Grading Code

Key

message	code
Good Job!	GJ
Nice Idea!	NI
Algebra Mistake	AM
Creative Strategy!	CS

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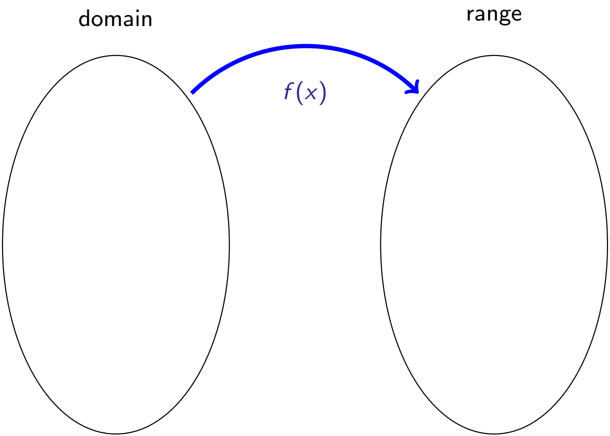
The longer code is not uniquely translatable; viewed as a function, it is not invertible.

Functions are Maps



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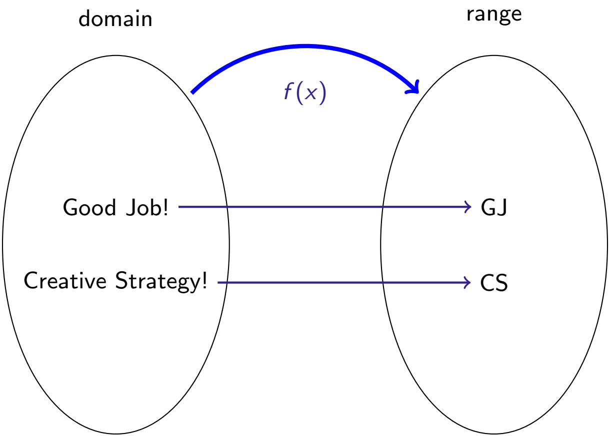

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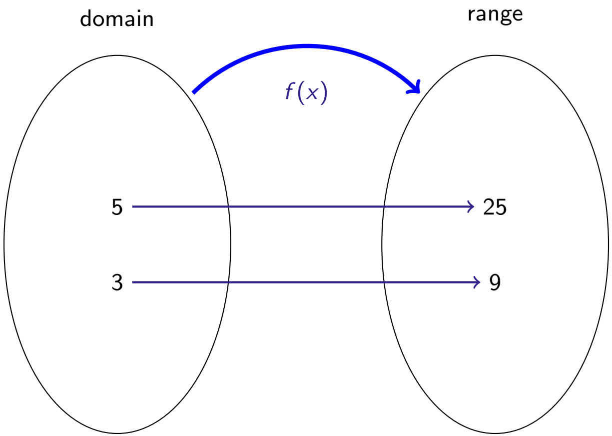

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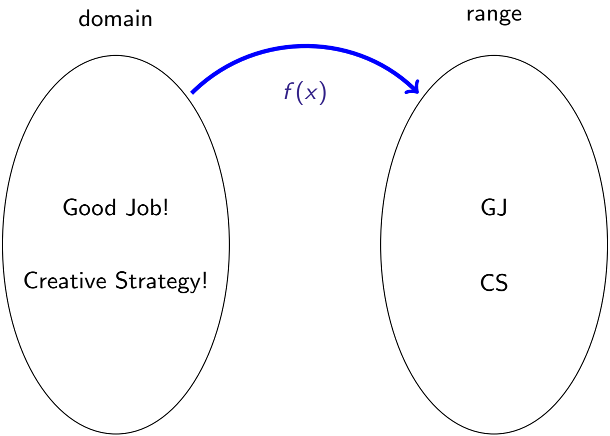

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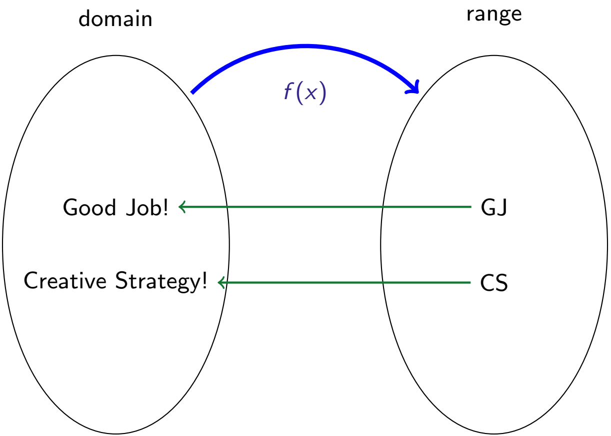

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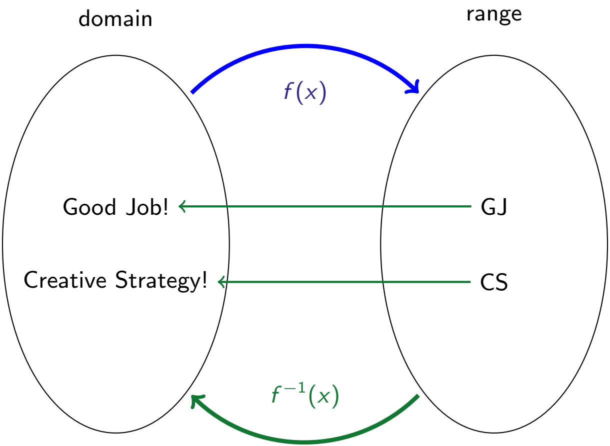

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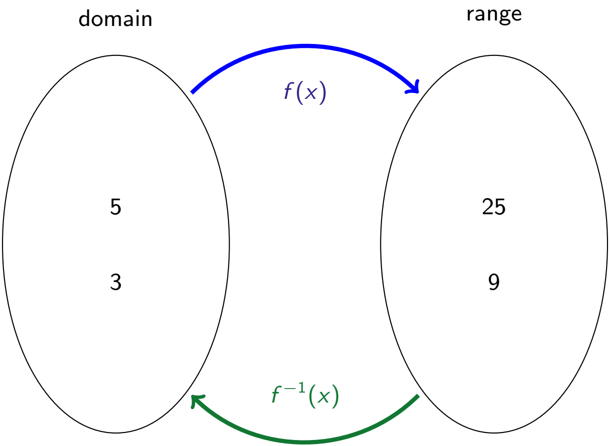

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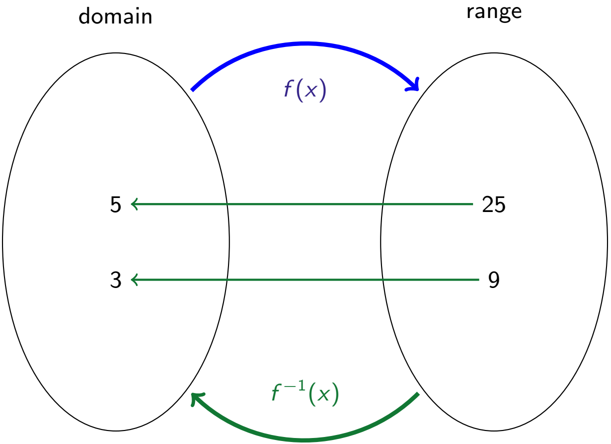

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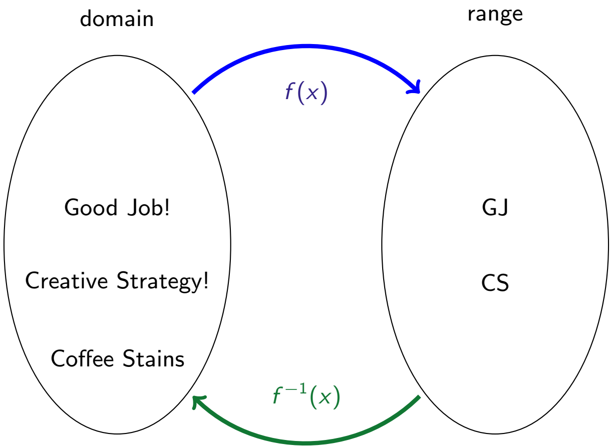

