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Solution

Let $\boxed{Q(t) = Ce^{-kt}}$, where k and C are constants. Then:

$$\frac{dQ}{dt} = C \cdot e^{-kt} \cdot (-k) = -kCe^{-kt} = -kQ$$

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Quantity of a Radioactive Isotope

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Seaborgium-266

Carbon Decay

The amount of ^{266}Sg (Seaborgium-266) in a sample at time t (measured in seconds) is given by

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According to Wikipedia, the half life of ^{266}Sg is 30 seconds. That is, every 30 seconds, the size of the sample halves.

What are C and k?

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$$\frac{1}{2}C = \frac{1}{2}Q(0) = Q(30) = Ce^{-30k}$$

$$k = \frac{\ln 2}{30}$$

Radioactive Decay

A sample of radioactive matter is stored in a lab in 2000. In the year 2002, it is tested and found to contain 10 units of a particular radioactive isotope. In the year 2005, it is tested and found to contain only 2 units of that same isotope. How many units of the isotope were present in the year 2000?

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The quantity of the isotope t years after 2000 is given by

$$Q(t) = Ce^{kt}$$

where $C = Q(0)$ is the amount in the initial sample. Then the question asks us to solve for C, give

$$10 = Q(2) = Ce^{2k} \quad \text{and} \quad 2 = Q(5) = Ce^{5k}$$

Then

$$C = 10e^{-2k} = 2e^{-5k}$$

$$5e^{3k} = 1 \qquad \implies \qquad e^{k} = \frac{1}{\sqrt[3]{5}}$$

$$C = 10e^{-2k} = 10\sqrt[3]{5}^{2} \approx 29$$

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Exponential Growth

Let $Q = Q(t)$ satisfy:

$$\frac{dQ}{dt} = kQ$$

for some constant k. Then

$$Q(t) = Ce^{kt}$$

where $C = Q(0)$ is a constant.

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Suppose $y(t)$ is a function with the properties that

$$\frac{dy}{dt} + 3y = 0 \quad \text{and} \quad y(1) = 2.$$

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What is $y(t)$? $y(t) = Ce^{-3t}$ by the result above; solving for C, we set $2 = y(1) = Ce^{-3}$ and find $C = 2e^3$. So, $y(t) = 2e^3 \cdot e^{-3t} = 2e^{3(1-t)}$.

Population Growth

Suppose a petri dish starts with a culture of 100 bacteria cells and a limited amount of food and space. The population of the culture at different times is given in the table below. At approximately what time did the culture start to show signs of limited resources?

time	population
0	100
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All the populations before $t = 5$ follow $B(t) = 100e^{t \ln 10} = 100 \cdot 10^t$. At $t = 5$ they do not; so some time between $t = 3$ and $t = 5$, the bacteria started reproducing at a slower rate.

Flu Season

The CDC keeps records (link) on the number of flu cases in the US by week. At the start of the flu season, the 40th week of 2014, there are 100 cases of a particular strain. Five weeks later (at week 45), there are 506 cases. What do you think was the first week to have 5,000 cases? What about 10,000 cases?

https://pixabay.com/p-156666/?no_redirect

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Let's let $t = 0$ be the 40th week of 2014. Then we can model the spread of the virus like so:

