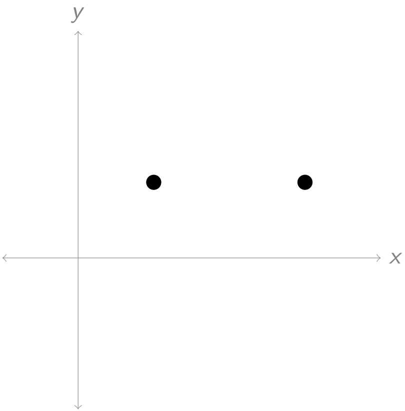
Rolle's Theorem



Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

-
-
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
-
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Rolle's Theorem

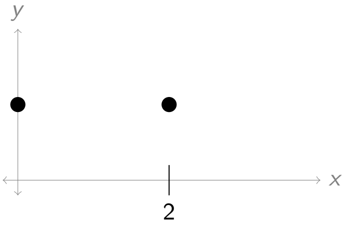
Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Example: Let $f(x) = x^3 - 2x^2 + 1$, and observe $f(2) = f(0) = 1$. Since $f(x)$ is a polynomial, it is continuous and differentiable everywhere.



Rolle's Theorem

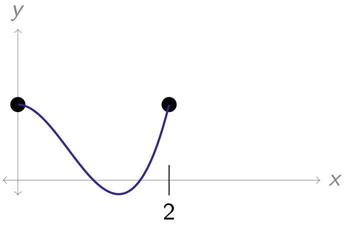
Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

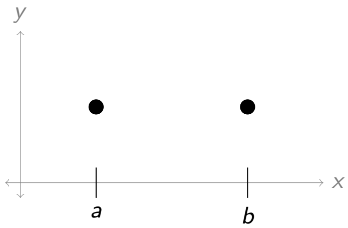
$$f'(c) = 0.$$

Example: Let $f(x) = x^3 - 2x^2 + 1$, and observe $f(2) = f(0) = 1$. Since $f(x)$ is a polynomial, it is continuous and differentiable everywhere.



$f'(4/3) = 0$

Rolle's Theorem



Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

How many different values of x between a and b have $f'(x) = 0$?

A. 0 or 1

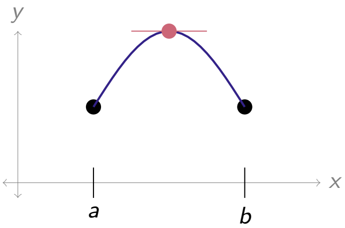
B. 1

C. 0, 1, or more

D. 1 or more

E. I'm not sure

Rolle's Theorem

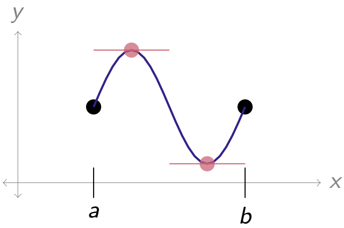


Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

How many different values of x between a and b have $f'(x) = 0$?

A. 0 or 1
B. 1
C. 0, 1, or more
D. 1 or more
E. I'm not sure

Rolle's Theorem



Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

How many different values of x between a and b have $f'(x) = 0$?

A. 0 or 1

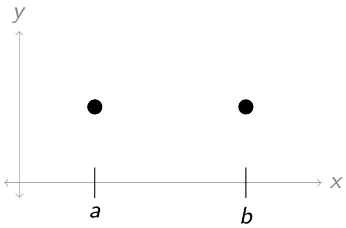
B. 1

C. 0, 1, or more

D. 1 or more

E. I'm not sure

Rolle's Theorem

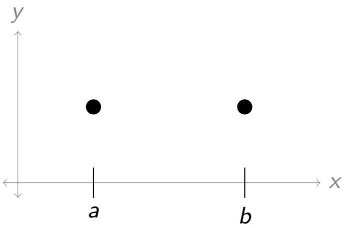


Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

How many different values of x between a and b have $f'(x) = 0$?

A. 0 or 1
B. 1
C. 0, 1, or more
D. 1 or more
E. I'm not sure

Rolle's Theorem

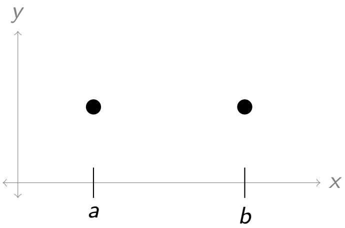


Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

How many different values of x between a and b have $f'(x) = 0$?