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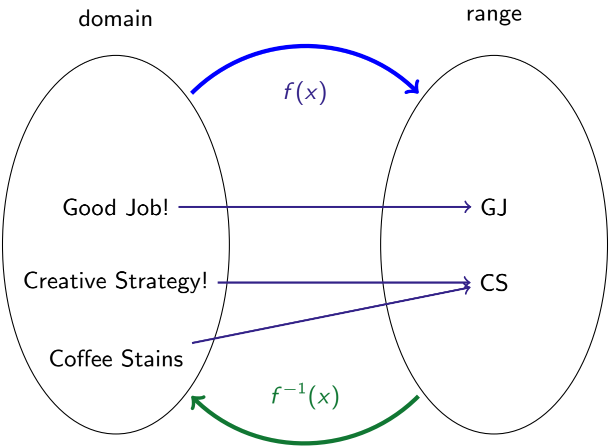

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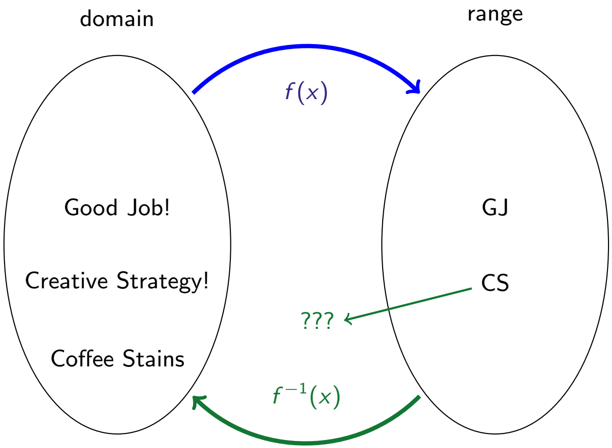

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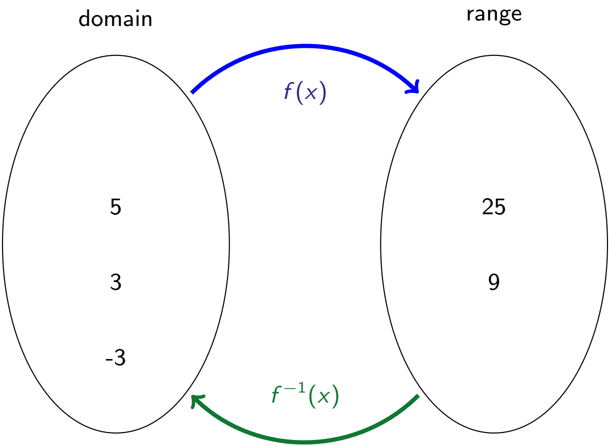

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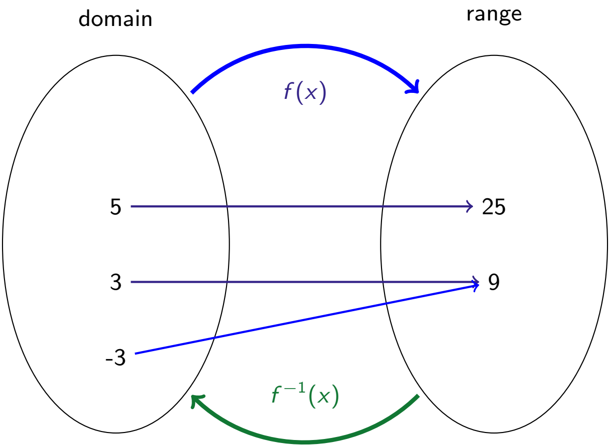

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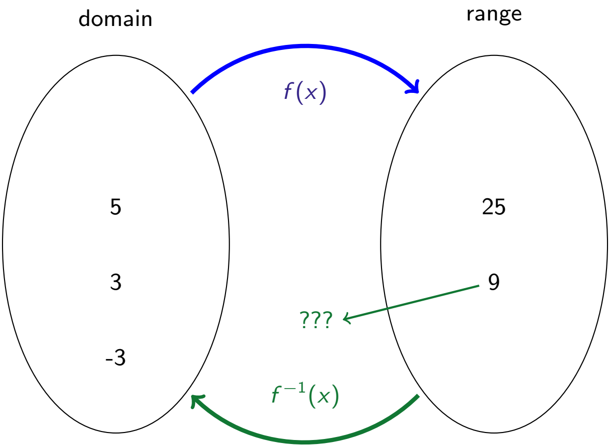
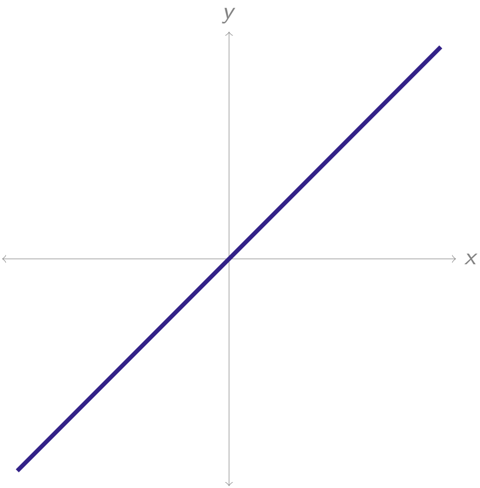
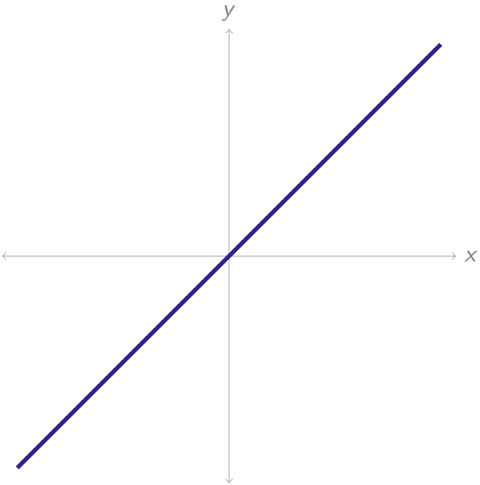




A. invertible B. not invertible C. not sure



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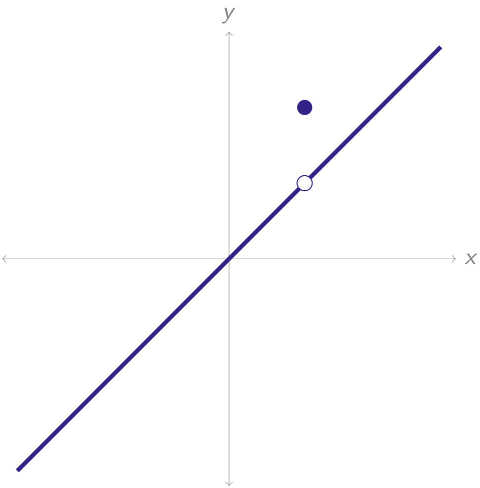

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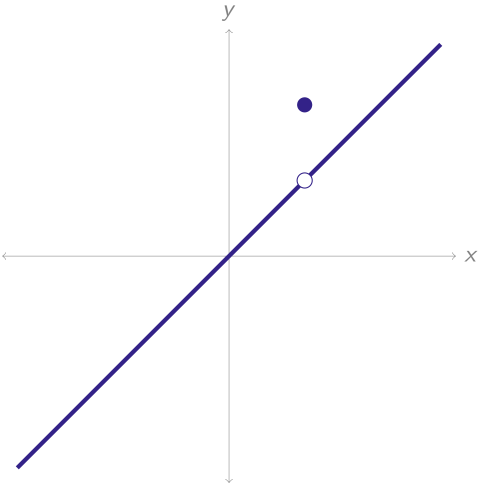

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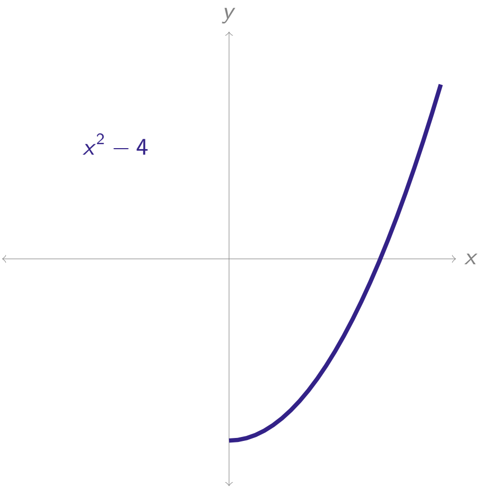

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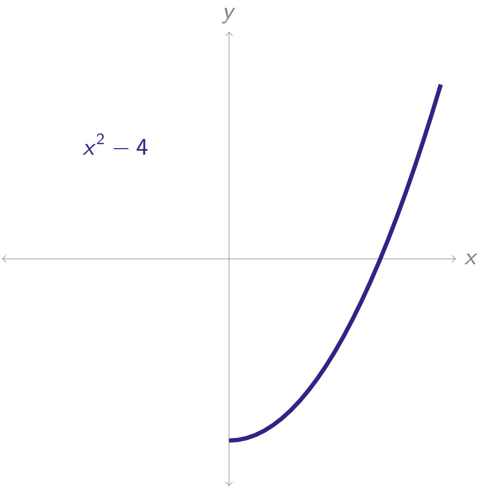

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Relationship between $f(x)$ and $f^{-1}(x)$

Let f be an invertible function.
What is $f^{-1}(f(x))$?

