to come back around, $s'(t) = 0$. If we can find the value of t that makes this true, then we plug it in to $s(t)$ to find the farthest to the right reached by the object.

$$s'(t) = [-t^2](1) + (-2t)(t-2) = -3t^2 + 4t = t(4-3t)$$

So, the object turns around when $t = \frac{4}{3}$.

Its position at that time is $s\left(\frac{4}{3}\right) = \frac{32}{27}$ units to the right of its starting position.

More About the Product Rule

$\frac{d}{dx}\{x^2\} = \frac{d}{dx}\{x \cdot x\}$

function	derivative
x	1

More About the Product Rule

$\frac{d}{dx}\{x^2\} = \frac{d}{dx}\{x \cdot x\} = x(1) + x(1)$

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With functions raised to a power, it's more complicated.

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With functions raised to a power, it's more complicated.
Example: differentiate $(2x+1)^2$.

$$\begin{aligned}\frac{d}{dx}\left\{(2x+1)^2\right\} &= \frac{d}{dx}\{(2x+1)(2x+1)\}\\ &= (2x+1)(2) + (2x+1)(2) = 4(2x+1)\end{aligned}$$

Power Rule

$$\frac{d}{dx}\{x^n\} = nx^{n-1}$$

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Differentiate $\dfrac{(x^4+1)(\sqrt[3]{x}+\sqrt[4]{x})}{2x+5}$.

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Differentiate $\dfrac{(x^4+1)(\sqrt[3]{x}+\sqrt[4]{x})}{2x+5}$.

$$\frac{d}{dx}\left\{\frac{(x^4+1)(\sqrt[3]{x}+\sqrt[4]{x})}{2x+5}\right\} = \frac{(2x+5)\cdot\frac{d}{dx}\left\{(x^4+1)(\sqrt[3]{x}+\sqrt[4]{x})\right\} - (x^4+1)(\sqrt[3]{x}+\sqrt[4]{x})(2)}{(2x+5)^2}$$

$$= \frac{(2x+5)\left[(x^4+1)\left(\frac{1}{3}x^{-2/3}+\frac{1}{4}x^{-3/4}\right)+4x^3(\sqrt[3]{x}+\sqrt[4]{x})\right]}{(2x+5)^2} - \frac{2(x^4+1)(\sqrt[3]{x}+\sqrt[4]{x})}{(2x+5)^2}$$

Suppose a motorist is driving their car, and their position is given by $s(t) = 10t^3 - 90t^2 + 180t$ kilometres. At $t = 1$ (t measured in hours), a police officer notices they are driving erratically. The motorist claims to have simply suffered a lack of attention: they were in the act of pressing the brakes even as the officer noticed their speed.

At $t = 1$, how fast was the motorist going, and were they pressing the gas or the brake?

What about at $t = 2$?

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At $t = 1$, how fast was the motorist going, and were they pressing the gas or the brake?
Velocity is the rate of change of position, so velocity of the car is given by:

$$s'(t) = 30t^2 - 180t + 180$$

When $t = 1$, $s'(1) = 30$, so the motorist was going 30 kph.

$$s''(t) = 60t - 180$$

When $t = 1$, the velocity of the car was changing by $s''(t) = -120$ kph per hour. Since the velocity was positive, but its rate of change is negative, the car was decelerating when $t = 1$.

What about at $t = 2$?

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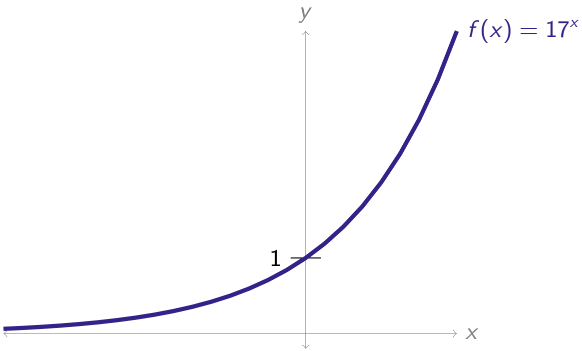
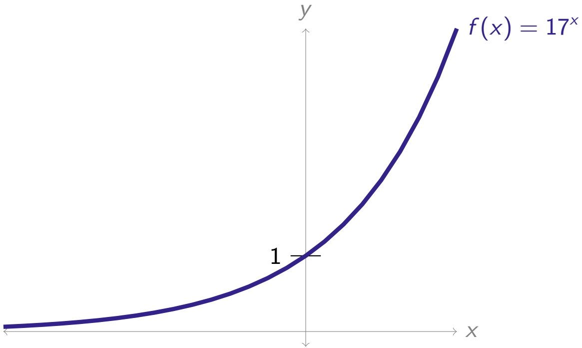
What about at $t = 2$?
$s'(2) = -60$, so the motorist is driving 60 kph.
$s''(2) = -60$, so the motorist's velocity is becoming increasingly more negative. Since it was negative to begin with, they are accelerating.

Recall that a sphere of radius r has volume $V = \frac{4}{3}\pi r^3$.
Suppose you are winding twine into a gigantic twine ball, filming the process, and trying to make a viral video. You can wrap one cubic meter of twine per hour. (In other words, when we have V cubic meters of twine, we're at time V hours.) How fast is the radius of your spherical twine ball increasing?

Exponential Functions

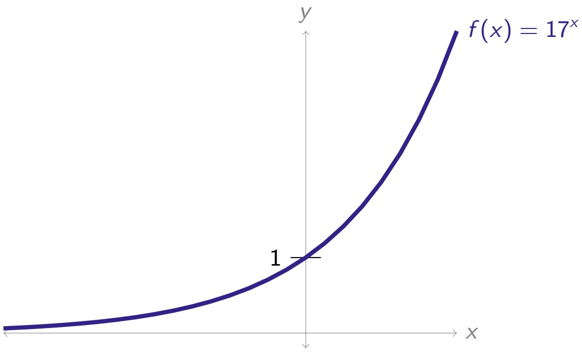


Exponential Functions



Consider $\frac{d}{dx}\left\{17^x\right\}$.

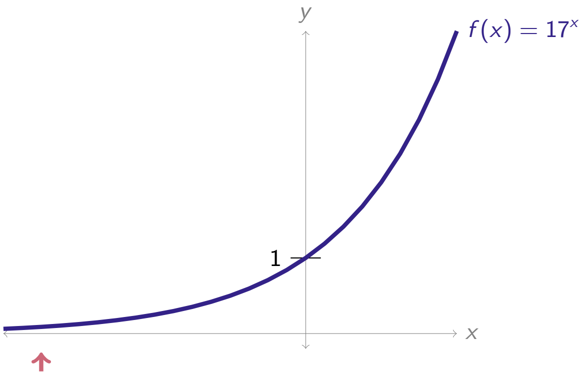
Exponential Functions



Consider $\frac{d}{dx}\{17^x\}$.

$f(x)$ is always increasing, so $f'(x)$ is always positive.

Exponential Functions



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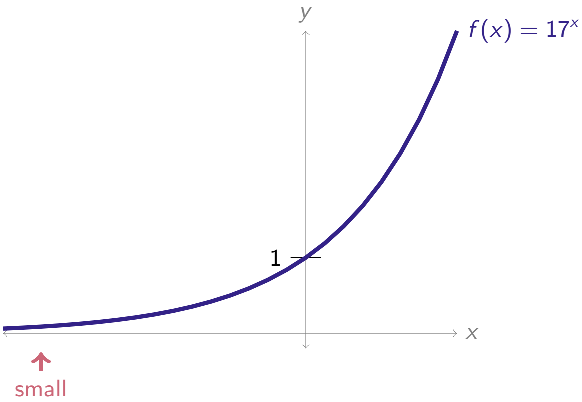

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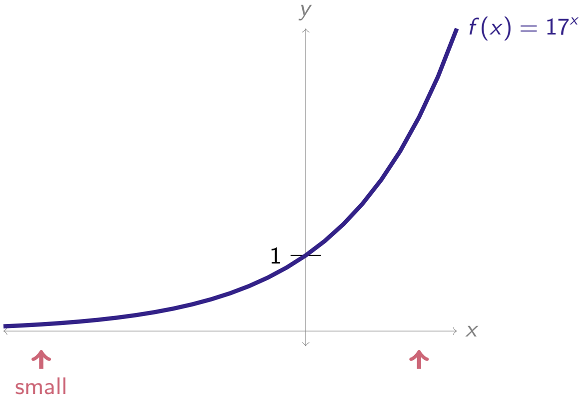