A. 0 or 1
B. 1
C. 0, 1, or more
D. 1 or more
E. I'm not sure

Rolle's Theorem

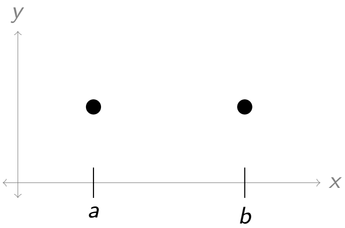


Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

Can f have an infinte number of points where $f'(x) = 0$ between a and b?

A. Sure! :D
B. No way! $>$:-[
C. Only if a and b are infinitely far apart
D. I'm not sure

Rolle's Theorem



Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

Can f have an infinte number of points where $f'(x) = 0$ between a and b?

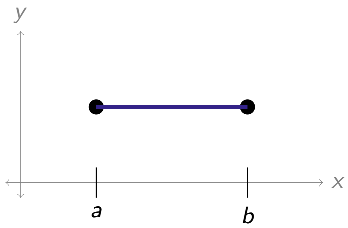
A. Sure! :D

B. No way! >:-[

C. Only if a and b are infinitely far apart

D. I'm not sure

Rolle's Theorem



Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

Can f have an infinte number of points where $f'(x) = 0$ between a and b?

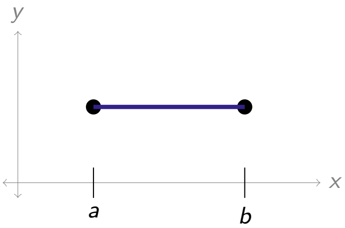
A. Sure! :D

B. No way! >:-[

C. Only if a and b are infinitely far apart

D. I'm not sure

Rolle's Theorem



Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

Can f have an infinte number of points where $f'(x) = 0$ between a and b?

A. Sure! :D
B. No way! >:-[
C. Only if a and b are infinitely far apart
D. I'm not sure

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f(x)$ has precisely seven roots. How many roots does $f'(x)$ have?

A. precisely six

B. precisely seven

C. at most seven

D. at least six

E. I don't know

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f(x)$ has precisely seven roots. How many roots does $f'(x)$ have?

A. precisely six

B. precisely seven

C. at most seven

D. at least six

E. I don't know

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f'(x)$ is also continuous and differentiable for all real numbers, and $f(x)$ has precisely seven roots. How many roots does $f''(x)$ have?

A. precisely six

B. precisely five

C. at most five

D. at least five

E. I don't know

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f'(x)$ is also continuous and differentiable for all real numbers, and $f(x)$ has precisely seven roots. How many roots does $f''(x)$ have?

A. precisely six

B. precisely five

C. at most five

D. at least five

E. I don't know

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Suppose $f(x)$ is continuous and differentiable for all real numbers, and there are precisely three places where $f'(x) = 0$. How many roots does $f(x)$ have?

A. at most three

B. at most four

C. at least three

D. at least four

E. I don't know

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Suppose $f(x)$ is continuous and differentiable for all real numbers, and there are precisely three places where $f'(x) = 0$. How many roots does $f(x)$ have?

A. at most three

B. at most four

C. at least three

D. at least four

E. I don't know

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f'(x) = 0$ for precisely three values of x. How many distinct values x exist with $f(x) = 17$?

A. at most three

B. at most four

C. at least three

D. at least four

E. I don't know

Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

- $f(x)$ is continuous for every x with $a \leq x \leq b$;
- $f(x)$ is differentiable for every x with $a < x < b$;
- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f'(x) = 0$ for precisely three values of x. How many distinct values x exist with $f(x) = 17$?

A. at most three

B. at most four

C. at least three

D. at least four

E. I don't know

What's the Use?

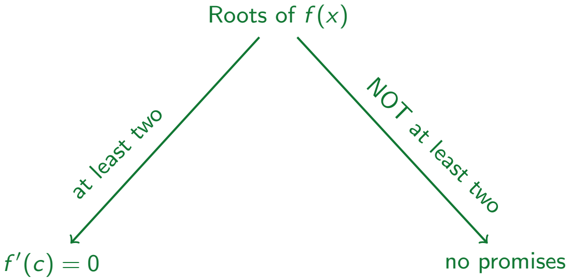
Example: MeanValue 1

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

What's the Use?