A. x

B. 1

C. 0

D. not sure

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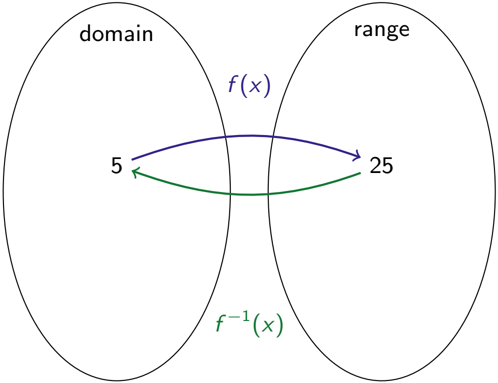

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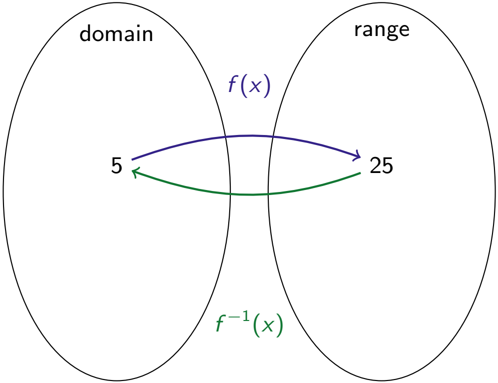

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What is $f(f^{-1}(x))$?

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Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)

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What is $f^{-1}(10)$? (do not simplify)

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What is $f^{-1}(10)$? (do not simplify)

What is $f^{-1}(x)$?

Invertibility

In order for a function to be invertible , different x values cannot map to the same y value.
We call such a function **one-to-one**, or **injective**.

Suppose $f(x) = \sqrt[3]{19 + x^3}$. What is $f^{-1}(3)$? (simplify your answer)
$f(2) = 3$, so $f^{-1}(3) = 2$

What is $f^{-1}(10)$? (do not simplify)
$\sqrt[3]{19 + y^3} = 10$ tells us $f^{-1}(10) = \sqrt[3]{10^3 - 19}$

What is $f^{-1}(x)$?
$\sqrt[3]{19 + y^3} = x$ tells us $f^{-1}(x) = \sqrt[3]{x^3 - 19}$

Example

$$\text{Let } f(x) = x^2 - x.$$

1. Sketch a graph of $f(x)$, and choose a domain over which it is invertible.

2. For the domain you chose, evaluate $f^{-1}(20)$.

3. For the domain you chose, evaluate $f^{-1}(x)$.

4. What are the domain and range of $f^{-1}(x)$? What are the (restricted) domain and range of $f(x)$?

Example

$$\text{Let } f(x) = x^2 - x.$$

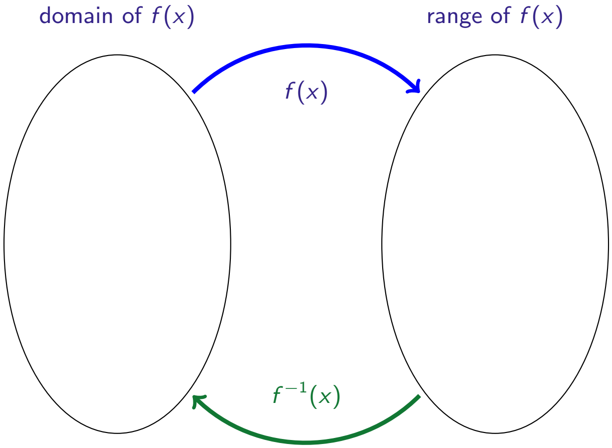
1. Sketch a graph of $f(x)$, and choose a domain over which it is invertible.
For instance, $(1/2, \infty)$

2. For the domain you chose, evaluate $f^{-1}(20)$.
This is a number y such that $y^2 - y = 20$. So y is either -4 or 5, depending on your choice of domain.

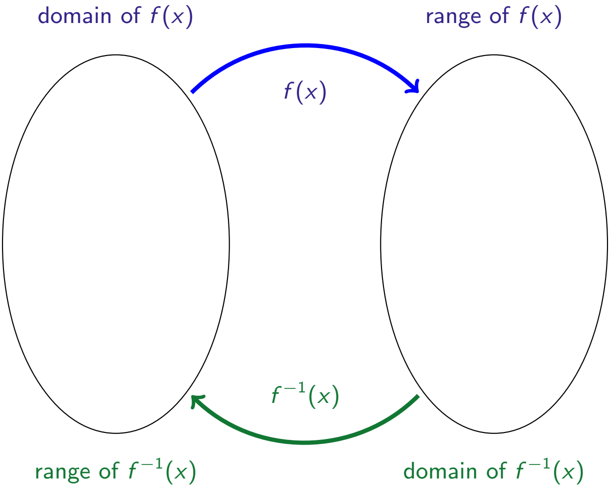
3. For the domain you chose, evaluate $f^{-1}(x)$.
$x = y^2 - y \quad \Rightarrow \quad y^2 - 1y - x = 0 \quad \Rightarrow \quad \frac{1 \pm \sqrt{1+4x}}{2}$, with plus or minus depending on domain chosen

4. What are the domain and range of $f^{-1}(x)$? What are the (restricted) domain and range of $f(x)$?

Domain and Range

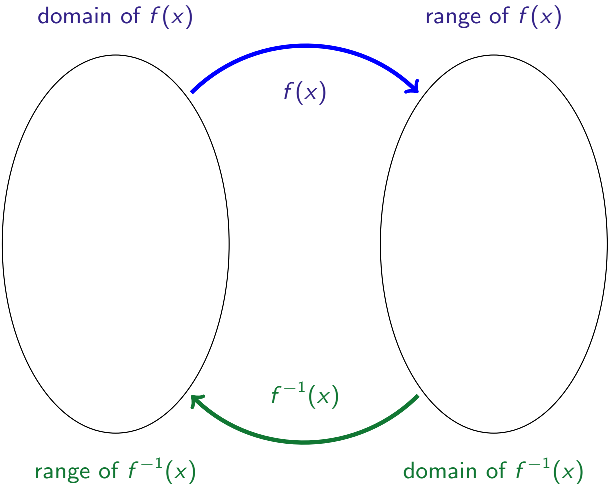


Domain and Range



Domain and Range

$f(x) = x^2 - x$, domain: $\left[\frac{1}{2}, \infty\right)$



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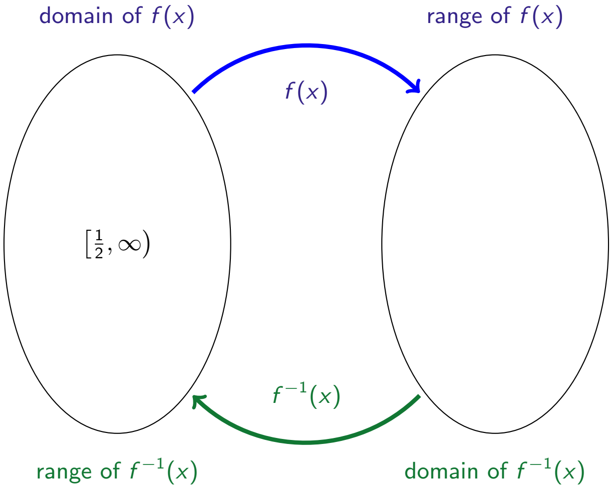

