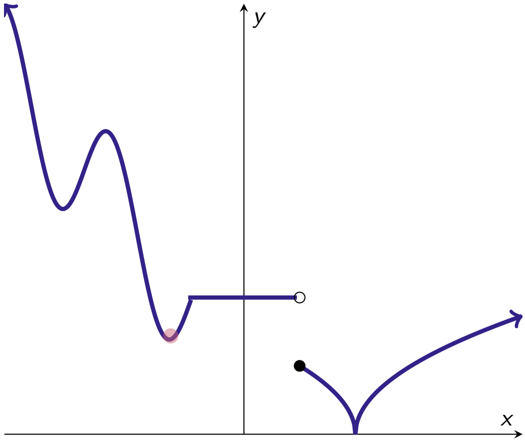
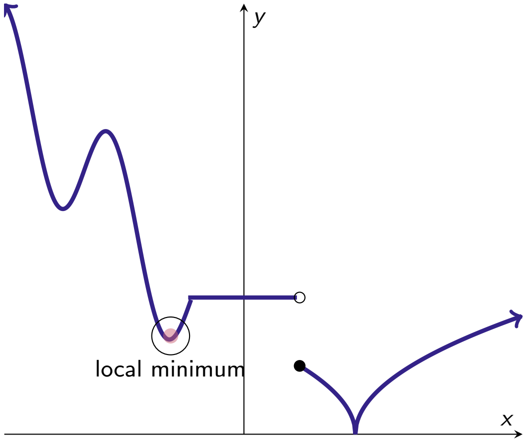
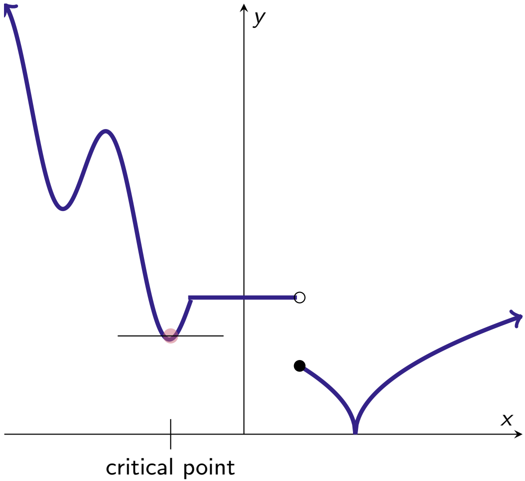
Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5

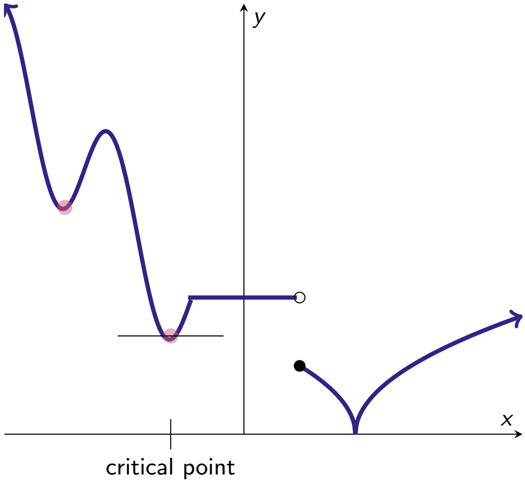


Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



c is a critical point if $f'(c) = 0$.

Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



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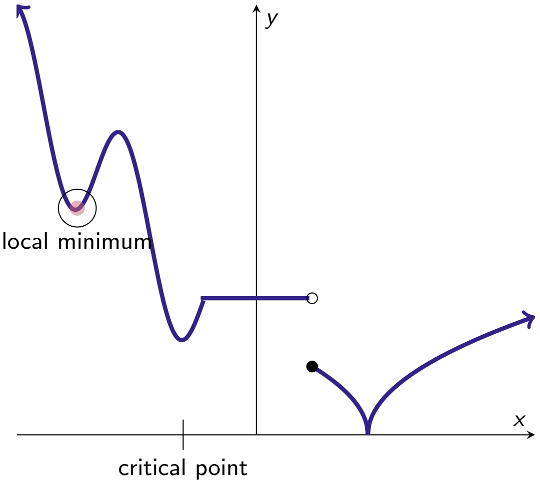
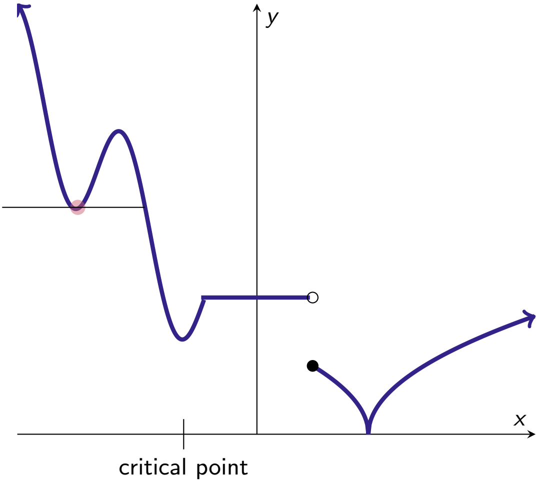
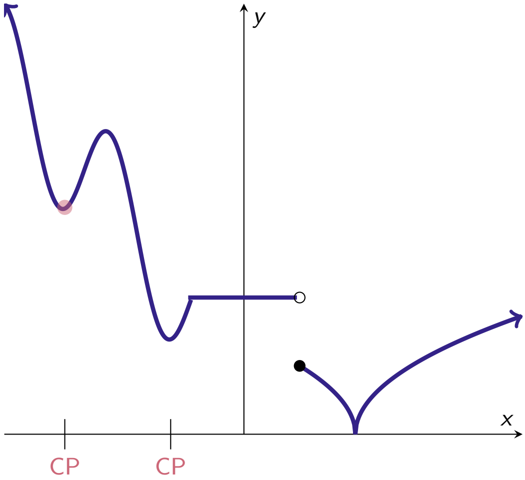


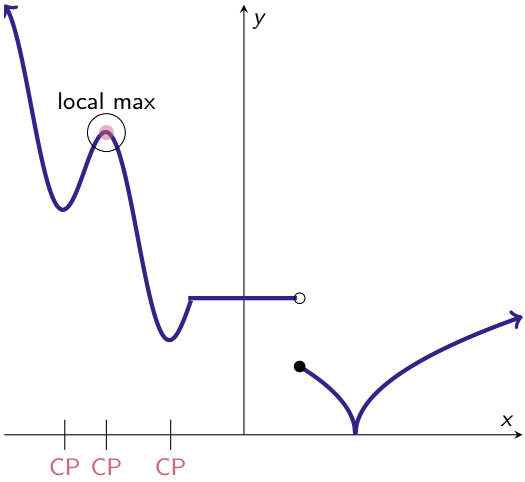
Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



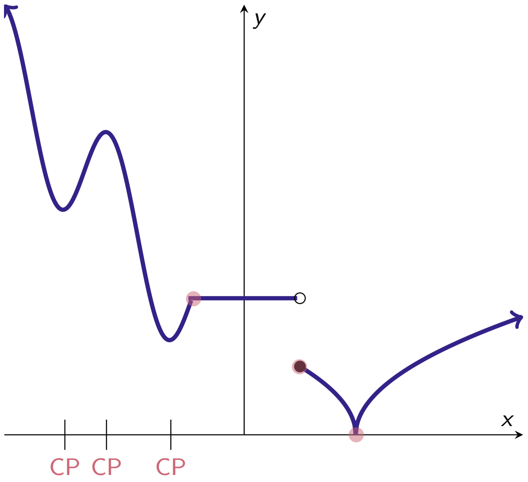
Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



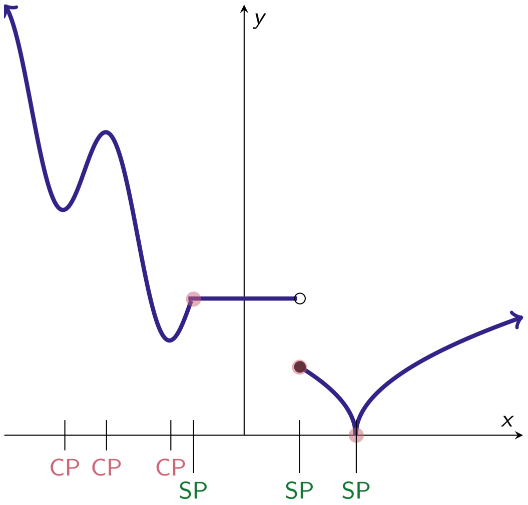
Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5