$$P(t) = 100e^{kt}$$

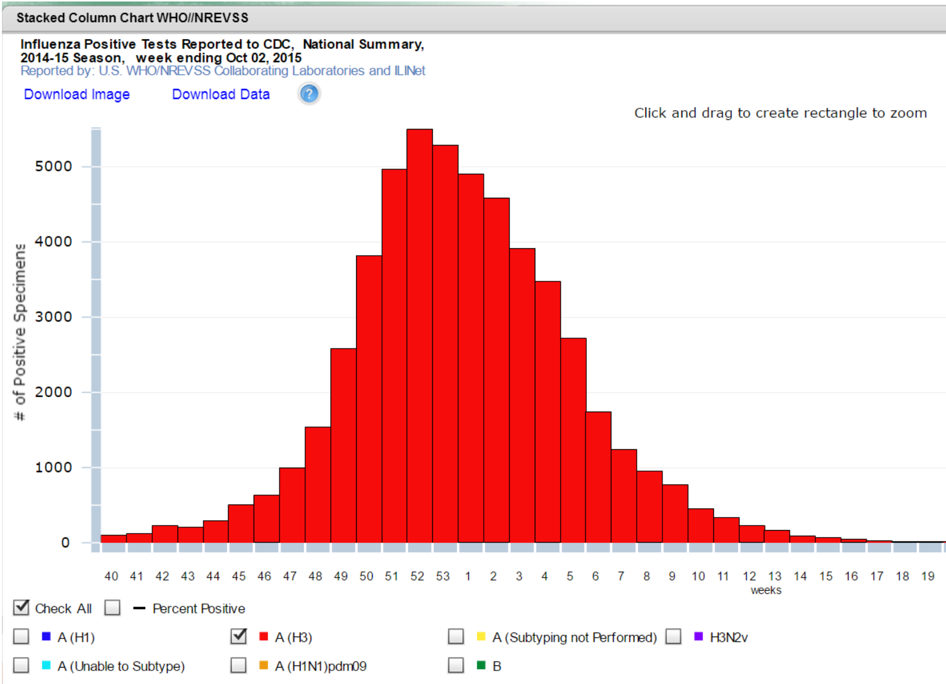
We have one other data point: $506 = P(5) = 100e^{5k}$, so we get $e^k = 5.06^{1/5}$. Now our equation is:

$$P(t) = 100(5.06)^{t/5}$$

We set it equal to 5000 and solve: $5000 = 100(5.06)^{t/5}$ implies $t = \frac{5 \ln 50}{\ln(5.06)} \approx 12.06$.

Data from the CDC says Week 51 ($t = 11$) had 4972 cases, and Week 52 ($t = 12$) had 5498 cases.

Using the same formula, $10000 = 100(5.06)^{t/5}$ yields $t = \frac{5 \ln 100}{\ln 5.06} \approx 14.2$ weeks; but the data shows that the flu season peaked with around 5,000 cases a week, and never got much higher.

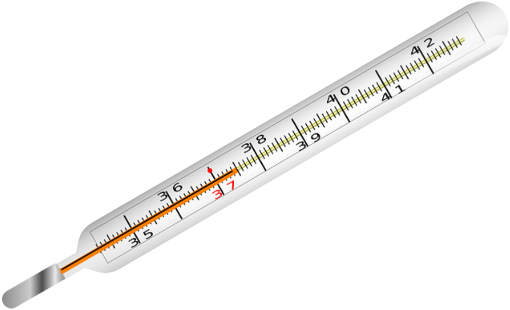


Stacked Column Chart WHO//NREVSS
Influenza Positive Tests Reported to CDC, National Summary,
2014-15 Season, week ending Oct 02, 2015
Reported by: U.S. WHO/NREVSS Collaborating Laboratories and ILINet
Download Image
Download Data
Click and drag to create rectangle to zoom
# of Positive Specimens
0
1000
2000
3000
4000
5000
40 41 42 43 44 45 46 47 48 49 50 51 52 53 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
weeks
Check All
Percent Positive
A (H1)
A (H3)
A (Subtyping not Performed)
H3N2v
A (Unable to Subtype)
A (H1N1)pdm09
B

Cooling

Newton's Law of Cooling

The rate of change of temperature of an object is proportional to the difference in temperature between that object and its surroundings.



https://pixabay.com/en/thermometer-temperature-fever-309120/

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$$\frac{dT}{dt} = K[T(t) - A]$$

where $T(t)$ is the temperature of the object at time t, A is the ambient temperature of the surroundings, and K is some constant depending on the object.

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Solution

$$T(t) = [T(0) - A]e^{Kt} + A$$

is the only equation satisfying Newton's Law of Cooling

Temperature of a Cooling Body

$$T(t) = [T(0) - A]e^{Kt} + A$$

Suppose $T(10) < A$. Then:

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Evaluate $\lim_{t \to \infty} T(t)$.

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Temperature of a Cooling Body

$$T(t) = [T(0) - A]e^{Kt} + A$$

What assumptions are we making that might not square with the real world?