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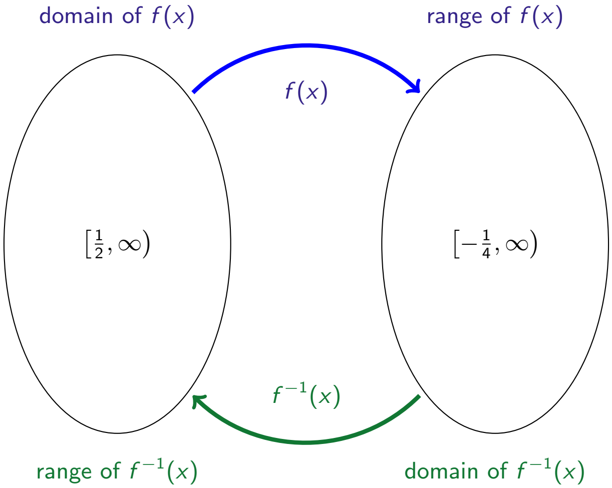
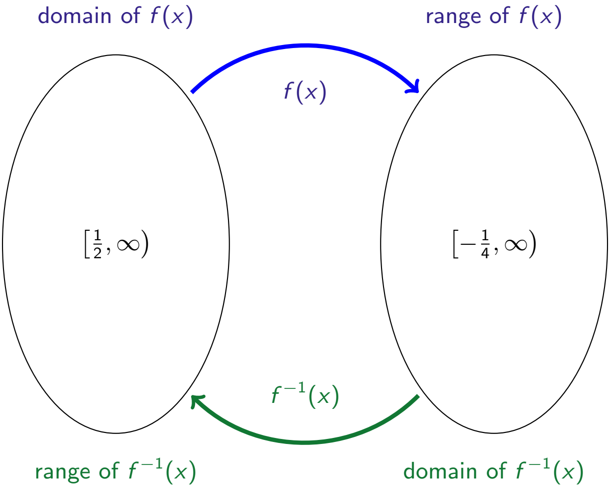


Domain and Range

$f(x) = x^2 - x$, domain: $\left[\frac{1}{2}, \infty\right)$

$$f^{-1}(x) = \frac{-1 + \sqrt{1 + 4x}}{2}$$



Exponents and Logarithms

$$f(x) = e^x \qquad f^{-1}(x) = \ln(x) = \log(x)$$

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x	e^x	
0	1	
1	e	
-1	$\frac{1}{e}$	
n	e^n	

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$\ln(1) =$

$\ln(e) =$

$\ln(\frac{1}{e}) =$

$\ln(e^n) =$

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x	e^x	ln fact $\leftrightarrow$ e fact
0	1	$\ln(1) = 0 \leftrightarrow e^0 = 1$
1	e	
-1	$\frac{1}{e}$	
n	e^n	

$\ln(1) = \boxed{0}$

$\ln(e) =$

$\ln(\frac{1}{e}) =$

$\ln(e^n) =$

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1	e	$\ln(e) = 1 \leftrightarrow e^1 = e$
-1	$\frac{1}{e}$	$\ln(\frac{1}{e}) = -1 \leftrightarrow e^{-1} = \frac{1}{e}$
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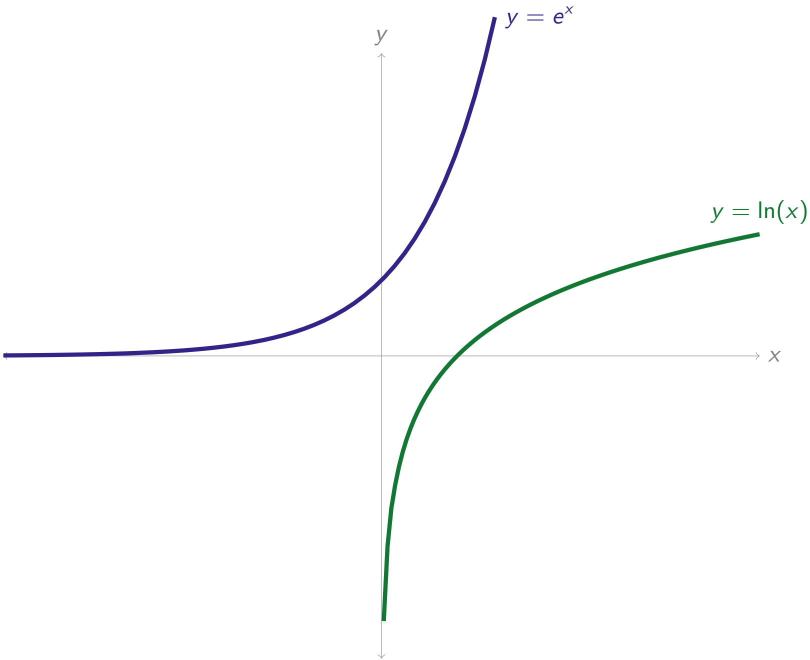
0	1	$\ln(1) = 0 \leftrightarrow e^0 = 1$
1	e	$\ln(e) = 1 \leftrightarrow e^1 = e$
-1	$\frac{1}{e}$	$\ln(\frac{1}{e}) = -1 \leftrightarrow e^{-1} = \frac{1}{e}$
n	e^n	$\ln(e^n) = n \leftrightarrow e^n = e^n$

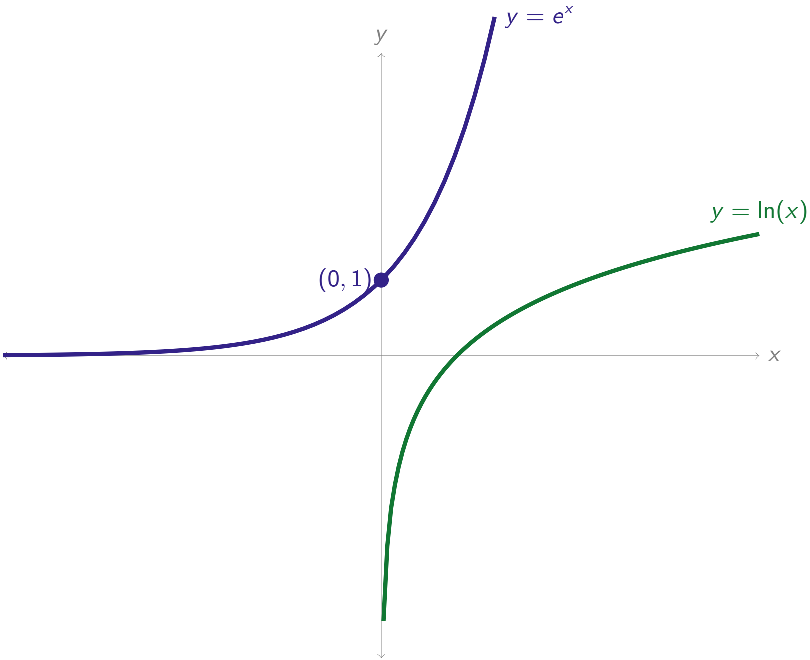
$\ln(1) = \boxed{0}$

$\ln(e) = \boxed{1}$

$\ln(\frac{1}{e}) = \boxed{-1}$

$\ln(e^n) = \boxed{n}$


$y = e^x$
y
$y = \ln(x)$
x


$y = e^x$
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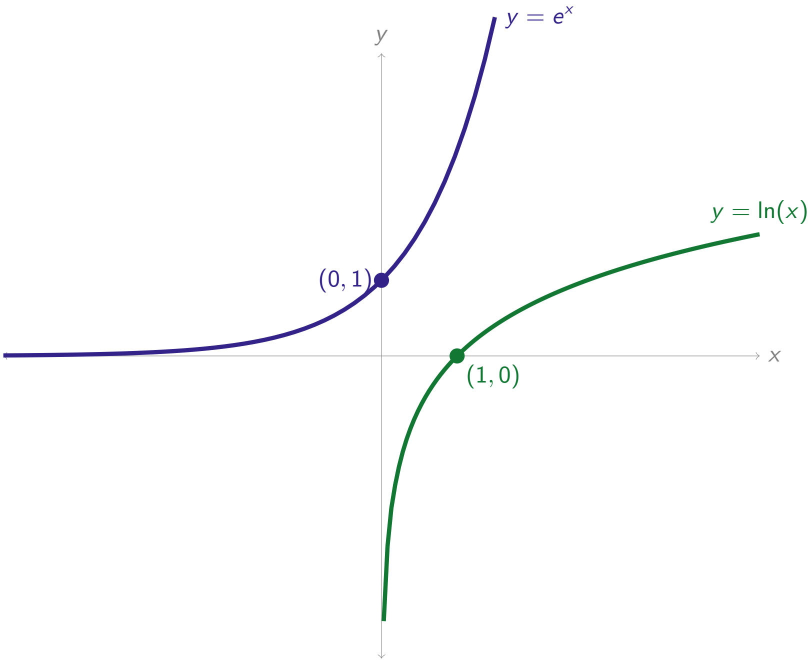
$(0, 1)$
x