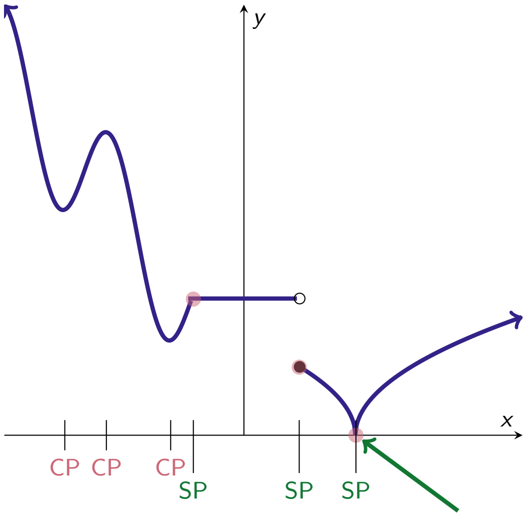


Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5

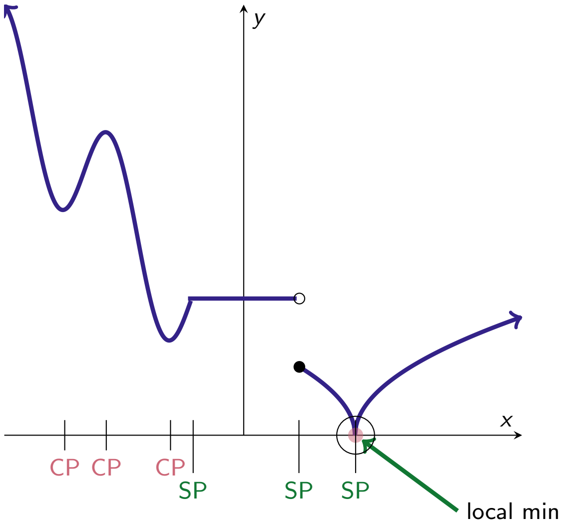


c is a singular point if $f'(c)$ does not exist.

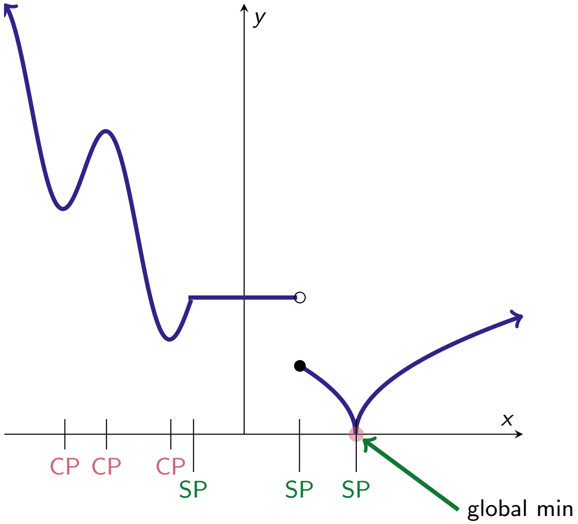
Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



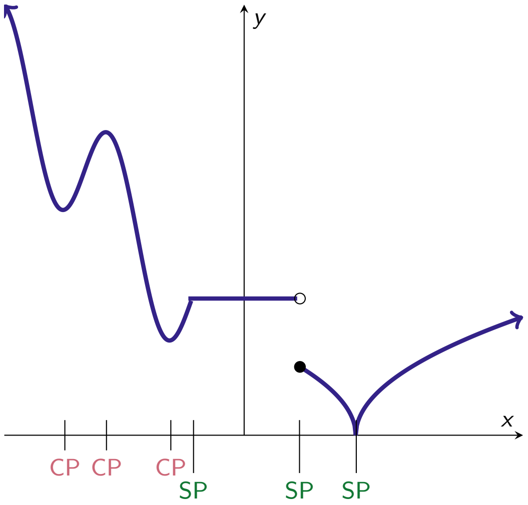
Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