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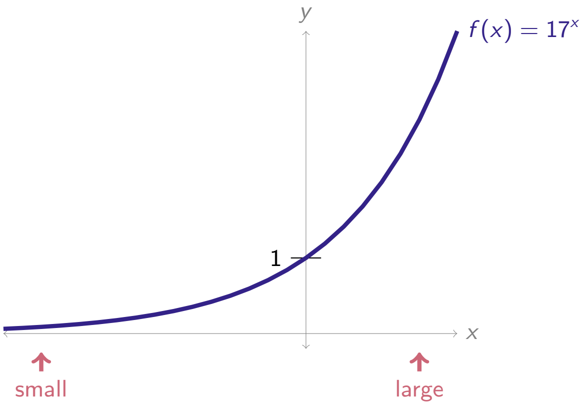

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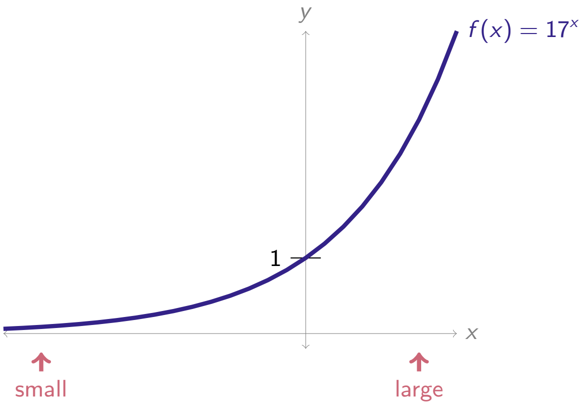

Consider $\frac{d}{dx}\{17^x\}$.
$f(x)$ is always increasing, so $f'(x)$ is always positive.
$f'(x)$ might look similar to $f(x)$.

Exponential Functions

$$\frac{d}{dx}\{17^x\} = \lim_{h \to 0} \frac{17^{x+h} - 17^x}{h}$$

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Exponential Functions

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Given what you know about $\frac{d}{dx}\{17^x\}$, **is it possible** that $\lim\limits_{h\to 0} \dfrac{17^h - 1}{h} = 0$?

A. Sure, there's no reason we've seen that would make it impossible.

B. No, it couldn't be 0, that wouldn't make sense.

C. I do not feel equipped to answer this question.

Exponential Functions

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$$= 17^x(\text{times a constant})$$

How could we find out what this limit is?

Exponential Functions

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h	$\frac{17^h - 1}{h}$
0.001	2.83723068608
0.00001	2.83325347992
0.0000001	2.83321374583
0.000000001	2.83321344163

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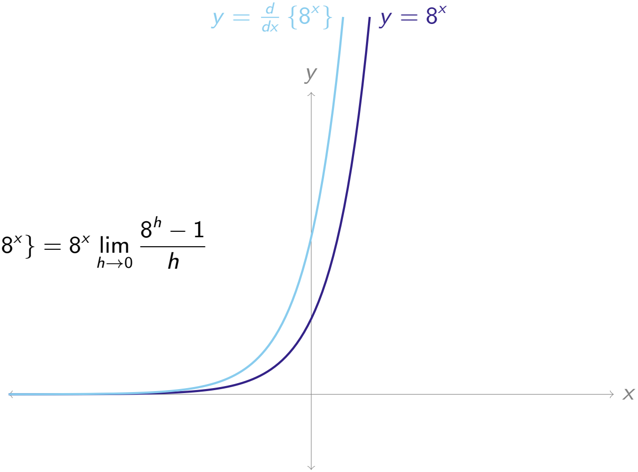
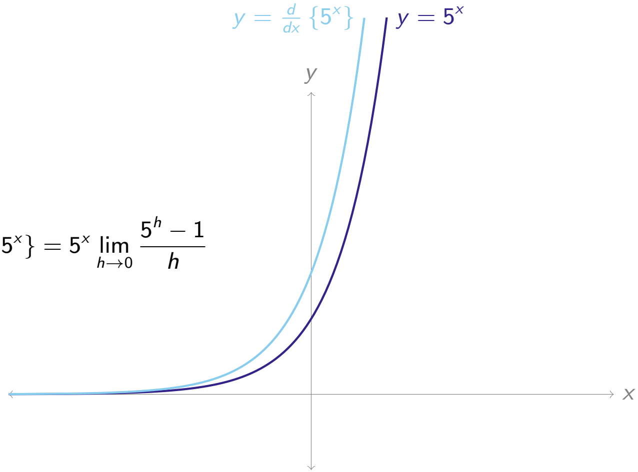
In general, $\frac{d}{dx}\{a^x\} = a^x \lim_{h\to 0}\frac{a^h-1}{h}$ for any positive number a.

Exponential Functions



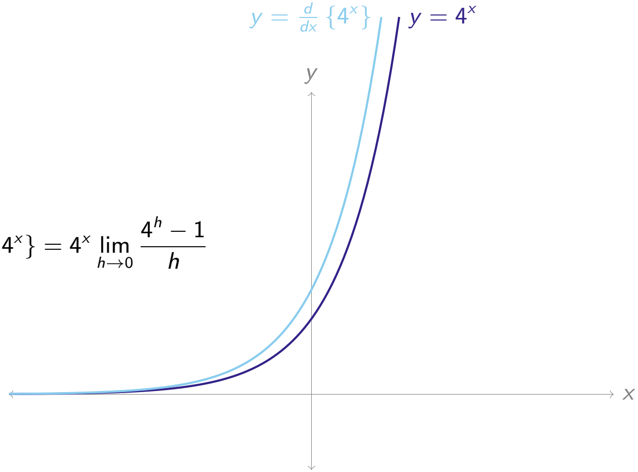
$$\frac{d}{dx}\left\{8^x\right\} = 8^x \lim_{h\to 0} \frac{8^h - 1}{h}$$

Exponential Functions



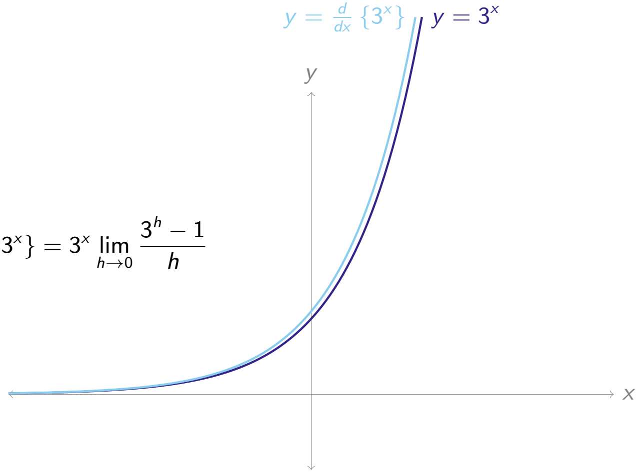
$$\frac{d}{dx}\{5^x\} = 5^x \lim_{h \to 0} \frac{5^h - 1}{h}$$

Exponential Functions



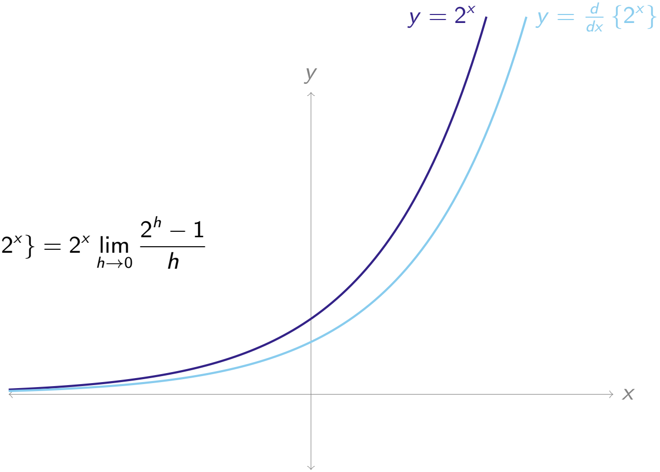
$$\frac{d}{dx}\{4^x\} = 4^x \lim_{h \to 0} \frac{4^h - 1}{h}$$

Exponential Functions



$$\frac{d}{dx}\{3^x\} = 3^x \lim_{h\to 0} \frac{3^h - 1}{h}$$

Exponential Functions



$$\frac{d}{dx}\left\{2^x\right\} = 2^x \lim_{h\to 0} \frac{2^h - 1}{h}$$

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In general, $\frac{d}{dx}\{a^x\} = a^x \lim_{h \to 0} \frac{a^h - 1}{h}$ for any positive number a.