Example: MeanValue 1

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.



Rolle's Theorem

What's the Use?

Example: MeanValue 1

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

Note that $f(x)$ is continuous and differentiable over all real numbers. So, by Rolle's Theorem, if it has two roots, then $f'(x) = 0$ for some x.

What's the Use?

Example: MeanValue 1

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

Note that $f(x)$ is continuous and differentiable over all real numbers. So, by Rolle's Theorem, if it has two roots, then $f'(x) = 0$ for some x.

$f'(x) = 3x^2 + 1$, and this is always positive, so it's never zero. Therefore, by Rolle's Theorem, $f(x)$ does not have two roots; so it has at most one.

What's the Use?

Example: MeanValue 1

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

Note that $f(x)$ is continuous and differentiable over all real numbers. So, by Rolle's Theorem, if it has two roots, then $f'(x) = 0$ for some x.

$f'(x) = 3x^2 + 1$, and this is always positive, so it's never zero. Therefore, by Rolle's Theorem, $f(x)$ does not have two roots; so it has at most one.

Logical Structure:

What's the Use?

Example: MeanValue 1

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

Note that $f(x)$ is continuous and differentiable over all real numbers. So, by Rolle's Theorem, if it has two roots, then $f'(x) = 0$ for some x.

$f'(x) = 3x^2 + 1$, and this is always positive, so it's never zero. Therefore, by Rolle's Theorem, $f(x)$ does not have two roots; so it has at most one.

Logical Structure:

- If A is true, then B is true.
- B is false.
- Therefore, A is false.

What's the Use?

Example: MeanValue 1

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

Note that $f(x)$ is continuous and differentiable over all real numbers. So, by Rolle's Theorem, if it has two roots, then $f'(x) = 0$ for some x.

$f'(x) = 3x^2 + 1$, and this is always positive, so it's never zero. Therefore, by Rolle's Theorem, $f(x)$ does not have two roots; so it has at most one.

Logical Structure:

- If A is true, then B is true.
- B is false.
- Therefore, A is false.

- If $f(x)$ has two (or more) roots, then $f'(x)$ has a root.
- $f'(x)$ does not have a root.
- Therefore, $f(x)$ does not have two (or more) roots.

What's the Use?

Example: MeanValue 1

Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

Note that $f(x)$ is continuous and differentiable over all real numbers. So, by Rolle's Theorem, if it has two roots, then $f'(x) = 0$ for some x.

$f'(x) = 3x^2 + 1$, and this is always positive, so it's never zero. Therefore, by Rolle's Theorem, $f(x)$ does not have two roots; so it has at most one.

Logical Structure:

- If A is true, then B is true.
- B is false.
- Therefore, A is false.

- If $f(x)$ has two (or more) roots, then $f'(x)$ has a root.
- $f'(x)$ does not have a root.
- Therefore, $f(x)$ does not have two (or more) roots.

How would you show that it has *precisely* one real root?

Example: MeanValue 2

Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

Example: MeanValue 2

Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

Again we use the structure:

- *If $f(x)$ has three roots, then $f'(x)$ has two roots.*
- *$f'(x)$ does not have two roots.*
- *Therefore, $f(x)$ does not have three roots.*

Example: MeanValue 2

Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

Again we use the structure:

- *If $f(x)$ has three roots, then $f'(x)$ has two roots.*
- *$f'(x)$ does not have two roots.*
- *Therefore, $f(x)$ does not have three roots.*

So, all we need to do is make sure the conditions of Rolle's Theorem are satisfied, and show that $f'(x)$ does not have three roots.

Example: MeanValue 2

Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

Again we use the structure:

- *If $f(x)$ has three roots, then $f'(x)$ has two roots.*
- *$f'(x)$ does not have two roots.*
- *Therefore, $f(x)$ does not have three roots.*

So, all we need to do is make sure the conditions of Rolle's Theorem are satisfied, and show that $f'(x)$ does not have three roots.

Since $f(x)$ is continuous and differentiable over all real numbers, the conditions of Rolle's Theorem are satisfied.