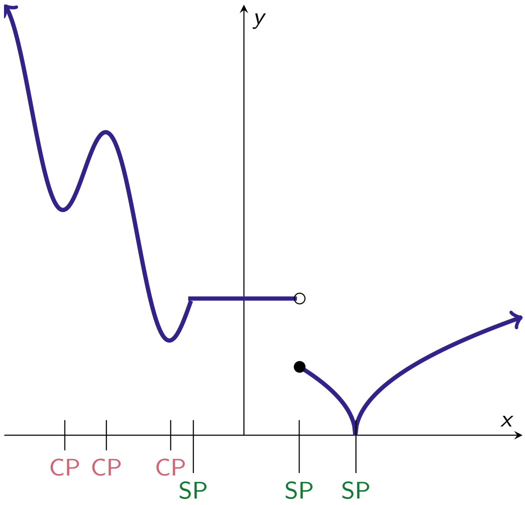
Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5

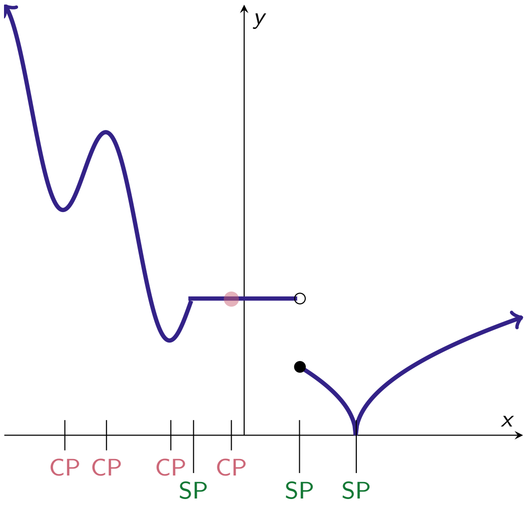


Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



This function as shown has no global maximum.

Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



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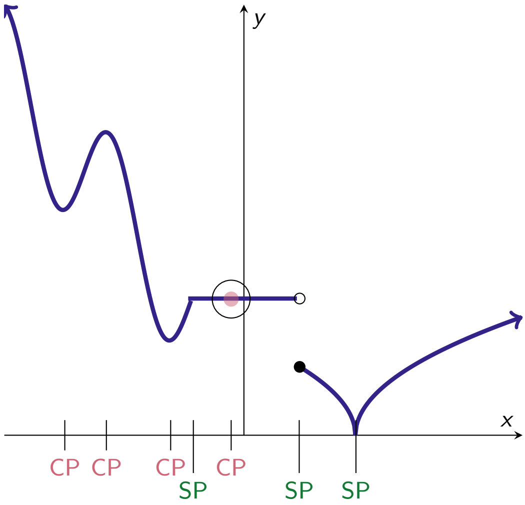

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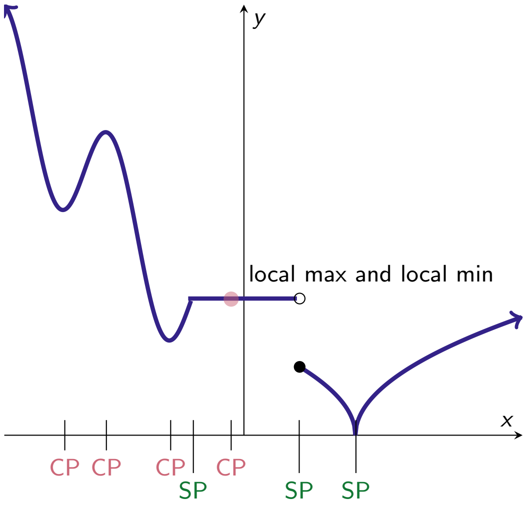
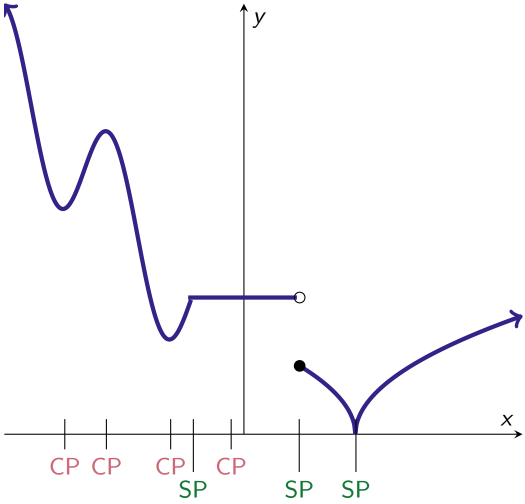