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We define e to be the unique number satisfying $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

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Derivatives of Exponential Functions

Using the definition of e,

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In general, $\lim_{h\to 0} \frac{a^h - 1}{h} = \ln(a)$, so $\frac{d}{dx}\{a^x\} = a^x \ln(a)$ [proof]

Quick Practice

Things to Have Memorized

$$\frac{d}{dx}\left\{e^x\right\} = e^x$$

When a is any constant,

$$\frac{d}{dx}\left\{a^x\right\} = a^x \log_e(a)$$

Let $f(x) = \dfrac{e^x}{3x^5}$. When is the tangent line to $f(x)$ horizontal?

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Horizontal tangent line $\Leftrightarrow$ slope of tangent line is zero $\Leftrightarrow$ $f'(x) = 0$

$$0 = f'(x) = \frac{3x^5 e^x - e^x(15x^4)}{(3x^5)^2} = \left(\frac{e^x}{9x^{10}}\right)\left(3x^4\right)(x-5)$$

$$x = 0 \text{ or } x = 5$$

But, since $f(x)$ is not defined at zero, the tangent line is only horizontal at

$$x = 5$$

Evaluate $\frac{d}{dx}\left\{e^{3x}\right\}$

Suppose the deficit, in millions, of a fictitious country is given by

$$f(x) = e^x(4x^3 - 12x^2 + 14x - 4)$$

where x is the number of years since the current leader took office. Suppose the leader has been in power for exactly two years.

1. Is the deficit increasing or decreasing?

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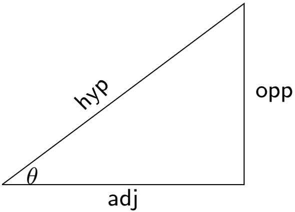
1. Is the deficit increasing or decreasing?

2. Is the rate at which the deficit is growing increasing or decreasing?

Trig Functions: Notation

Basic Trig Functions



$$\sin(\theta) = \frac{opp}{hyp}; \qquad \cos(\theta) = \frac{adj}{hyp}; \qquad \tan(\theta) = \frac{opp}{adj};$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}; \qquad \sec(\theta) = \frac{1}{\cos(\theta)}; \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Trig Facts

Commonly used facts:

- Graphs of sine, cosine, tangent

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- Identities: $\sin^2 x + \cos^2 x = 1$; $\tan^2 x + 1 - \sec^2 x$;
  $\sin^2 x = \dfrac{1 - \cos(2x)}{2}$; $\cos^2 x = \dfrac{1 + \cos 2x}{2}$

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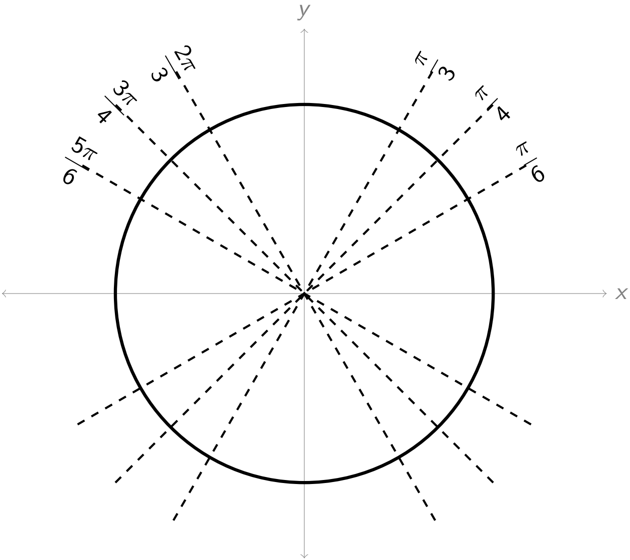
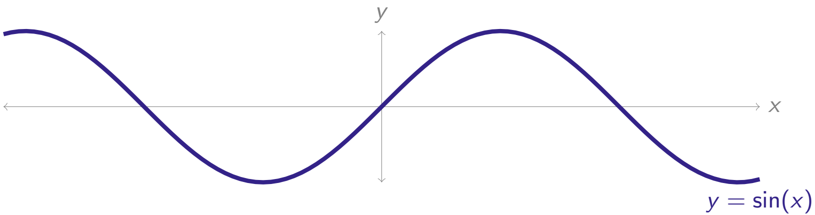
- Identities: $\sin^2 x + \cos^2 x = 1$; $\quad \tan^2 x + 1 - \sec^2 x$;
  $\sin^2 x = \dfrac{1 - \cos(2x)}{2}$; $\quad \cos^2 x = \dfrac{1 + \cos 2x}{2}$
- Conversion between radians and degrees

CLP notes has an appendix on high school trigonometry that you should be familiar with.

Trig Facts



Derivative of Sine



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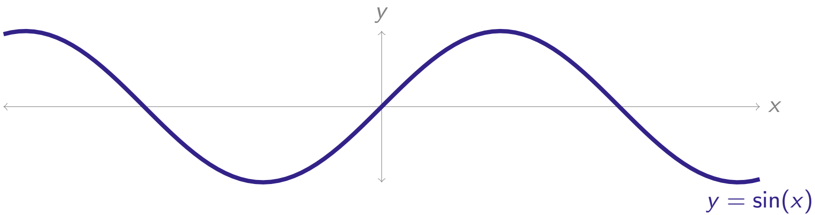

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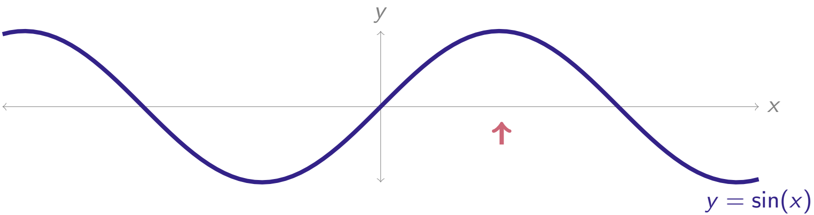

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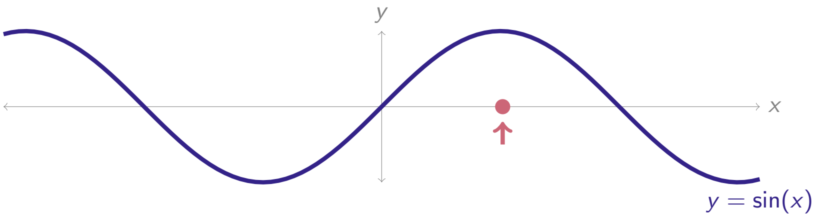

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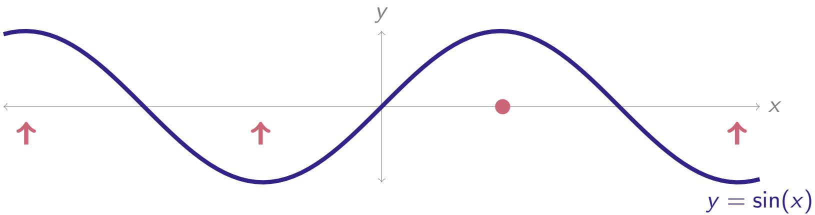

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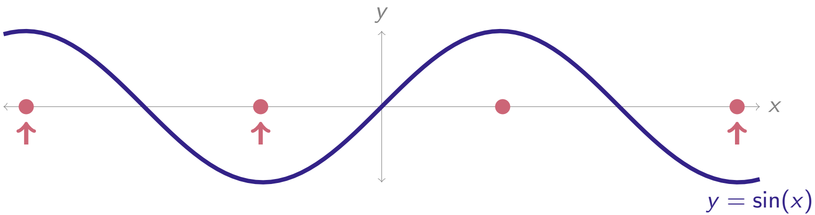

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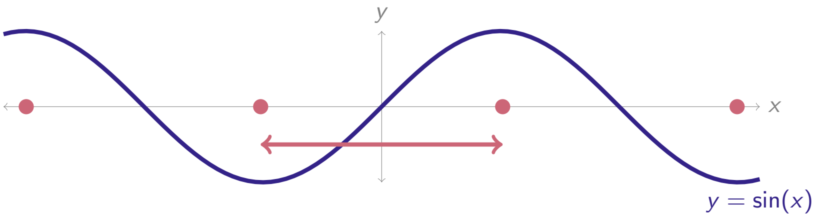
Derivative of Sine