Example: MeanValue 2

Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

Again we use the structure:

- *If $f(x)$ has three roots, then $f'(x)$ has two roots.*
- *$f'(x)$ does not have two roots.*
- *Therefore, $f(x)$ does not have three roots.*

So, all we need to do is make sure the conditions of Rolle's Theorem are satisfied, and show that $f'(x)$ does not have three roots.

Since $f(x)$ is continuous and differentiable over all real numbers, the conditions of Rolle's Theorem are satisfied.

$f'(x) = x^2 + 6x + 9 = (x + 3)^2$, which only has ONE root.

Example: MeanValue 2

Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

Again we use the structure:

- *If $f(x)$ has three roots, then $f'(x)$ has two roots.*
- *$f'(x)$ does not have two roots.*
- *Therefore, $f(x)$ does not have three roots.*

So, all we need to do is make sure the conditions of Rolle's Theorem are satisfied, and show that $f'(x)$ does not have three roots.

Since $f(x)$ is continuous and differentiable over all real numbers, the conditions of Rolle's Theorem are satisfied.

$f'(x) = x^2 + 6x + 9 = (x+3)^2$, which only has ONE root.

Therefore, $f'(x)$ does not have two roots, so $f(x)$ does not have three roots.

Example: MeanValue 2

Use Rolle's Theorem to show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

Again we use the structure:

- *If $f(x)$ has three roots, then $f'(x)$ has two roots.*
- *$f'(x)$ does not have two roots.*
- *Therefore, $f(x)$ does not have three roots.*

So, all we need to do is make sure the conditions of Rolle's Theorem are satisfied, and show that $f'(x)$ does not have three roots.

Since $f(x)$ is continuous and differentiable over all real numbers, the conditions of Rolle's Theorem are satisfied.

$f'(x) = x^2 + 6x + 9 = (x+3)^2$, which only has ONE root.

Therefore, $f'(x)$ does not have two roots, so $f(x)$ does not have three roots.

So, $f(x)$ has at most two roots.

Example: MeanValue 3

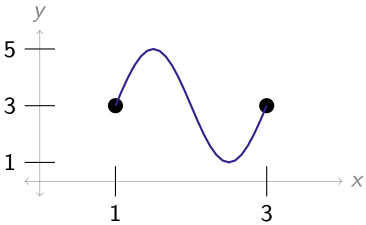
Show that the function $f(x) = \frac{1}{4}x^4 + x + 9$ has no real roots.

Example: MeanValue 3

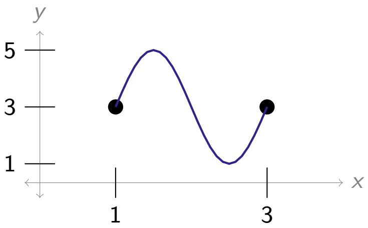
Show that the function $f(x) = \frac{1}{4}x^4 + x + 9$ has no real roots.

Rolle's Theorem can't help us show that there are *no* real roots. We find the global minimum instead. Since $\lim_{x\to\pm\infty} f(x) = \infty$, there is no global maximum. There are n singular points, and the only critical point is at $x = -1$, and this is the global minimum. Since $f(-1) = \frac{1}{4} - 1 + 9 > 0$, we conclude $f(x)$ has no roots.

Average Rate of Change



Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 1$ to $x = 3$?

A. 0

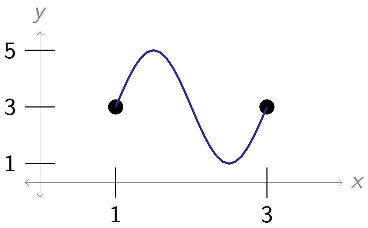
B. 1

C. 2

D. 4

E. I'm not sure

Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 1$ to $x = 3$?

A. 0 $\frac{\Delta y}{\Delta x} = \frac{3-3}{3-1} = \frac{0}{2} = 0$

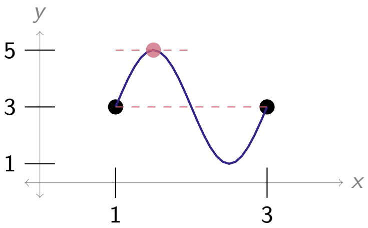
B. 1

C. 2

D. 4

E. I'm not sure

Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 1$ to $x = 3$?