Anatomy of a Function: CLP Definitions 3.5.3, 3.5.5



Multiple Choice

Suppose $f(x)$ has domain $(-\infty, \infty)$.

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Suppose $f(x)$ has domain $(-\infty, \infty)$.
If f has a local minimum at $x = 2$, then:

A. $f'(2) = 0$

B. $f'(2)$ DNE

C. $f'(2) = 0$ OR $f'(2)$ DNE

D. $f(2) = 0$

E. Not necessarily any of the above

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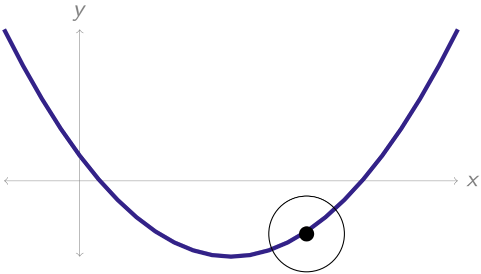

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Multiple Choice

Suppose $f(x)$ has domain $(-\infty, \infty)$.
If $f'(5) = 0$, then:

A. $f'(5)$ DNE

B. f has a local maximum or minimum at 5

C. f may or may not have a local extremum (max or min) at 5

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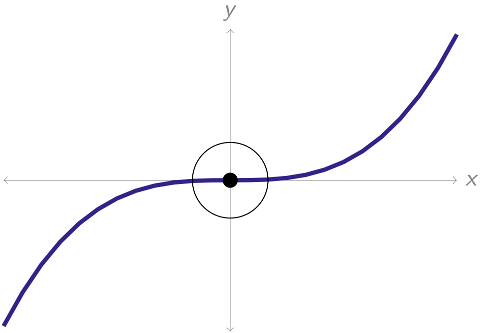

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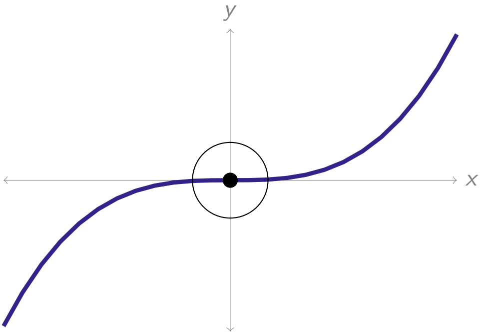

Draw a continuous function $f(x)$ with a local maximum at $x = 3$ and a local minimum at $x = -1$.

Draw a continuous function $f(x)$ with a local maximum at $x = 3$ and a local minimum at $x = -1$, but $f(3) < f(-1)$.

Draw a function $f(x)$ with a singular point at $x = 2$ that is NOT a local maximum, or a local minimum.

Example: MaxMin 1

Suppose $f'(x) = (x+5)^2(x-5)$. Then f has no singular points, and its critical points are ± 5. Identify whether the critical points are local maxima, local minima, or neither.

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Second Derivative Test:

Suppose $f'(a) = 0$ and $f''(a) > 0$. Then $x = a$ is a local

Suppose $f'(a) = 0$ and $f''(a) < 0$. Then $x = a$ is a local

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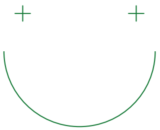
Example: MaxMin 1

Suppose $f'(x) = (x+5)^2(x-5)$. Then f has no singular points, and its critical points are ± 5. Identify whether the critical points are local maxima, local minima, or neither.

Second Derivative Test:



Suppose $f'(a) = 0$ and $f''(a) > 0$. Then $x = a$ is a local **minimum**.

Suppose $f'(a) = 0$ and $f''(a) < 0$. Then $x = a$ is a local **maximum**.