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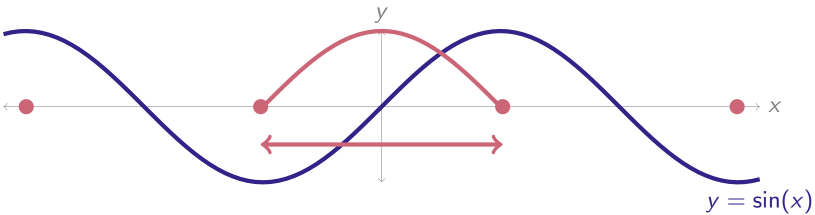

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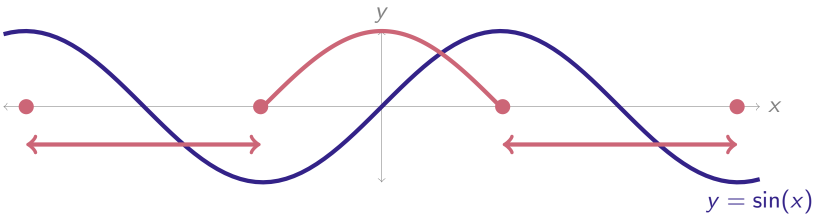

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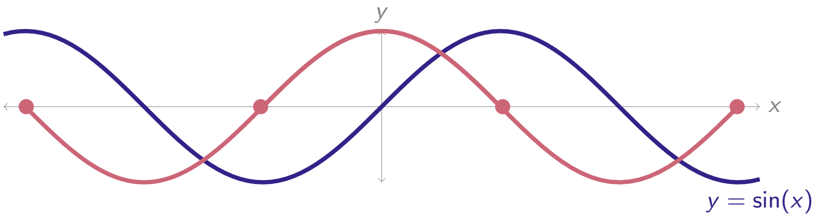

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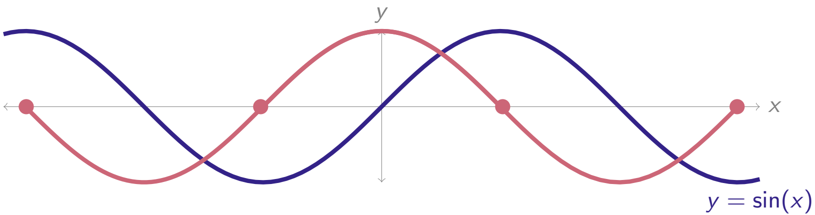

Consider the derivative of $f(x) = \sin(x)$.

$$\frac{d}{dx}\{\sin(x)\} = \cos(x).$$

Derivative of Sine

$$\frac{d}{dx}\{\sin x\} = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

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$$\tfrac{d}{dx}\{\sin x\} = \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$$

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$$= \lim_{h\to 0} \frac{\sin(x)(\cos(h)-1)+\cos(x)\sin(h)}{h}$$

Derivative of Sine

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Derivative of Sine

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Derivative of Sine

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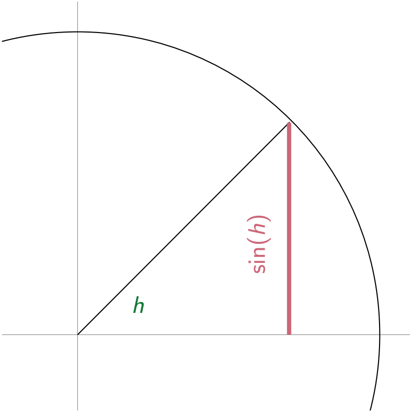
$$= \sin(x) \frac{d}{dx}\{\cos(x)\}\Big|_{x=0} + \cos(x) \lim_{h\to 0} \frac{\sin(h)}{h} \qquad = \qquad \boxed{\cos(x) \lim_{h\to 0} \frac{\sin(h)}{h}}$$

$$\frac{\sin h}{h}$$



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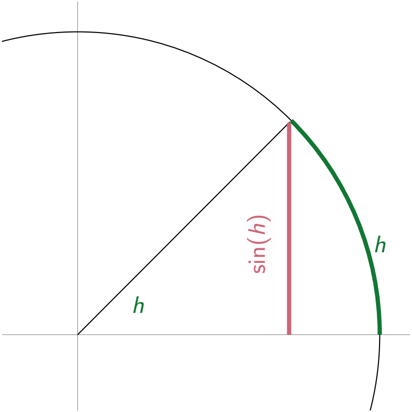

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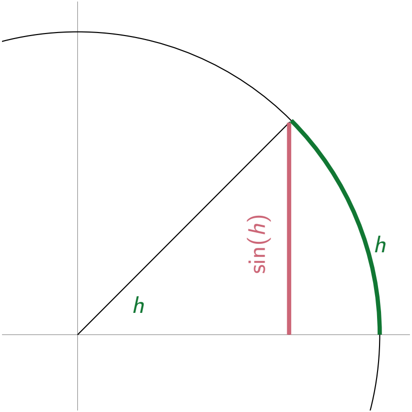
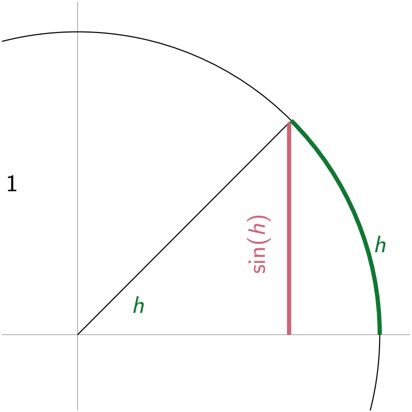


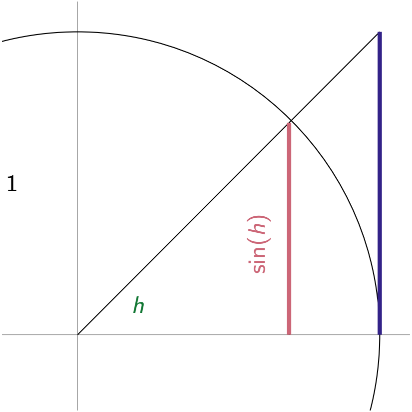
$\sin(h) \leq h$

$$\frac{\sin h}{h} \leq 1$$



$\sin(h) \leq h$ so $\boxed{\frac{\sin(h)}{h} \leq 1}$

$$\frac{\sin h}{h} \leq 1$$



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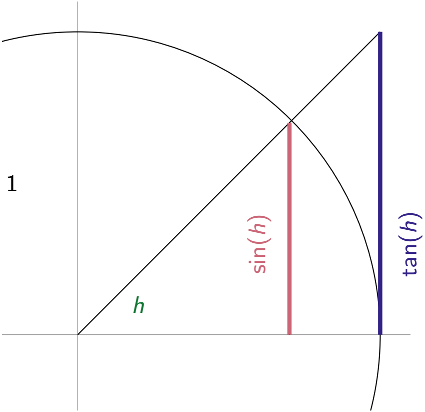

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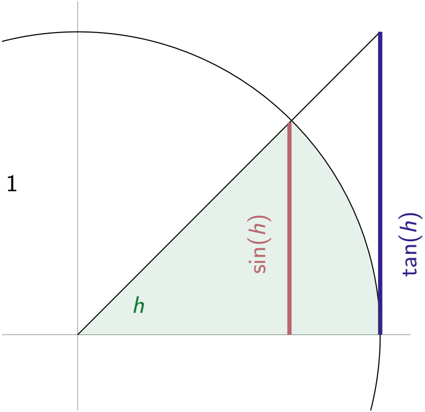

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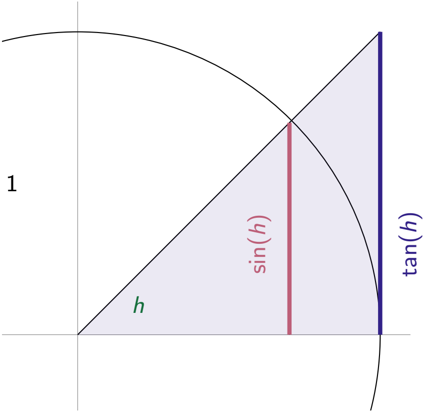