A. 0

B. 1

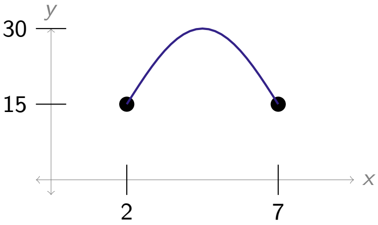
C. 2

D. 4

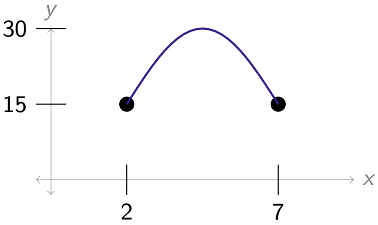
E. I'm not sure

$\frac{\Delta y}{\Delta x} = \frac{3-3}{3-1} = \frac{0}{2} = 0$

Average Rate of Change



Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 2$ to $x = 7$?

A. 0

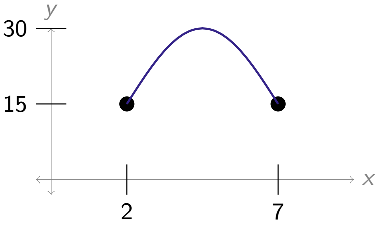
B. 3

C. 5

D. 15

E. I'm not sure

Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 2$ to $x = 7$?

A. 0

$\frac{\Delta y}{\Delta x} = \frac{15-15}{7-2} = \frac{0}{5} = 0$

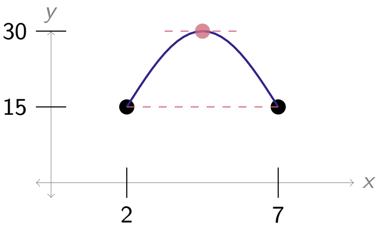
B. 3

C. 5

D. 15

E. I'm not sure

Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 2$ to $x = 7$?

A. 0

$\frac{\Delta y}{\Delta x} = \frac{15-15}{7-2} = \frac{0}{5} = 0$

B. 3

C. 5

D. 15

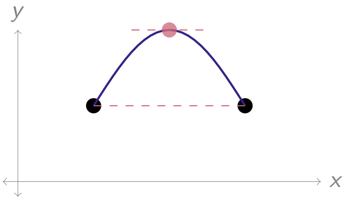
E. I'm not sure

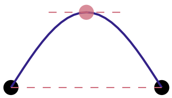
Rolle's Theorem and Average Rate of Change

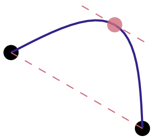
Suppose $f(x)$ is continuous on the interval $[a, b]$, differentiable on the interval (a, b), and $f(a) = f(b)$. Then there exists a number c strictly between a and b such that

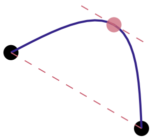
$$f'(c) = 0 = \frac{f(b) - f(a)}{b - a}.$$

So there exists a point where the derivative is the same as the average rate of change.

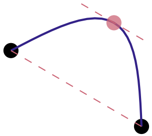

y
x







Mean Value Theorem



Mean Value Theorem

Let $f(x)$ be continuous on the interval $[a, b]$ and differentiable on (a, b). Then there is a number c between a and b such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

That is: there is some point c strictly between a and b where the instantaneous rate of change of the function is equal to the average rate of change of the function on the interval $[a, b]$.

Rolle's Theorem

Let $f(x)$ be continuous on the interval $[a, b]$, differentiable on (a, b), and let $f(a) = f(b)$. Then there is a number c strictly between a and b such that:

$$f'(c) = 0 = \frac{f(b) - f(a)}{b - a}$$

Example: MeanValue 4

Suppose you are driving along a long, straight highway with no shortcuts. The speed limit is 100 kph. A police officer notices your car going 90 kph, and uploads your plate and the time they saw you to their database. 150 km down this same straight road, 75 minutes later, another police officer notices your car going 85kph, and uploads your plates to the database. Then they pull you over, and give you a speeding ticket. Why were they justified?