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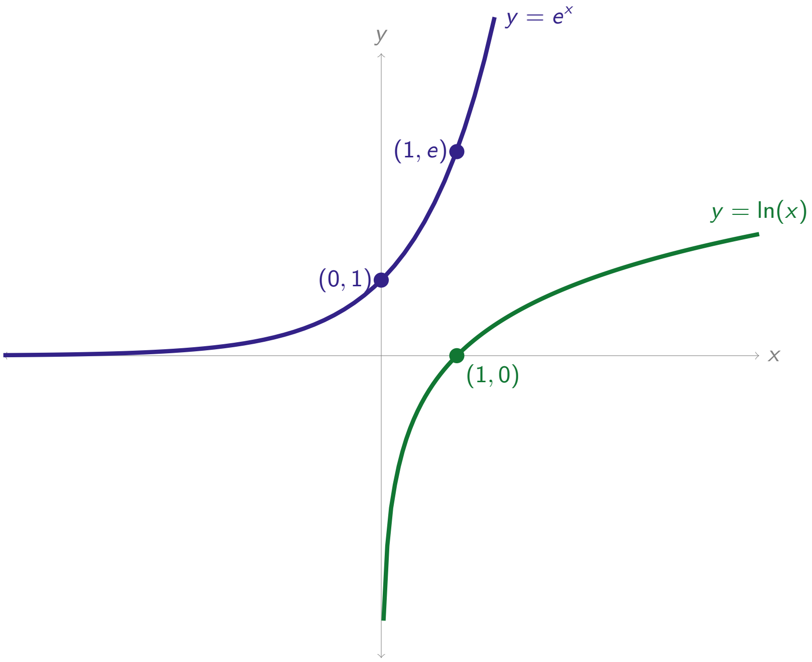
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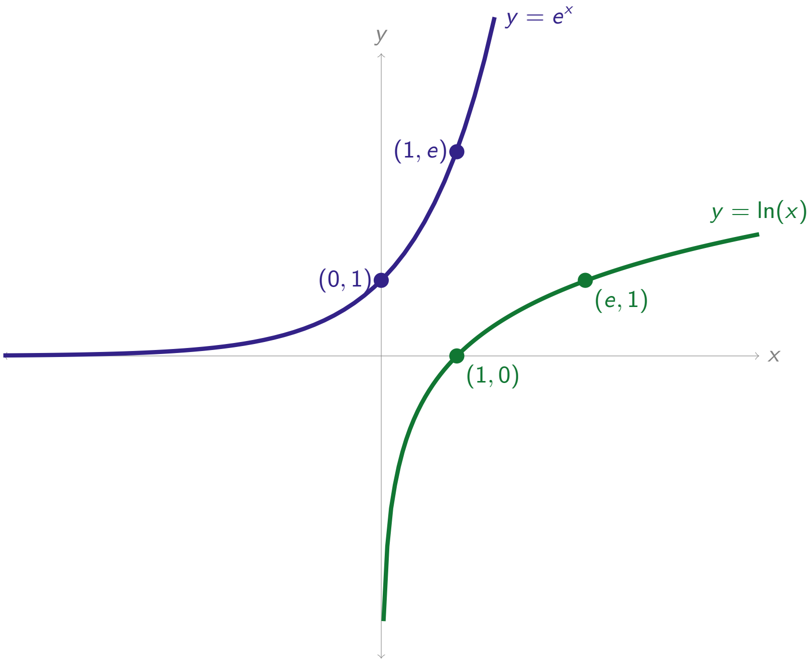

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$(e, 1)$
x
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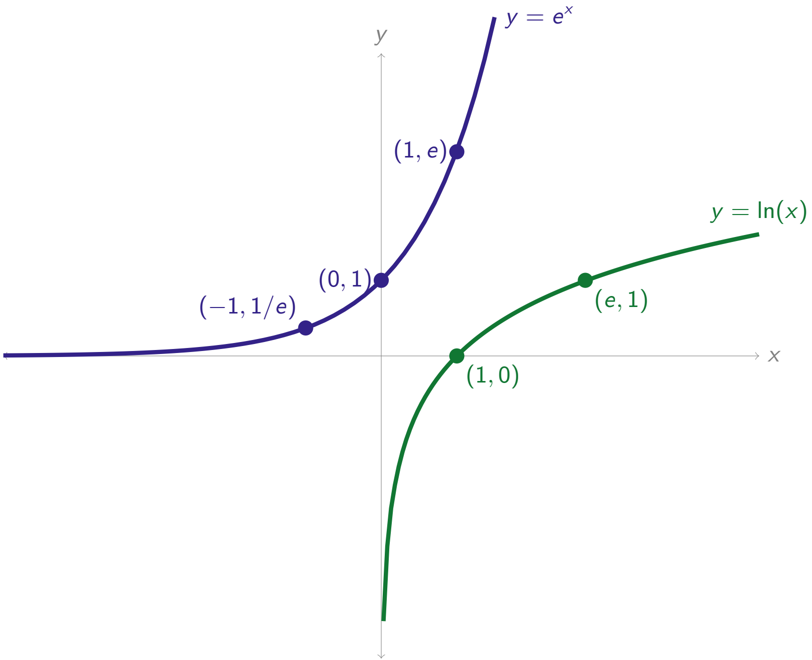
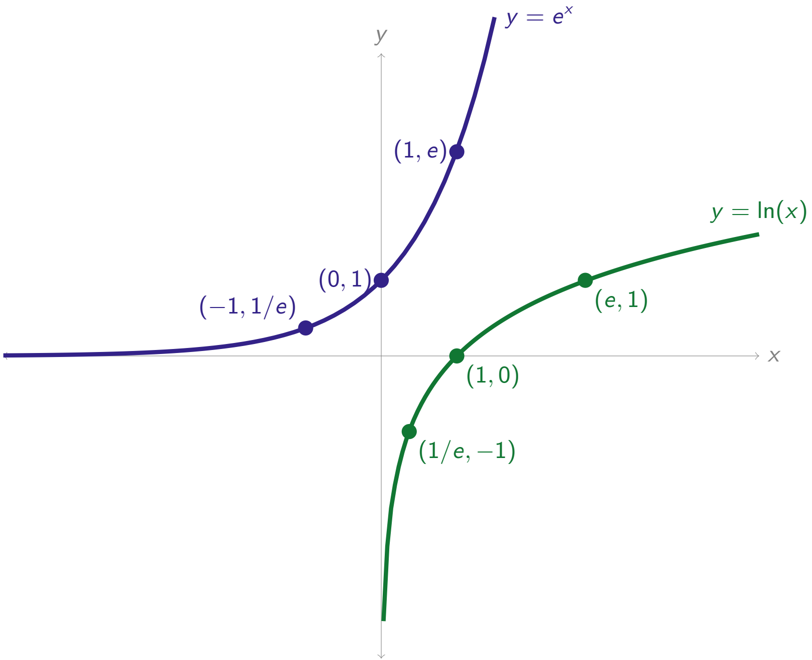

$y = e^x$
y
$(1, e)$
$y = \ln(x)$
$(0, 1)$
$(-1, 1/e)$
$(e, 1)$
x
$(1, 0)$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$y = \ln(x)$
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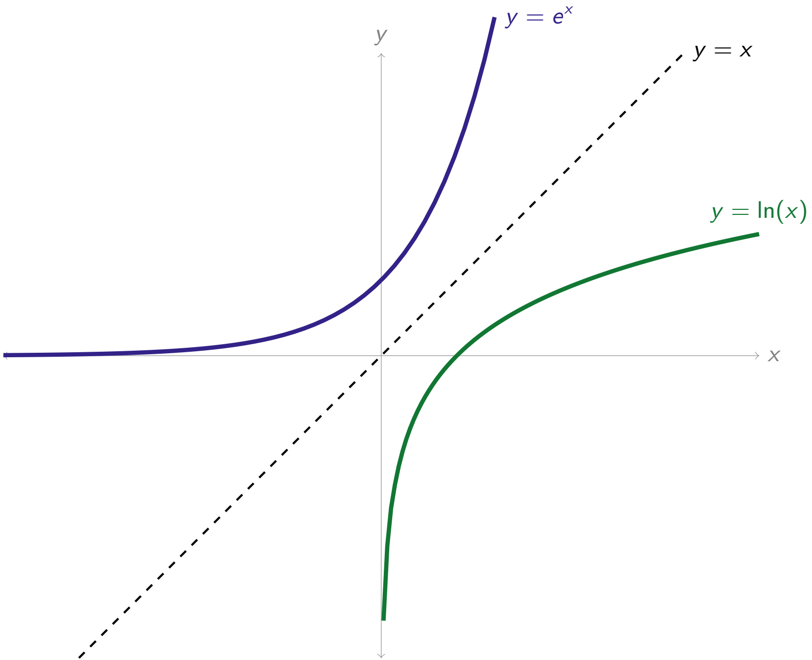
$(-1, 1/e)$
$(e, 1)$
x
$(1, 0)$
$(1/e, -1)$