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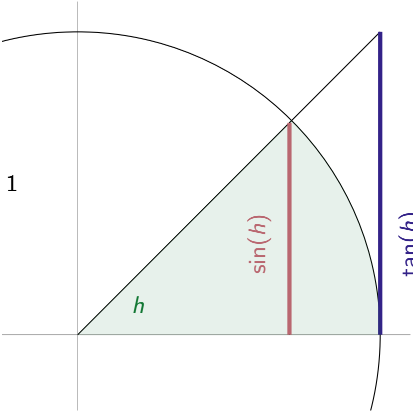
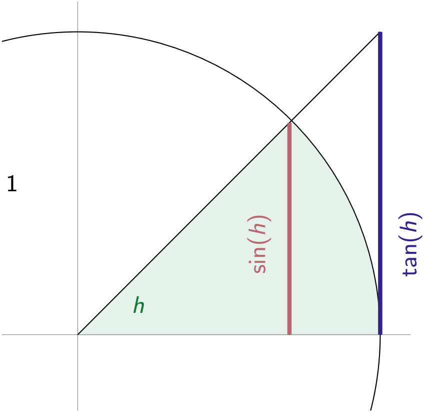


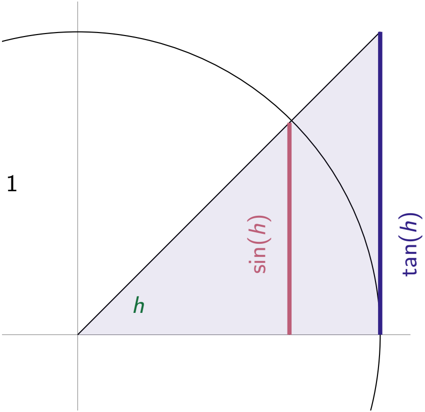
Green area:

$$\frac{\sin h}{h} \leq 1$$



Green area: $\frac{h}{2}$.

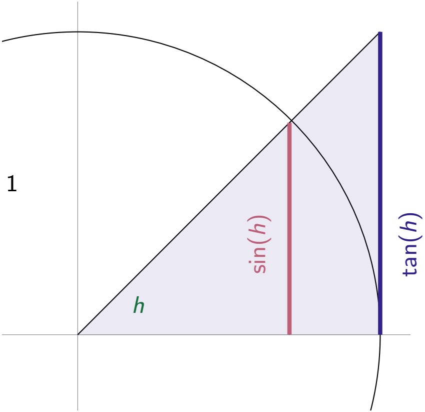
$$\frac{\sin h}{h} \leq 1$$



Green area: $\frac{h}{2}$.

Blue area:

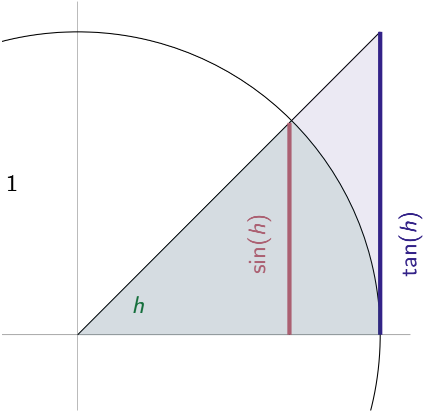
$$\frac{\sin h}{h} \leq 1$$



Green area: $\frac{h}{2}$.

Blue area: $\frac{\tan h}{2}$

$$\frac{\sin h}{h} \leq 1$$

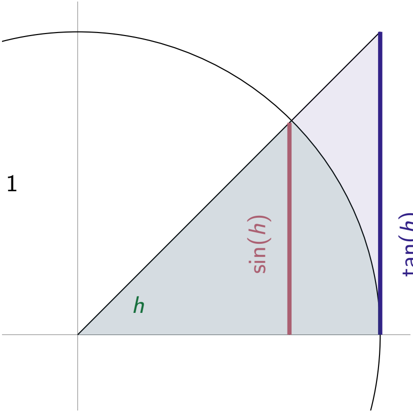


Green area: $\frac{h}{2}$.

$$\frac{h}{2} \leq \frac{\tan(h)}{2}$$

Blue area: $\frac{\tan h}{2}$

$$\cos h \leq \frac{\sin h}{h} \leq 1$$



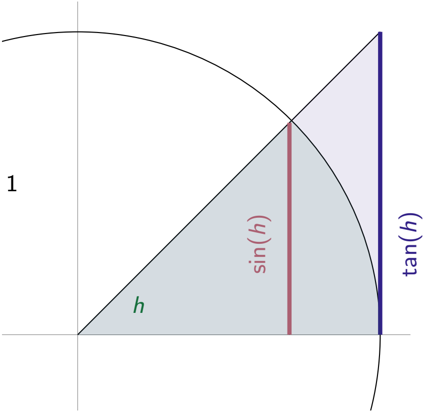
Green area: $\frac{h}{2}$.

$$\frac{h}{2} \leq \frac{\tan(h)}{2}$$

Blue area: $\frac{\tan h}{2}$

$$\boxed{\cos(h) \leq \frac{\sin(h)}{h}}$$

$$\cos h \leq \frac{\sin h}{h} \leq 1$$



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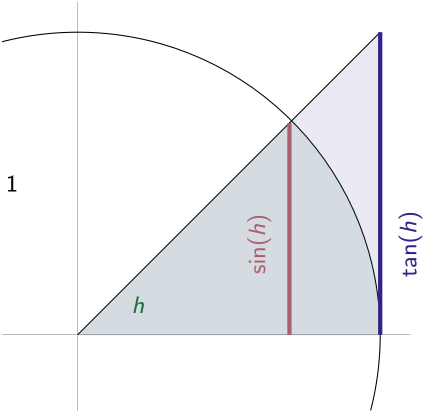

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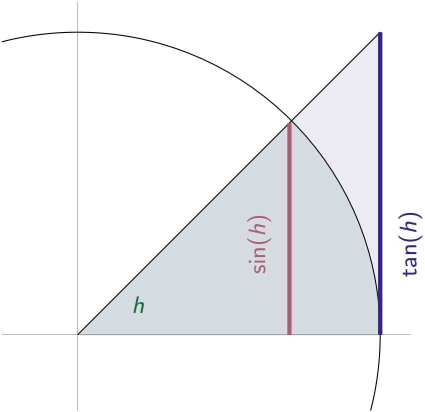
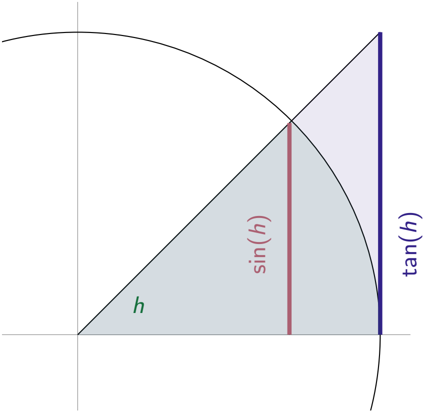
$$\frac{\sin h}{h}$$



$$\cos h \quad \leq \quad \frac{\sin h}{h} \quad \leq \quad 1$$

$$\lim_{h \to 0} \cos h = 1 \qquad \lim_{h \to 0} 1 = 1$$

$$\frac{\sin h}{h}$$



$$\cos h \leq \frac{\sin h}{h} \leq 1$$

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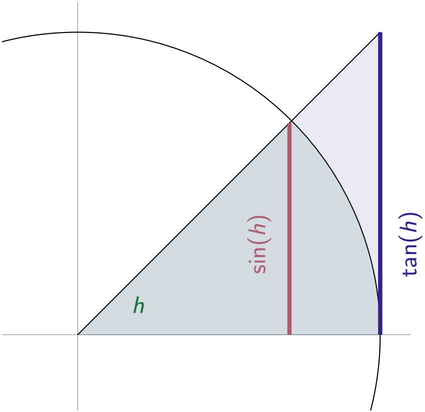
By the Squeeze Theorem,

$$\frac{\sin h}{h}$$



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Derivatives of Sine and Cosine

From before,

$$\frac{d}{dx}\{\sin(x)\} = \cos(x)\lim_{h\to 0}\frac{\sin(h)}{h} =$$

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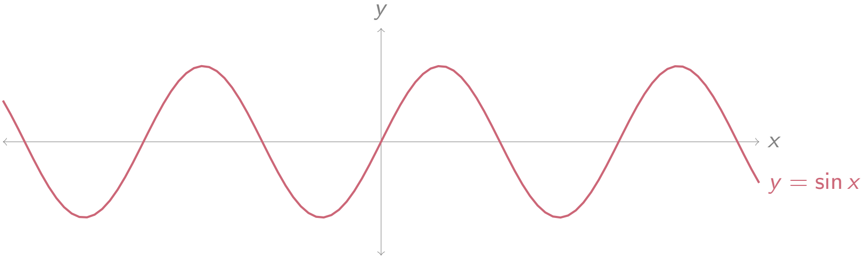