A farrier forms a horseshoe heated to 400° C, then dunks it in a pool of room-temperature (25° C) water. The water near the horseshoe boils for 30 seconds, but the temperature of the pool as a whole hasn't changed appreciably. The horseshoe is safe for the horse when it's 40° C. When can the farrier put on the horseshoe?

https://upload.wikimedia.org/wikipedia/commons/thumb/9/90/West_Show_Jersey_2010_farrier_f.jpg/794px-West_Show_Jersey_2010_farrier_f.jpg

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We know: $T(0) = 400$, $T(30) = 100$, and $A = 25$. We want to find K.

$$100 = T(30) = [T(0) - A]e^{30K} + 25 = 375e^{30k} + 25$$

$$\Rightarrow 75 = 375e^{30K} \Rightarrow \frac{1}{5} = e^{30k} \Rightarrow K = \frac{-\ln 5}{30}$$

Now, we set $T(t) = 40$ and solve for t:

$$40 = T(t) = 375e^{\frac{-\ln 5}{30}t} + 25$$

$$15 = 375e^{\frac{-ln5}{30}t} = 375 \cdot 5^{-t/30}$$

$$\frac{1}{25} = 5^{-t/30}$$

$$25 = 5^{t/30}$$

$$2 = t/30$$

So the farrier can put the shoe on after 60 seconds in the water.

A glass of just-boiled tea is put on a porch outside. After ten minutes, the tea is 40°, and after 20 minutes, the tea is 25°. What is the temperature outside?

https://c1.staticflickr.com/3/2546/4211619112_53d74cc974.jpg

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$T(0) = 100$, so $T(10) = [100 - A]e^{10k} + A = 40$ and $T(20) = [100 - A]e^{20k} + A = 25$.

Solving both for A, we get $A = \dfrac{40 - 100e^{10k}}{1 - e^{10k}} = \dfrac{25 - 100e^{20k}}{1 - e^{20k}}$

Although this looks complicated, if we set $x = e^{10k}$, it simplifies to something we can easily solve.

$$A = \frac{40 - 100e^{10k}}{1 - e^{10k}} = \frac{25 - 100e^{20k}}{1 - e^{20k}}$$
$$A = \frac{40 - 100x}{1 - x} = \frac{25 - 100x^2}{1 - x^2}$$
$$(40 - 100x)(1 - x^2) = (25 - 100x^2)(1 - x)$$
$$(40 - 100x)(1 + x)(1 - x) = (25 - 100x^2)(1 - x)$$
$$(40 - 100x)(1 + x) = 25 - 100x^2$$
$$40 - 60x - 100x^2 = 25 - 100x^2$$
$$40 - 60x = 25$$
$$x = \frac{1}{4}$$
$$A = \frac{40 - 100x}{1 - x} = \frac{40 - \frac{100}{4}}{1 - \frac{1}{4}} = 20$$

It is 20 degrees outside.

Population

In 1963, the US Fish and Wildlife Service recorded a bald eagle population of 487 breeding pairs. In 1993, that number was 4015. How many breeding pairs would you expect there were in 2006? What about 2015?

Source: `http://www.fws.gov/midwest/eagle/population/chtofprs.html`

Image: `https://pixabay.com/p-527426/?no_redirect`

Since we don't have a better model, let's assume the population P of nesting pairs follows:

$$P(t) = P(0)e^{kt}$$

for some constant k. To fit the data we have, let $t = 0$ represent 1963, so $P(0) = 487$. Then $4015 = P(30) = 487e^{30k}$, so $e^k = \left(\frac{4015}{487}\right)^{1/30}$. Now we use this to predict $P(43)$ (since 2006 is 43 years after 1963) and $P(52)$ (since 2015 is 52 years after 1963).

$P(43) = 487(e^k)^{43} = 487\left(\frac{4015}{487}\right)^{43/30} \approx 10016$

So we guess in 2016 there were about 10 016 breeding pairs in the lower 48.

$P(52) = 487(e^k)^{52} = 487\left(\frac{4015}{487}\right)^{52/30} \approx 18860$

Wood Bison Restoration in Alaska, Alaska Department of Fish and Game

Excerpt: "Based on experience with reintroduced populations elsewhere, wood bison would be expected to increase at a rate of 15%-25% annually after becoming established.... With an average annual growth rate of 20%, an intial precalving population of 50 bison would increase to 500 in approximately 13 years."