$y = e^x$
y
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x

Logs of Other Bases

$\log_{10} 10^8 =$

A. 0

B. 8

C. 10

D. other

E. I'm not sure

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$\log_2 16 =$

A. 1
B. 2
C. 3
D. other
E. I'm not sure

Logs of Other Bases

$\log_{10} 10^8 =$

A. 0
B. 8
C. 10
D. other
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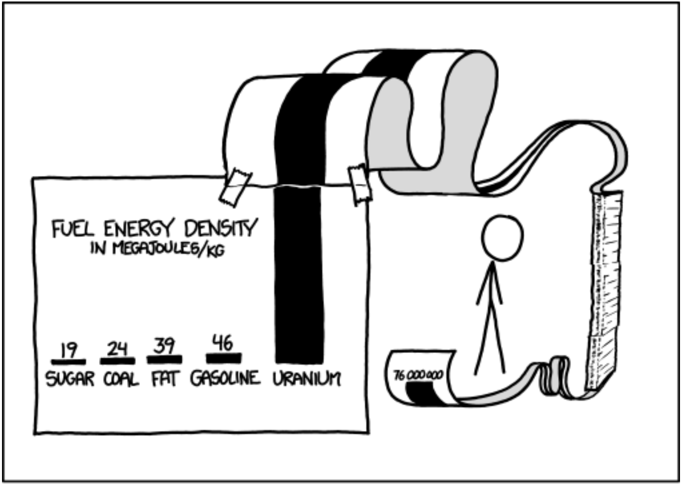
$\log_2 16 =$

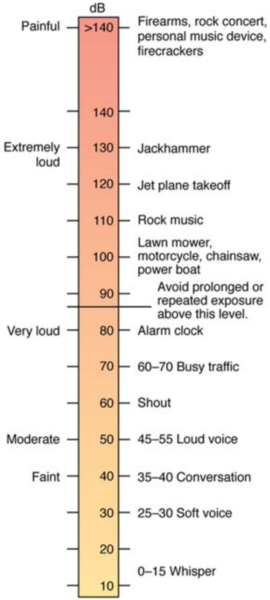
A. 1
B. 2
C. 3
D. other: $2^4 = 16$ so $\log_2 16 = 4$
E. I'm not sure



SCIENCE TIP: LOG SCALES ARE FOR QUITTERS WHO CAN'T FIND ENOUGH PAPER TO MAKE THEIR POINT *PROPERLY.*

`https://xkcd.com/1162/`

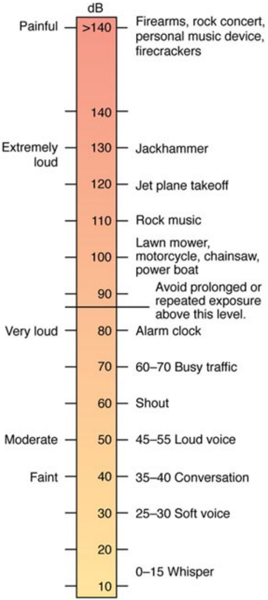
Log scale in action: `https://xkcd.com/482/`



Decibels: For a particular measure of the power P of a sound wave, the decibels of that sound is:

$$10\log_{10}(P)$$

So, every ten decibels corresponds to a sound being ten times louder.



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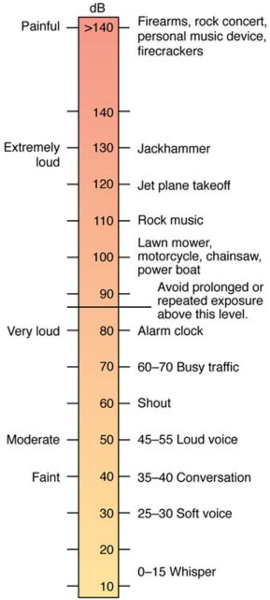
A lawnmower emits a 100dB sound. How much sound will two lawnmowers make?

A. 100 dB
B. 110 dB
C. 200 dB
D. other
E. I'm not sure

http://biology-forums.com/index.php?action=gallery;sa=view;id=6156



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So, every ten decibels corresponds to a sound being ten times louder.

A lawnmower emits a 100dB sound. How much sound will two lawnmowers make?

A. 100 dB
B. 110 dB
C. 200 dB
D. other: more than 100, less than 110
E. I'm not sure

http://biology-forums.com/index.php?action=gallery;sa=view;id=6156

Logarithm Rules

Let A and B be positive, and let n be any real number.

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Simplify into a single logarithm:

$$f(x) = \ln\left(\frac{10}{x^2}\right) + 2\ln x + \ln(10 + x)$$

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$$\begin{aligned}
f(x) &= \ln\left(\frac{10}{x^2}\right) + 2\ln x + \ln(10+x) \\
&= \ln 10 - \ln(x^2) + 2\ln x + \ln(10+x) \\
&= \ln 10 - 2\ln x + 2\ln x + \ln(10+x) \\
&= \ln 10 + \ln(10+x) \\
&= \ln(10(10+x))) \\
&= \ln(100+10x)
\end{aligned}$$

Base Change

$$b^{\log_b(a)} = a$$

Base Change

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\begin{aligned}
b^{\log_b(a)} &= a \\
\Rightarrow \ln\left(b^{\log_b(a)}\right) &= \ln(a) \\
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Suppose your calculator can only compute logarithms base 10. What would you enter to calculate $\ln(17)$?

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Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\ln(2)$?

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Suppose your calculator can only compute logarithms base 2. What would you enter to calculate $\ln(2)$? $\frac{\log_2 2}{\log_2 e} = \frac{1}{\log_2 e}$

Differentiating the Natural Logarithm

Calculate $\frac{d}{dx}\{\ln x\}$.

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Derivative of Natural Logarithm

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Differentiate: $f(x) = \ln|\cot x|$

We use the chain rule:

$$\begin{aligned}\frac{d}{dx}\left\{\ln\left|\boxed{\cot x}\right|\right\} &= \frac{1}{\cot x}\cdot\left(-\csc^2 x\right)\\ &= \frac{-\csc^2 x}{\cot x}\end{aligned}$$

Logarithmic Differentiation

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In general, if $f(x) \neq 0$, $\dfrac{d}{dx}\{\ln|f(x)|\} = \dfrac{f'(x)}{f(x)}$.

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Find $f'(x)$.

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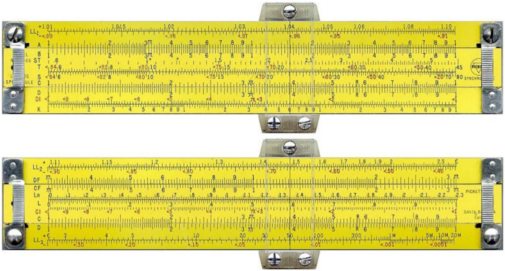

http://sliderulemuseum.com/SR_Course.htm

$$f(x) = \frac{(x^2+17)(32x^{10}-8)}{\sin x + 2}$$

$$\begin{aligned}
\frac{(x^2+17)(32x^{10}-8)}{\sin x+2} &= y \\
\ln\left|\frac{(x^2+17)(32x^{10}-8)}{\sin x+2}\right| &= \ln|y| && \text{ln both sides} \\
\ln|x^2+17| + \ln|32x^{10}-8| - \ln|\sin x+2| &= \ln|y| && \text{log rules} \\
\frac{d}{dx}\left[\ln|x^2+17| + \ln|32x^{10}-8| - \ln|\sin x+2|\right] &= \frac{d}{dx}[\ln|y|] && \text{differentiate} \\
\frac{2x}{x^2+17} + \frac{320x^9}{32x^{10}-8} - \frac{\cos x}{\sin x} &= \frac{y'}{y} \\
y\left(\frac{2x}{x^2+17} + \frac{320x^9}{32x^{10}-8} - \frac{\cos x}{\sin x+2}\right) &= y' \\
\left(\frac{2x}{x^2+17} + \frac{320x^9}{32x^{10}-8} - \frac{\cos x}{\sin x+2}\right)\cdot\frac{(x^2+17)(32x^{10}-8)}{\sin x+2} &= y' && \text{plug in } y
\end{aligned}$$

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$$\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x} = y$$

$$\ln \left| \frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x} \right| = \ln |y|$$

$$\left[\ln |x^8 - e^x| + \ln |x^{1/2} + 5| - 5 \ln |\csc x| \right] = \ln |y|$$

$$\frac{d}{dx} \left[\ln |x^8 - e^x| + \ln |x^{1/2} + 5| - 5 \ln |\csc x| \right] = \frac{d}{dx} \ln |y|$$

$$\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2} x^{-1/2}}{x^{1/2} + 5} - 5 \frac{-\csc x \cot x}{\csc x} = \frac{y'}{y}$$

$$\frac{(x^8 - e^x)(\sqrt{x} + 5)}{\csc^5 x} \left(\frac{8x^7 - e^x}{x^8 - e^x} + \frac{\frac{1}{2} x^{-1/2}}{x^{1/2} + 5} - 5 \frac{-\csc x \cot x}{\csc x} \right) = y'$$

Logarithmic Differentiation

$$f(x) = (x^2 + 17)(32x^5 - 8)(x^{98} - x^{57} + 32x^2)^4(32x^{10} - 10x^{32})$$

Find $f'(x)$.