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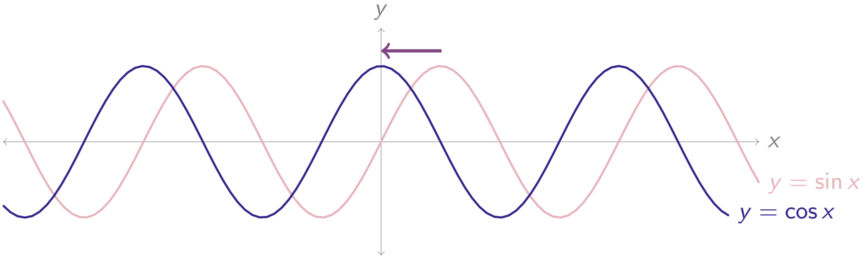

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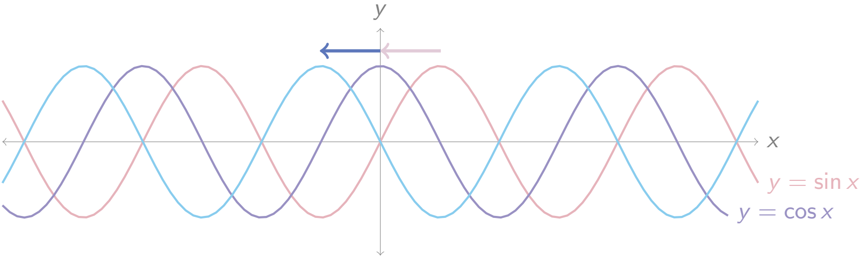

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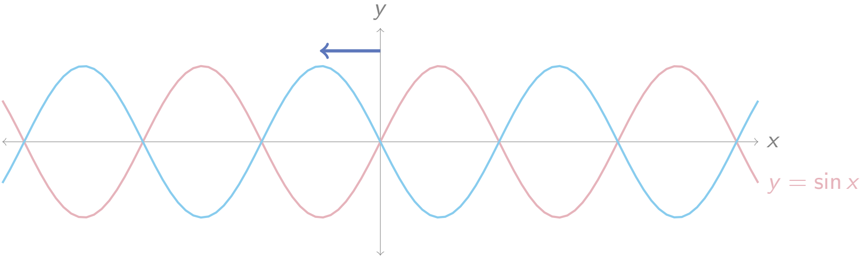

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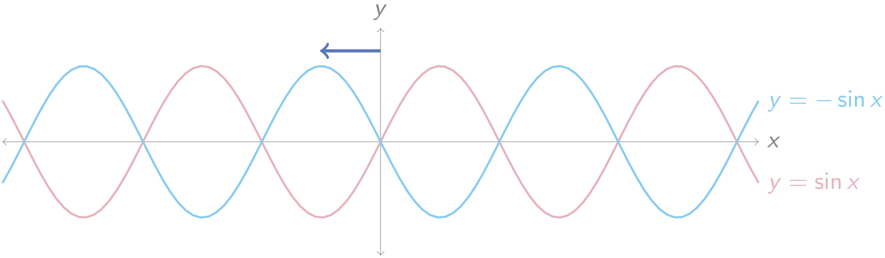

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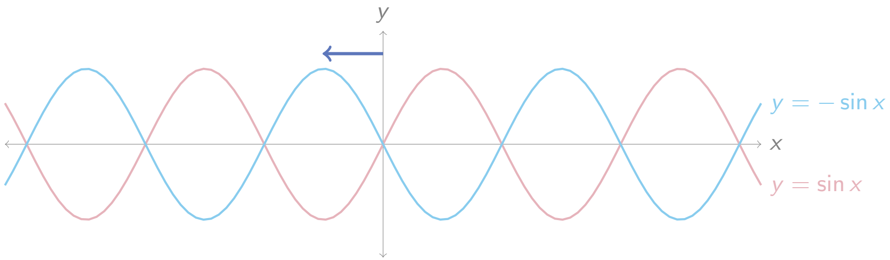

We might reasonably expect: $\frac{d}{dx}\{\cos x\} = -\sin x.$

Derivatives of Sine and Cosine

From before,

$$\frac{d}{dx}\{\sin(x)\} = \cos(x)\lim_{h\to 0}\frac{\sin(h)}{h} = \cos(x)$$

$$\frac{d}{dx}\{\cos(x)\} = \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$

Derivatives of Sine and Cosine

From before,

$$\frac{d}{dx}\{\sin(x)\} = \cos(x)\lim_{h\to 0}\frac{\sin(h)}{h} = \cos(x)$$

$$\begin{aligned}
\frac{d}{dx}\{\cos(x)\} &= \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h} \\
&= \lim_{h\to 0}\frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h} \\
&= \cos(x)\lim_{h\to 0}\frac{\cos(h)-1}{h} - \sin(x)\lim_{h\to 0}\frac{\sin(h)}{h} \\
&= \cos(x)(0)-\sin(x)(1) = -\sin(x)
\end{aligned}$$

Derivatives of Trig Functions

$\frac{d}{dx}\{\sin(x)\} = \cos(x)$

$\frac{d}{dx}\{\cos(x)\} = -\sin(x)$

$\frac{d}{dx}\{\tan(x)\} =$

$\frac{d}{dx}\{\sec(x)\} =$

$\frac{d}{dx}\{\csc(x)\} =$

$\frac{d}{dx}\{\cot(x)\} =$

Honorable Mention

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Derivatives of Trig Functions

$\frac{d}{dx}\{\sin(x)\} = \cos(x)$

$\frac{d}{dx}\{\cos(x)\} = -\sin(x)$

$\frac{d}{dx}\{\tan(x)\} = \sec^2(x)$

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$$\frac{d}{dx}\{\sec(x)\} = \sec(x)\tan(x)$$
$$\frac{d}{dx}\{\csc(x)\} = -\csc(x)\cot(x)$$
$$\frac{d}{dx}\{\cot(x)\} = -\csc^2(x)$$

Honorable Mention

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Other Trig Functions

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$= \frac{d}{dx}[\tan(x)] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right]$$

$$= \frac{\cos(x)\cos(x) - \sin(x)[-\sin(x)]}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$

Other Trig Functions

$$\sec(x) = \frac{1}{\cos(x)}$$

$$= \frac{d}{dx}[\sec(x)] = \frac{d}{dx}\left[\frac{1}{\cos(x)}\right]$$

$$= \frac{\cos(x)(0) - (1)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos(x)}\frac{\sin(x)}{\cos(x)}$$

$$= \sec(x)\tan(x)$$

Other Trig Functions

$$\csc(x) = \frac{1}{\sin(x)}$$

$$= \frac{d}{dx}[\csc(x)] = \frac{d}{dx}\left[\frac{1}{\sin(x)}\right]$$

$$= \frac{\sin(x)(0) - (1)\cos(x)}{\sin^2(x)}$$

$$= \frac{-\cos(x)}{\sin^2(x)}$$

$$= \frac{-1}{\sin(x)}\frac{\cos(x)}{\sin(x)}$$

$$= -\csc(x)\cot(x)$$

Other Trig Functions

$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$= \frac{d}{dx}[\cot(x)] = \frac{d}{dx}\left[\frac{\cos(x)}{\sin(x)}\right]$$

$$= \frac{\sin(x)(-\sin(x)) - \cos(x)\cos(x)}{\sin^2(x)}$$

$$= \frac{-1}{\sin^2(x)}$$

$$= -\csc^2(x)$$

Stuff to Know

$\frac{d}{dx}\{\sin(x)\} = \cos(x)$

$\frac{d}{dx}\{\cos(x)\} = -\sin(x)$

$\frac{d}{dx}\{\tan(x)\} = \sec^2(x)$

$\frac{d}{dx}\{\sec(x)\} = \sec(x)\tan(x)$

$\frac{d}{dx}\{\csc(x)\} = -\csc(x)\cot(x)$

$\frac{d}{dx}\{\cot(x)\} = -\csc^2(x)$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

Stuff to Know

$$\frac{d}{dx}\{\sin(x)\} = \cos(x)$$
$$\frac{d}{dx}\{\cos(x)\} = -\sin(x)$$
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$$\frac{d}{dx}\{\csc(x)\} = -\csc(x)\cot(x)$$
$$\frac{d}{dx}\{\cot(x)\} = -\csc^2(x)$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

Let $f(x) = \dfrac{x\tan(x^2+7)}{15e^x}$. Use the definition of the derivative to find $f'(0)$.

Differentiate $(e^x + \cot x)\left(5x^6 - \csc x\right)$. (No need to simplify.)