Example: MaxMin 1

Suppose $f'(x) = (x+5)^2(x-5)$. Then f has no singular points, and its critical points are ± 5. Identify whether the critical points are local maxima, local minima, or neither.

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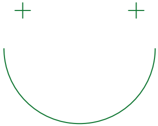
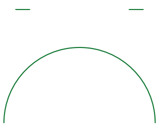


Suppose $f'(a) = 0$ and $f''(a) > 0$. Then $x = a$ is a local **minimum**.



Suppose $f'(a) = 0$ and $f''(a) < 0$. Then $x = a$ is a local **maximum**.

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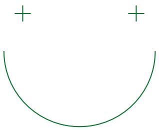
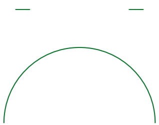
Suppose $f'(x) = (x+5)^2(x-5)$. Then f has no singular points, and its critical points are ± 5. Identify whether the critical points are local maxima, local minima, or neither.

Second Derivative Test:



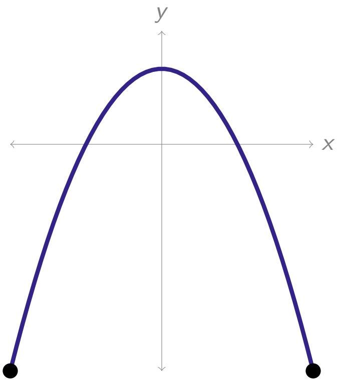
Suppose $f'(a) = 0$ and $f''(a) > 0$. Then $x = a$ is a local **minimum**.



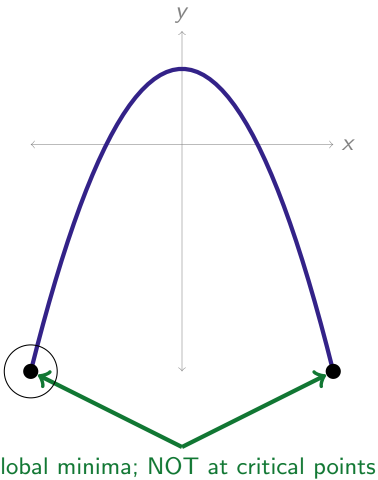
Suppose $f'(a) = 0$ and $f''(a) < 0$. Then $x = a$ is a local **maximum**.

We see that, when we are close to -5, whether x is less than or greater than -5, still $f'(x)$ is negative. So, $f(x)$ is decreasing before $x- = 5$ and also after it. So, -5 is not a local max or a local min.
Now consider $x = 5$. When x is a little less, $f'(x)$ is negative; when x is a little more than 5, $f'(x)$ is positive. So, f is decreasing till 5, then increasing after: so 5 is a local min. Indeed, $x = 5$ is the site of a global min.

Endpoints

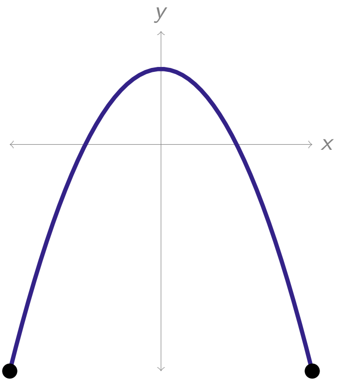


Endpoints



CLP Notes: Definition 3.5.3

Endpoints



Theorems 3.5.10, 3.5.11: A function that is continuous on the interval $[a, b]$ (where a and b are real numbers–not infinite) has a global max and min, and they occur at endpoints, critical points, or singular points.

Determining Extrema

Determining Extrema

To find local extrema:

- Could be at critical points ($f'(x) = 0$)
- Could be at singular points ($f'(x)$ DNE)
- At these points, check whether there is some interval around x where $f(x)$ is no larger than the other numbers, or no smaller. (A sketch helps. The signs of the derivatives on either side of x are also a clue.)

To find global extrema:

- Could be at critical points ($f'(x) = 0$)
- Could be at singular points ($f'(x)$ DNE)
- Could be at endpoints;
  also check the limit as the function goes to $\pm\infty$.
- Check the value of the function at all of these, and compare.