Logarithmic Differentiation

$$f(x) = (x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})$$

Find $f'(x)$.

$$(x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32}) = y$$

$$\ln\left|(x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})\right| = \ln|y|$$

$$\ln|x^2+17| + \ln|32x^5-8| + 4\ln|x^{98}-x^{57}+32x^4| + \ln|32x^{10}-10x^{32}| = \ln y$$

$$\frac{d}{dx}\left[\ln|x^2+17| + \ln|32x^5-8| + 4\ln|x^{98}-x^{57}+32x^4| + \ln|32x^{10}-10x^{32}|\right] = \frac{d}{dx}[\ln y]$$

$$\frac{2x}{x^2+17} + \frac{160x^4}{32x^5-8} + 4\frac{98x^{97}-57x^{56}+64x}{x^{98}-x^{57}+32x^2} + \frac{320x^9-320x^{31}}{32x^{10}-10x^{32}} = \frac{y'}{y}$$

$$\left((x^2+17)(32x^5-8)(x^{98}-x^{57}+32x^2)^4(32x^{10}-10x^{32})\right)\cdot$$
$$\left(\frac{2x}{x^2+17} + \frac{160x^4}{32x^5-8} + 4\frac{98x^{97}-57x^{56}+64x}{x^{98}-x^{57}+32x^2} + \frac{320x^9-320x^{31}}{32x^{10}-10x^{32}}\right) = y'$$

Logarithmic Differentiation

Let $f(x) = x^{\cos x}$, where $x \geq 0$.

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Let $f(x) = x^{\cos x}$, where $x \geq 0$. Do the local peaks occur where $\cos x = 1$?

Logarithmic Differentiation

Let $f(x) = x^{\cos x}$, where $x \geq 0$. Do the local peaks occur where $\cos x = 1$?

First, find the derivative.

$$\begin{aligned}
x^{\cos x} &= y \\
\ln\left(x^{\cos x}\right) &= \ln y \\
\cos x \ln x &= \ln y \\
\frac{d}{dx}[\cos x \ln x] &= \frac{d}{dx}[\ln y] \\
\cos x \frac{1}{x} + \ln x(-\sin x) &= \frac{y'}{y} \\
\frac{\cos x}{x} - \ln x \sin x &= \frac{y'}{y} \\
y\left(\frac{\cos x}{x} - \ln x \sin x\right) &= y' \\
x^{\cos x}\left(\frac{\cos x}{x} - \ln x \sin x\right) &= y'
\end{aligned}$$

If the peaks occur when $\cos x = 1$, then the derivative should be zero there. In particular, $\cos x = 1$ when $x = 2\pi n$. Then:

$$f'(2\pi n) = (2\pi n)^1 \left(\frac{1}{2\pi n} - \ln(2\pi n) \sin(2\pi n) \right)$$
$$= 2\pi n \left(\frac{1}{2\pi n} - 0 \right) = 1 \neq 0$$

So the peaks do NOT occur exactly at the places where $\cos x = 1$.

Logarithmic Differentiation

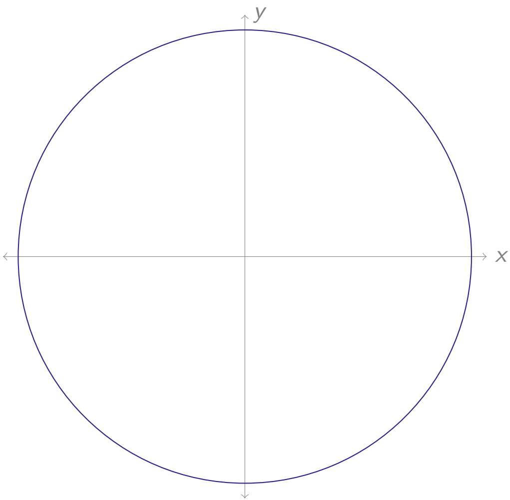
Find the derivative of x^x.

Logarithmic Differentiation

Find the derivative of x^x.

$$\begin{aligned} x^x &= y \\ \ln(x^x) &= \ln y \\ x\ln x &= \ln y \\ x\cdot\frac{1}{x} + \ln x &= \frac{y'}{y} \\ 1+\ln x &= \frac{y'}{y} \\ y' &= y(1+\ln x) = x^x(1+\ln x) \end{aligned}$$

$x^2 + y^2 = 1$



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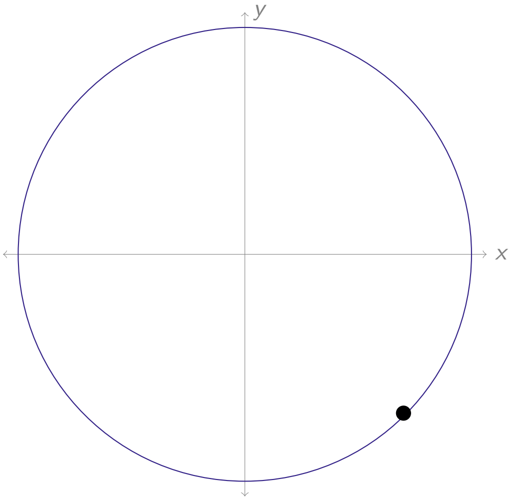

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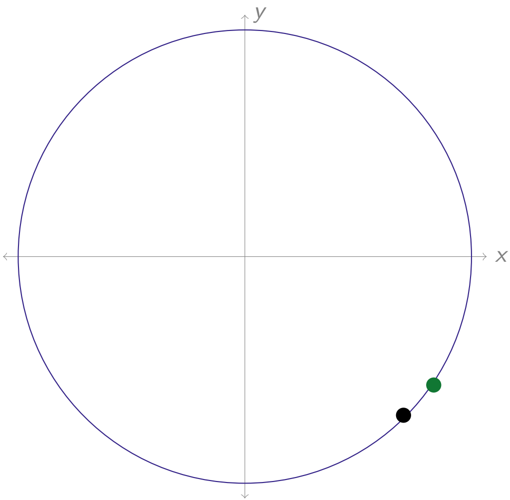

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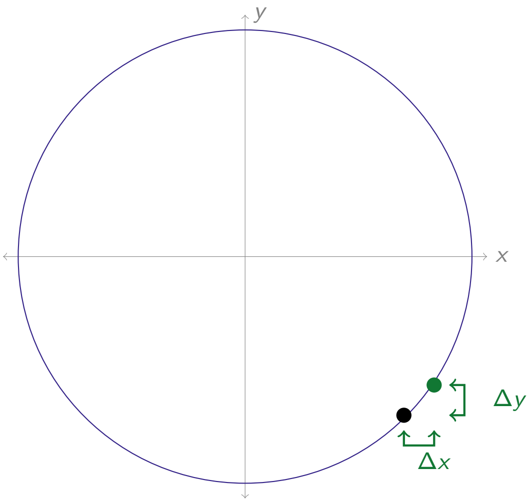

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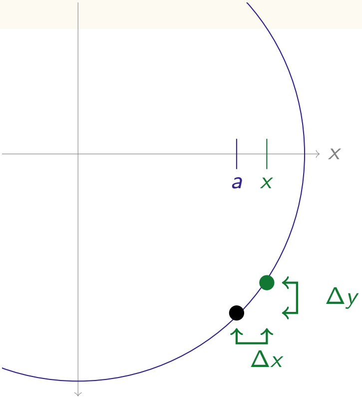
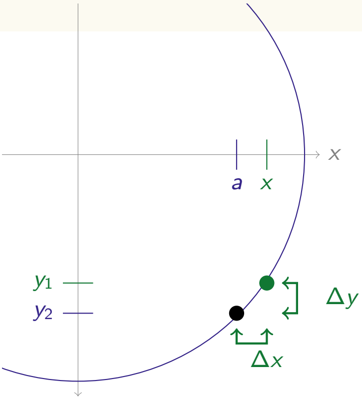