Are they using our same model?

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Our model gives the same result.

Compound Interest

Suppose you invest \$10,000 in to an account that accrues compound interest. After one month, your balance (with interest) is \$10,100. How much money will be in your account after a year?

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$$\begin{aligned} 10000e^{r\cdot 1} &= 10100 \\ e^{r} &= \frac{10100}{10000} = 1.01 \\ 10000e^{12r} &= 10000\cdot(e^{r})^{12} = 10000\cdot 1.01^{12} \approx 11268.25 \end{aligned}$$

Carrying Capacity

For a population with unrestricted access to resources, let β be the average number of offspring each breeding pair produces per generation, where a generation has length t_g. Then $b = \frac{\beta - 2}{2t_g}$ is the net birthrate (births minus deaths) per member per unit time. This yields $\frac{dP}{dt} = bP(t)$, hence:

$$P(t) = P(0)e^{bt}$$

But as resources grow scarce, b might change. If K is the carrying capacity of an ecosystem, we can model $b = b_0(1 - \frac{P}{K})$. Then:

$$\frac{dP}{dt} = b_0 \left(1 - \frac{P(t)}{K}\right) P(t)$$

Radiocarbon Dating

Researchers at Charlie Lake in BC have found evidence[1] of habitation dating back to around 8500 BCE. For instance, a butchered bison bone was radiocarbon dated to about 10,500 years ago.

Suppose a comparable bone of a bison alive today contains 1mg of ^{14}C. If the half-life of ^{14}C is about 5730 years, how much ^{14}C do you think the researchers found in the sample?

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Make a rough estimate first.

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B. About $\frac{1}{4}$ mg
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E. I'm not sure how to estimate this

First, an estimate; 10500 is not so far off from 2(5730), so we might guess that there is roughly a quarter of a miligram left.

We know $Q(t) = Ce^{-kt} = e^{-kt}$ mg. We want to find $Q(10500)$, so we need to solve for k. Since we know the half-life: to do this, solve $\frac{1}{2} = e^{-k\cdot 5730}$ to get $k = \frac{\ln 2}{5730}$. Now: $Q(10500) = e^{-\frac{\ln 2}{5730}\cdot 10500} = 2^{-\frac{10500}{5730}} \approx 0.28$ mg.

[1] http://pubs.aina.ucalgary.ca/arctic/Arctic49-3-265.pdf

Suppose a body is discovered at 3:45 pm, in a room held at 20°, and the body's temperature is 27°: not the normal 37°. At 5:45 pm, the temperature of the body has dropped to 25.3°. When did the owner of the body die?

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Set our time so that $t = 0$ is 3:45pm and $t = 2$ is 5:45pm. Then $T(0) = 27$, $T(2) = 25.4$, and $A = 20$. Now:

$$T(t) = [27 - 20]e^{Kt} + 20 = 7e^{Kt} + 20$$

Using what we know about 5:45pm: $7e^{2K} + 20 = T(2) = 25.3$, so $7e^{2K} = 5.3$, hence $e^{2K} = \frac{5.3}{7}$ and $e^K = \left(\frac{5.3}{7}\right)^{1/2}$.
Now:

$$T(t) = 7e^{Kt} + 20 = 7\left(\frac{5.3}{7}\right)^{t/2} + 20$$

So we set $T(t) = 37$ and solve for t.

$7\left(\frac{5.3}{7}\right)^{t/2} + 20 = 37$

$7\left(\frac{5.3}{7}\right)^{t/2} = 17$

$\left(\frac{5.3}{7}\right)^{t/2} = \frac{17}{7}$

$\frac{t}{2} = \log_{\frac{5.3}{7}}\left(\frac{17}{7}\right) = \dfrac{\ln(17/7)}{\ln(5.3/7)}$

$t = 2\dfrac{\ln(17/7)}{\ln(5.3/7)} \approx -6.4$

So the person died about 6.4 hours before 3:45pm. 0.4 hours is 24 minutes, so 6 hours and 24 minutes before 3:45 pm is 6 hours before 3:21pm, which is 9:21 am.