link, Wikimedia commons, creative commons

Example: MeanValue 4

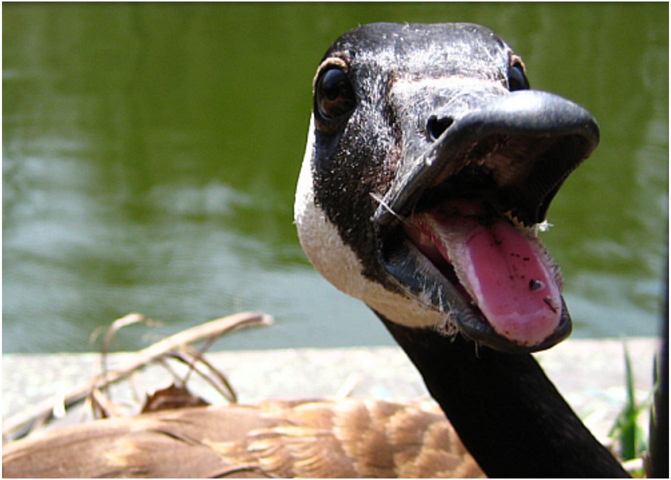
Suppose you are driving along a long, straight highway with no shortcuts. The speed limit is 100 kph. A police officer notices your car going 90 kph, and uploads your plate and the time they saw you to their database. 150 km down this same straight road, 75 minutes later, another police officer notices your car going 85kph, and uploads your plates to the database. Then they pull you over, and give you a speeding ticket. Why were they justified?

You travelled 150 km in 75 minutes. Since a moving car has a position that is continuous and differentiable, the MVT tells us that at some point, your instantaneous velocity was $\frac{150}{75}$ kilometers per minute, which works out to $\frac{150 \cdot 60}{75} = 120$ kph. So even though you weren't speeding when the officers saw you, you were definitely speeding some time in between.

Alternately, if you were going at most 100kph, then you would travel 150 kilometers in at least 90 minutes.

Example: MeanValue 5

According to this website, Canada geese may fly 1500 miles in a single day under favorable conditions. It also says their top speed is around 70mph. Does this seem like a typo? (If it contradicts the Mean Value Theorem, it's probably a typo.)



Credit: This Incredible World, link, unedited, creative commons license

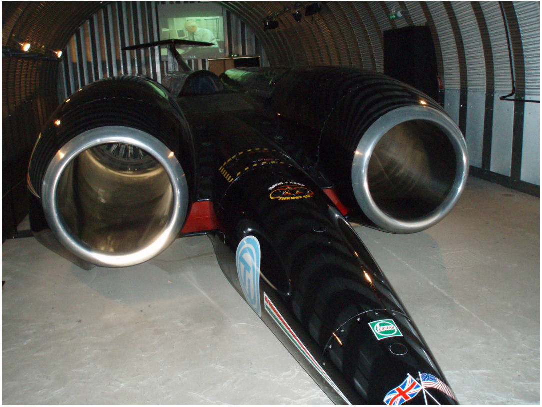
Example: MeanValue 5

According to this website, Canada geese may fly 1500 miles in a single day under favorable conditions. It also says their top speed is around 70mph. Does this seem like a typo? (If it contradicts the Mean Value Theorem, it's probably a typo.)

We can assume that the position of a goose is continuous and differentiable. Then the MVT tells us that a goose that travels 1500 miles in a day (24 hours) achieves, at some instant, a speed of $\frac{1500}{24}$ mph. Since $\frac{1500}{24} = 62.5$, these two facts seem compatible (and amazing!).

Example: MeanValue 6

The record for fastest wheel-driven land speed is around 700 kph. [1] However, non-wheel driven cars (such as those powered by jet engines) have achieved higher speeds. [2]
Suppose a driver of a jet-powered car starts a 10km race at 12:00, and finishes at 12:01. Did they beat 700kph?



[1] George Poteet, https://en.wikipedia.org/wiki/Wheel-driven_land_speed_record
[2] record-holder ThrustSSC shown

Example: MeanValue 6

The record for fastest wheel-driven land speed is around 700 kph. [1] However, non-wheel driven cars (such as those powered by jet engines) have achieved higher speeds. [2]
Suppose a driver of a jet-powered car starts a 10km race at 12:00, and finishes at 12:01. Did they beat 700kph?

Maybe, but not necessarily. We are guaranteed by the MVT that at some point they reached the following speed: $\frac{10}{(1/60)} = 600$ kph.