Suppose $h(x) = \begin{cases} \frac{\sin x}{x} & , \quad x < 0 \\ \frac{ax+b}{\cos x} & , \quad x \geq 0 \end{cases}$

Which values of a and b make $h(x)$ continuous at $x = 0$?

Practice

$$f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

Is $f(x)$ differentiable at $x = 0$?

$$g(x) = \begin{cases} e^{\frac{\sin x}{x}} & , \quad x < 0 \\ (x-a)^2 & , \quad x \geq 0 \end{cases}$$

What values of a makes $g(x)$ continuous at $x = 0$?

Practice

A ladder 3 meters long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall, measured in radians, and let y be the height of the top of the ladder. If the ladder slides away from the wall, how fast does y change with respect to θ? When is the top of the ladder sinking the fastest? The slowest?

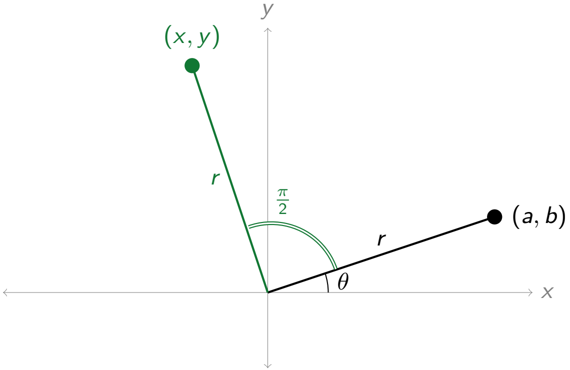
We want to find how fast y is changing with respect to θ, so we want $\frac{dy}{d\theta}$, or $y'(\theta)$. To calculate that, we need to find y as a function of θ. Note that the ladder forms a right triangle with the wall, and y is the side adjacent to θ, while 3 is the hypotenuse. So, $\cos(\theta) = \frac{y}{3}$, hence $y = 3\cos(\theta)$. Now we differentiate, and see

$$\frac{dy}{d\theta} = -3\sin(\theta)$$

To answer the other questions, note that θ never gets larger than $\pi/2$, since at that point the ladder is lying on the ground. When $0 \leq \theta \leq \pi/2$, the smaller θ gives the smaller rate of change (in absolute value); so the top of the ladder is sinking slowly at first, then faster and faster, fastest just as it hits the ground.

Practice

Suppose a point in the plane that is r centimeters from the origin, at an angle of θ, is rotated $\pi/2$ radians. What is its new coordinate (x, y)? When is the x coordinate changing fastest and slowest with respect to θ? (For simplicity, you may assume the original point is in the first quadrant; that is, $0 \leq \theta < \pi/2$.)



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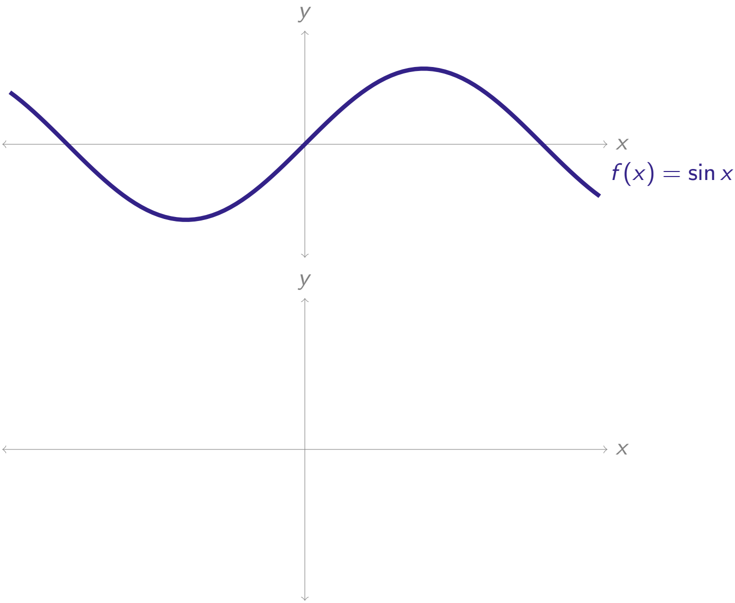
link: explanation of 2d rotation

$x = -r\sin(\theta)$ and $y = r\cos(\theta)$. To find how fast x is changing with respect to θ, we take $x'(\theta) = -r\cos(\theta)$. We see that when $\theta = 0$, x changes a lot when θ changes; and when $\theta = \pi/2$, x only changes a little when θ changes.

Intuition: $\sin x$ versus $\sin(2x)$



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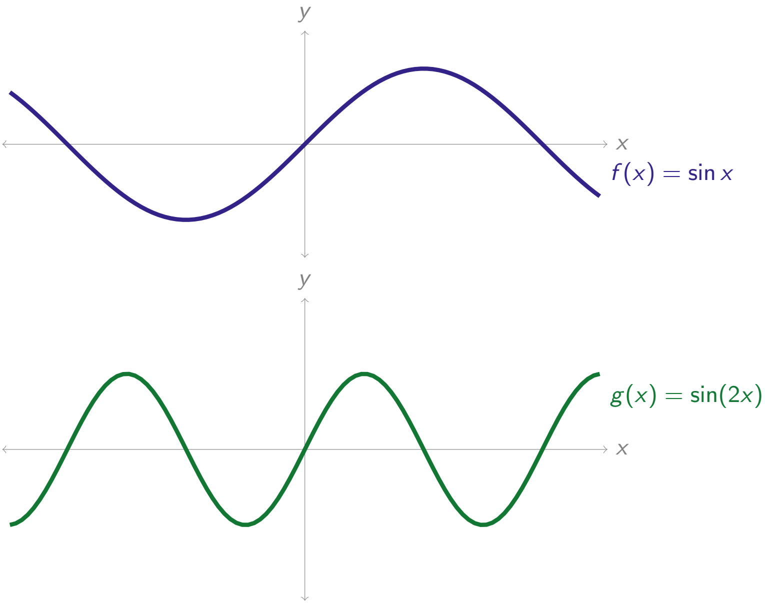

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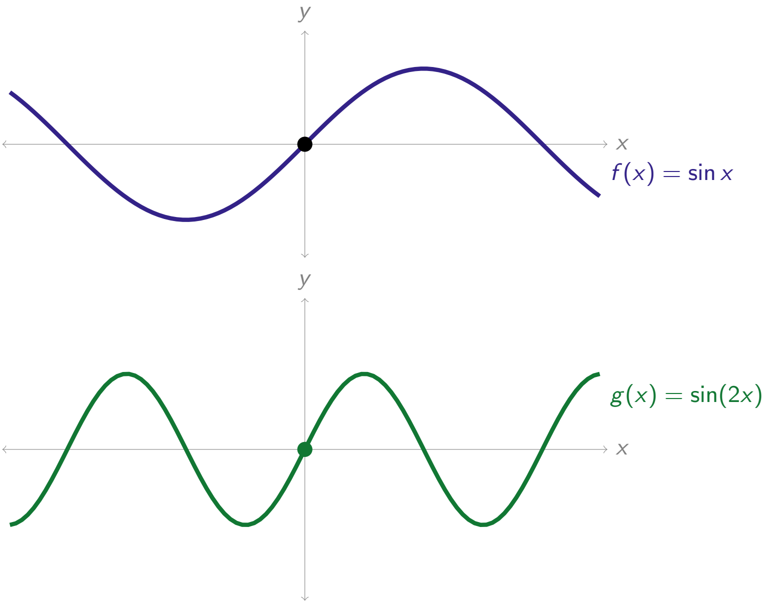

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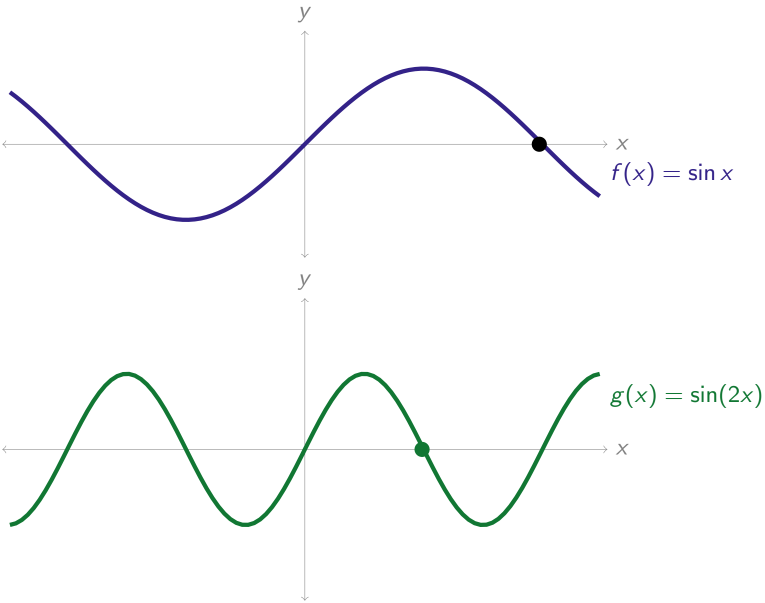