Example: MaxMin 2

Find All Extrema[1]:

$$f(x) = x^3 - 3x$$

[1] Extrema: local and global maxima and minima

Example: MaxMin 2

Find All Extrema[1]:

$$f(x) = x^3 - 3x$$

Since there are no endpoints, we only need to find critical points and singular points. $f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x+1)(x-1)$. So there are no singular points, and the critical points are ± 1.

We know that cubic functions grow hugely positive in one direction, and hugely negative in the other. So, there's no global max or min. We need only decide whether $x = 1$ and $x = -1$ are local extrema.

We can easily graph $f'(x)$, and we see it is an upwards-pointing parabola. It is positive to the left of $x = -1$ and positive to its right, so f is increasing up till $x = -1$, then decreasing after; so $x = 1$ is a local max.

Likewise, $f'(x)$ is negative to the left of $x = 1$ and positive to the right of it; so it's decreasing till $x = 1$ and increasing after. Thus $x = 1$ is a local min.

[1]Extrema: local and global maxima and minima

Example: MaxMin 3

Find All Extrema

$$f(x) = \sqrt[3]{x^2 - 64}, \qquad x \text{ in } [-1, 10]$$

Example: MaxMin 3

Find All Extrema

$$f(x)=\sqrt[3]{x^2-64}, \qquad x \text{ in } [-1,10]$$

The endpoints are -1 and 10. We differentiate to identify critical points and singular points: $f'(x)=\frac{1}{3}(x^2-64)^{-2/3}(2x)=\frac{2}{3}x(x^2-64)^{-2/3}$. So the critical point is $x=0$ and the singular points are $x=\pm 8$; but since $x=-8$ is not our domain, we don't have to worry about it.
The global extrema are found by simply comparing the value of the function at the various interesting points.
$f(0)=\sqrt[3]{-64}=-4$; $f(8)=0$; $f(-1)=-\sqrt[3]{63}$; and $f(10)=\sqrt[3]{100-64}=\sqrt[3]{36}$. Of these, -4 is the smallest and $\sqrt[3]{36}$ is the largest, so the global max is $\sqrt[3]{36}$ at $x=10$, and the global min is -4 at $x=0$.
Then it's pretty clear that $x=0$ is a local min. Since -1 and 10 are endpoints, they can't be local mins. So, what of $x=8$? When x is slightly smaller than 8, or slightly larger than 8, $f'(x)$ is positive; so $f(x)$ is increasing to the left of 8 and also to the right of 8. Then 8 is neither a local max nor a local min.

Example: MaxMin 4

Find the largest and smallest value of $f(x) = x^4 - 18x^2$.

Example: MaxMin 4

Find the largest and smallest value of $f(x) = x^4 - 18x^2$.

There are no endpoints given, so we take the domain to be the domain of the function, which is all real numbers. As x goes to infinity or negative infinity, $f(x)$ goes to infinity, so there is no global max, hence no largest value.
To find the global min, we differentiate: $f'(x) = 4x^3 - 36x = 4x(x^2 - 9)$. So the critical points are 0 and ± 3, and there are no singular points.
$f(0) = 0$, and $f(3) = f(-3) = -81$, so the smallest value (and global min) is -81, and it occurs twice (which is fine): at 3 and -3.

Example: MaxMin 5

Find the largest and smallest values of $f(x) = \sin^2 x - \cos x$.

Example: MaxMin 5

Find the largest and smallest values of $f(x) = \sin^2 x - \cos x$.

Since this function is periodic, we can restrict our search to x values in $[0, 2\pi)$. $f'(x) = 2\sin x \cos x + \sin x = \sin x(2\cos x + 1)$. So our critical points occur when $\sin x = 0$ and when $\cos x = -1/2$. That is, when x is $0, \pi, 2\pi/3$, or $4\pi/3$. We plug these in to find $f(0) = -1$, $f(\pi) = 1$, and $f(2\pi/3) = f(4\pi/3) = \frac{5}{4}$. So the biggest this function gets is 1.25, and this occurs at $x = (2+6n)\pi/3$ and $(4+6n)\pi/3$ for any integer n. The smallest $f(x)$ gets is -1, and this occurs at $x = 2\pi n$, for any integer n.