[1] George Poteet, https://en.wikipedia.org/wiki/Wheel-driven_land_speed_record
[2] record-holder ThrustSSC shown

Example: MeanValue 7

Suppose you want to download a file that is 3000 MB (slightly under 3GB). Your internet provider guarantees you that your download speeds will always be between 1 MBPS (MB per second) and 5 MBPS (because you bought the cheap plan). Using the Mean Value Theorem, give an upper and lower bound for how long the download can take (assuming your providers aren't lying, and your device is performing adequately).

Example: MeanValue 7

Suppose you want to download a file that is 3000 MB (slightly under 3GB). Your internet provider guarantees you that your download speeds will always be between 1 MBPS (MB per second) and 5 MBPS (because you bought the cheap plan). Using the Mean Value Theorem, give an upper and lower bound for how long the download can take (assuming your providers aren't lying, and your device is performing adequately).

We assume the download is continuous and differentiable, so we can use the MVT. Let T be the time (in seconds) the download takes. The MVT tells us that at some point, our speed was exactly $\frac{3000}{T}$, so it must be true that

$$1 \leq \frac{3000}{T} \leq 5$$

So, $\frac{3000}{5} \leq T \leq 3000$. That is, T is between 600 and 3000 seconds, or between 10 and 50 minutes.

Example: MeanValue 8

Suppose $1 \leq f'(t) \leq 5$ for all values of t, and $f(0) = 0$. What are the possible solutions to $f(t) = 3000$?

Example: MeanValue 8

Suppose $1 \leq f'(t) \leq 5$ for all values of t, and $f(0) = 0$. What are the possible solutions to $f(t) = 3000$?
Notice: since the derivative exists for all real numbers, $f(x)$ is differentiable and continuous for all real numbers!

Example: MeanValue 8

Suppose $1 \leq f'(t) \leq 5$ for all values of t, and $f(0) = 0$. What are the possible solutions to $f(t) = 3000$?
Notice: since the derivative exists for all real numbers, $f(x)$ is differentiable and continuous for all real numbers!

Since f is continuous and differentiable, we can use the MVT.

$$\frac{f(t) - f(0)}{t - 0} = \frac{3000}{t} = f'(c)$$

for some value c between 0 and t.
So,

$$1 \leq \frac{3000}{t} \leq 5$$

hence

$$600 \leq t \leq 3000$$

Corollaries to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = 0$ for all x in (a, b), then

If $f'(x) = g'(x)$, then

If $f'(x) > 0$ for all x in (a, b), then

If $f'(x) < 0$ for all x in (a, b), then

Corollaries to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = 0$ for all x in (a, b), then $f(x)$ is constant in that interval. That is, $f(c) = f(d)$ for all c, d in $[a, b]$.

If $f'(x) = g'(x)$, then

If $f'(x) > 0$ for all x in (a, b), then

If $f'(x) < 0$ for all x in (a, b), then

Corollaries to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = 0$ for all x in (a, b), then $f(x)$ is constant in that interval. That is, $f(c) = f(d)$ for all c, d in $[a, b]$.

If $f'(x) = g'(x)$, then $f(x) = g(x) + A$ for some constant value A.

If $f'(x) > 0$ for all x in (a, b), then

If $f'(x) < 0$ for all x in (a, b), then

Corollaries to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = 0$ for all x in (a, b), then $f(x)$ is constant in that interval. That is, $f(c) = f(d)$ for all c, d in $[a, b]$.

If $f'(x) = g'(x)$, then $f(x) = g(x) + A$ for some constant value A.

If $f'(x) > 0$ for all x in (a, b), then $f(x)$ is increasing. That is, $f(c) < f(d)$ for all $c < d$ in $[a, b]$.

If $f'(x) < 0$ for all x in (a, b), then

Corollaries to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = 0$ for all x in (a, b), then $f(x)$ is constant in that interval. That is, $f(c) = f(d)$ for all c, d in $[a, b]$.

If $f'(x) = g'(x)$, then $f(x) = g(x) + A$ for some constant value A.

If $f'(x) > 0$ for all x in (a, b), then $f(x)$ is increasing. That is, $f(c) < f(d)$ for all $c < d$ in $[a, b]$.

If $f'(x) < 0$ for all x in (a, b), then $f(x)$ is decreasing. That is, $f(d) < f(c)$ for all $c < d$ in $[a, b]$.