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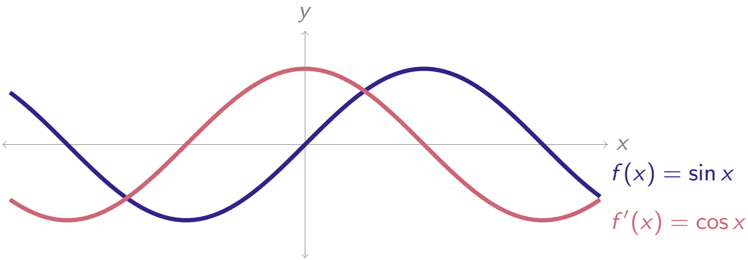

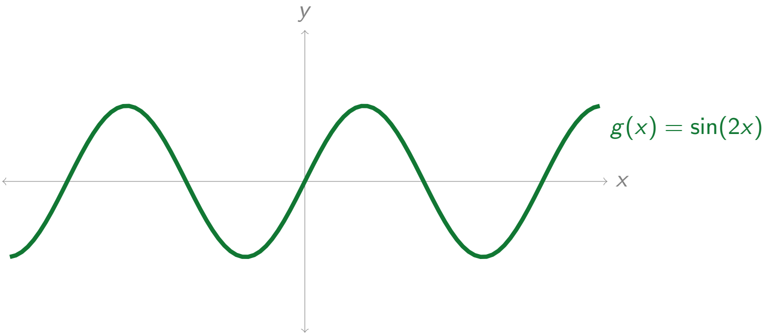

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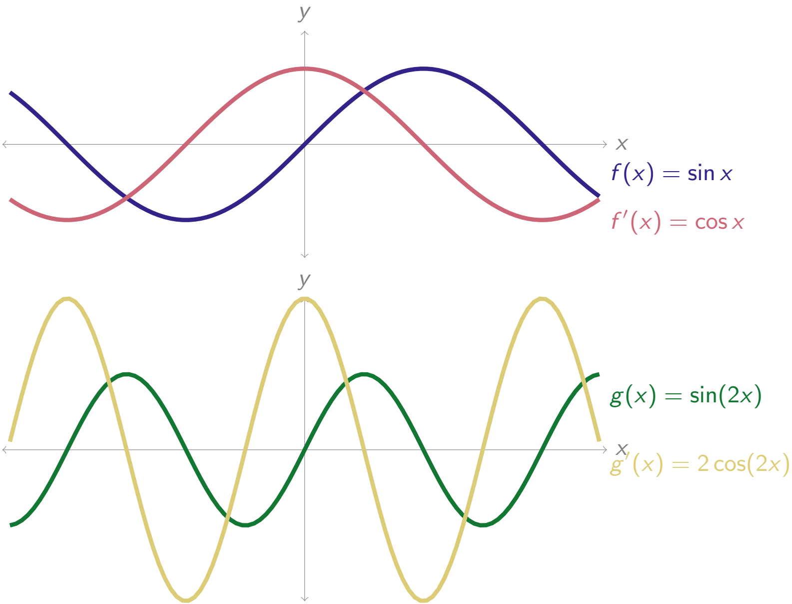

Compound Functions

Video: 2:27-3:50

Kelp Population

k	kelp population
u	urchin population
o	otter population

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$$
\begin{aligned}
\frac{d}{dx}\{f(g(x))\} &= \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{h} \\
&= \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{h} \left(\frac{g(x+h)-g(x)}{g(x+h)-g(x)} \right) \\
&= \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h)-g(x)} \cdot \frac{g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h)-g(x)} \cdot \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(\boxed{g(x+h)}) - f(\boxed{g(x)})}{\boxed{g(x+h)} - \boxed{g(x)}} \cdot g'(x) \\
\text{Set } H = g(x+h) - g(x) \qquad &= \lim_{H\to 0} \frac{f(g(x)+H) - f(g(x))}{H} \cdot g'(x) \\
&= f'(g(x)) \cdot g'(x)
\end{aligned}
$$

Chain Rule

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Suppose f and g are differentiable functions. Then

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) = \frac{df}{dg}\frac{dg}{dx}$$

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So, $f'(x) = \cos(e^x + x^2)(e^2 + x^2)$

Another Example

$$F(v) = \left(\frac{v}{v^3 + 1}\right)^6$$

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$$F'(v) = 6\left(\boxed{\frac{v}{v^3+1}}\right)^5 \cdot \frac{(v^3+1)(1)-(v)(3v^2)}{(v^3+1)^2}$$

$$= 6\left(\boxed{\frac{v}{v^3+1}}\right)^5 \cdot \frac{-2v^3+1}{(v^3+1)^2}$$

More Examples

Let $f(x) = (10^x + \csc x)^{1/2}$. Find $f'(x)$.

Suppose $o(t) = e^t$, $u(o) = \frac{1}{o+\sin(o)}$, and $t \geq 10$ (so all these functions are defined). Using the chain rule, find $u'(t)$. *Note:* your answer should depend only on t: not o.

More Examples

Let $f(x) = (10^x + \csc x)^{1/2}$. Find $f'(x)$.

$f(x) = (\boxed{10^x + \csc x})^{1/2}$, so using the chain rule,

$$f'(x) = \frac{1}{2}(\boxed{10^x + \csc x})^{-1/2}(10^x \ln 10 - \csc x \cot x)$$

$$= \frac{10^x \ln 10 - \csc x \cot x}{2\sqrt{10^x + \csc x}}$$

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$o'(t) = e^t$ and $u'(o) = \dfrac{(o + \sin o)(0) - (1)(1 + \cos o)}{(o + \sin o)^2} = \dfrac{-(1 + coso)}{(o + \sin o)^2}$. Then,

$$u'(t) = -e^t \left(\frac{1 + \cos(e^t)}{(e^t + \sin(e^t))^2} \right)$$

More Examples

Evaluate $\frac{d}{dx}\left\{x^2+\sec\left(x^2+\frac{1}{x}\right)\right\}$

Evaluate $\frac{d}{dx}\left\{\frac{1}{x+\frac{1}{x+\frac{1}{x}}}\right\}$

More Examples

Evaluate $\dfrac{d}{dx}\left\{x^2+\sec\left(x^2+\dfrac{1}{x}\right)\right\}$

$$\begin{aligned}\frac{d}{dx}\left\{x^2+\sec\left(\boxed{x^2+\frac{1}{x}}\right)\right\} &= 2x+\sec\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\tan\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\frac{d}{dx}\left\{\boxed{x^2+\frac{1}{x}}\right\}\\ &= 2x+\sec\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\tan\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\frac{d}{dx}\left\{\boxed{x^2+x^{-1}}\right\}\\ &= 2x+\sec\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\tan\left(\boxed{x^2+\frac{1}{x}}\right)\cdot\left(2x-x^{-2}\right)\end{aligned}$$

Notice: That first term, $2x$, is not multiplied by anything else.

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$$\frac{d}{dx}\left\{\frac{1}{x + \frac{1}{x + \frac{1}{x}}}\right\} = \frac{d}{dx}\left\{\left(x + \left(x + x^{-1}\right)^{-1}\right)^{-1}\right\}$$

$$= -\left(\boxed{x + \left(x + x^{-1}\right)^{-1}}\right)^{-2} \cdot \frac{d}{dx}\left\{\boxed{x + \left(x + x^{-1}\right)^{-1}}\right\}$$

$$= -\left(\boxed{x + \left(x + x^{-1}\right)^{-1}}\right)^{-2} \cdot \left[1 + (-1)\left(\boxed{x + x^{-1}}\right)^{-2} \cdot \frac{d}{dx}\left\{\boxed{x + x^{-1}}\right\}\right]$$

$$= -\left(\boxed{x + \left(x + x^{-1}\right)^{-1}}\right)^{-2} \cdot \left[1 + (-1)\left(\boxed{x + x^{-1}}\right)^{-2} \cdot (1 - x^{-2})\right]$$