Compare:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$x^2 + y^2 = 1$



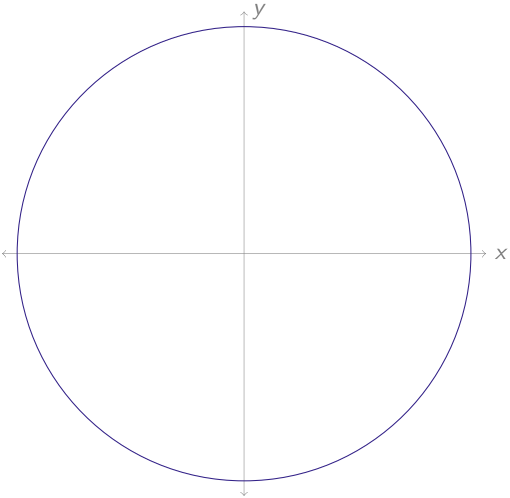
Compare:

$$\lim_{x \to a} \cancel{\frac{f(x) - f(a)}{x - a}}$$

$$\lim_{x \to a} \frac{y_1 - y_2}{x - a}$$

$x^2 + y^2 = 1$

Find the slope of the tangent line to the unit circle at point (x, y).
Verify your answer by determining when the tangent line is horizontal and when it is vertical.



$x^2 + y^2 = 1$

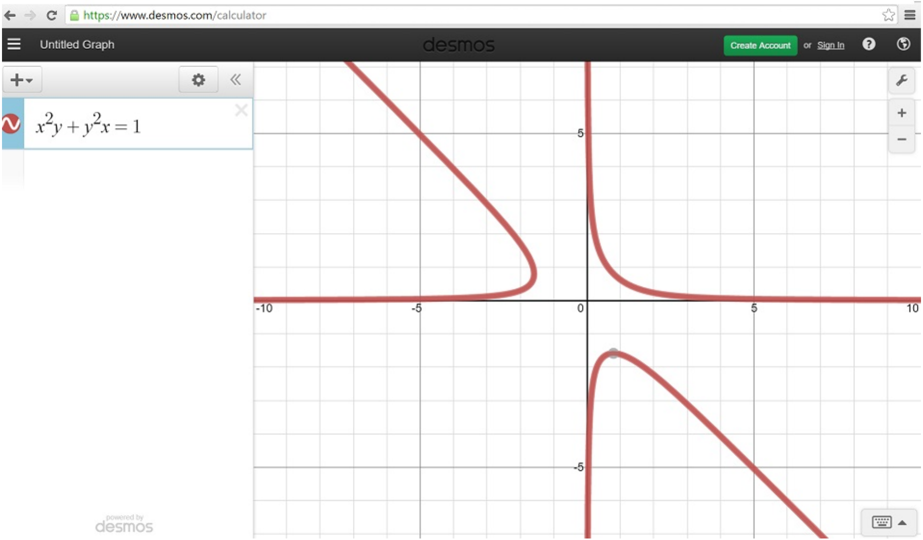
Find the slope of the tangent line to the unit circle at point (x, y).
Verify your answer by determining when the tangent line is horizontal and when it is vertical.

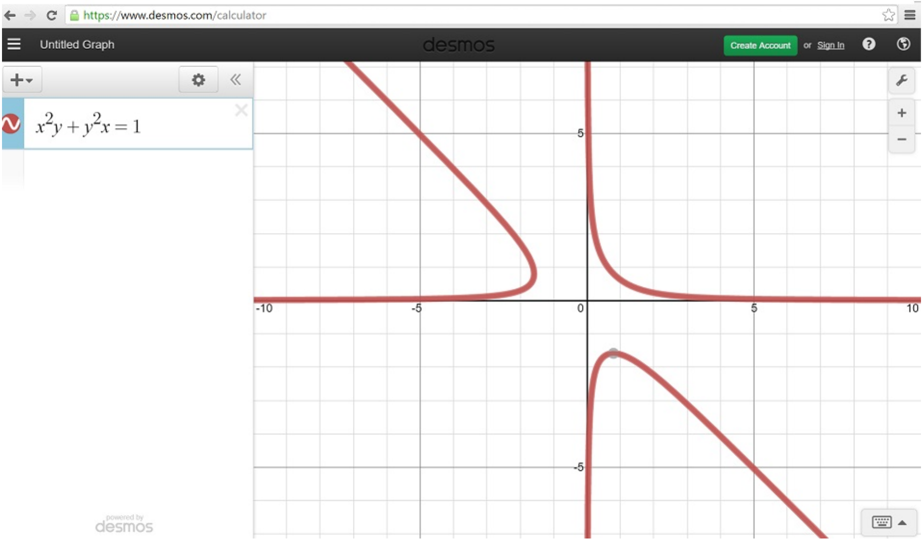
$x^2 + y^2 = 1 \;\Rightarrow\; \frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}[1]$

$\Rightarrow\; 2x + 2y\frac{dy}{dx} = 0 \;\Rightarrow\; 2y\frac{dy}{dx} = -2x$

$\Rightarrow\; \frac{dy}{dx} = -\frac{x}{y}$

So, $\frac{dy}{dx}$ is 0 when $x = 0$ and undefined when $y = 0$. This fits with the picture.


https://www.desmos.com/calculator
Untitled Graph
desmos
Create Account
or
Sign In
$x^2y + y^2x = 1$
5
-5
-10
0
10
powered by desmos



What are the coordinates of the right-most bump on the left?

$$x^2 y + y^2 x = 1$$

$$x^2y + y^2x = 1$$

First, find $\frac{dy}{dx}$, and where it doesn't exist.

$$x^2y + y^2x = 1$$

First, find $\frac{dy}{dx}$, and where it doesn't exist.
$\frac{d}{dx}[x^2y + y^2x] = \frac{d}{dx}[1]$
$(x^2)\frac{dy}{dx} + y(2x) + y^2(1) + x\left(2y\frac{dy}{dx}\right) = 0$
$\frac{dy}{dx}\left[x^2 + 2xy\right] = -2xy - y^2$
$\frac{dy}{dx} = \dfrac{-2xy - y^2}{x^2 + 2xy}$
This doesn't exist when $x^2 + 2xy = 0$, that is, when $x(x + 2y) = 0$, so when $x = 0$ or $x = 2y$. Notice though, then $x = 0$, the function is undefined. So our only candidate is when $x = -2y$.

$$x^2y + y^2x = 1$$

$$x = -2y$$

$$x^2y + y^2x = 1$$
$$x = -2y$$

Now, find the point on the curve corresponding to the bump.

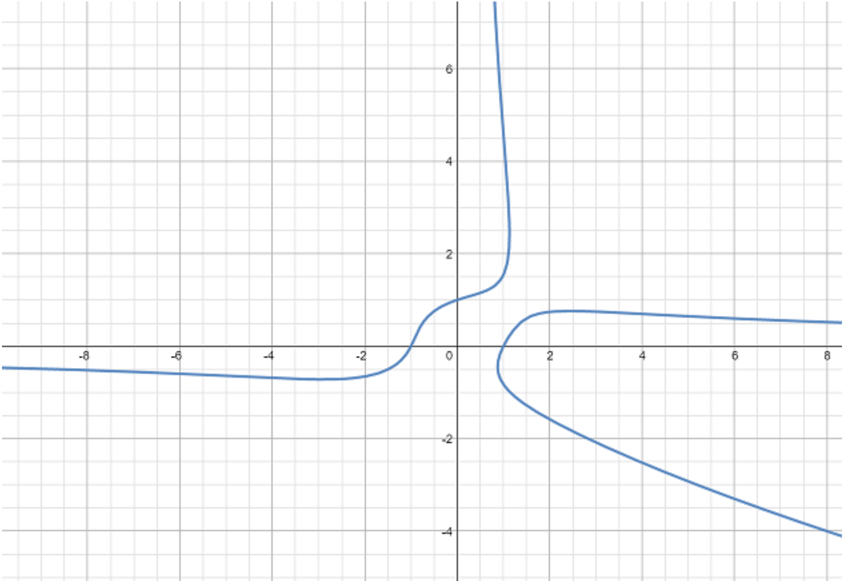
$(-2y)^2y + y^2(-2y) = 1 \Rightarrow 2y^3 = 1 \Rightarrow y = \frac{1}{\sqrt[3]{2}}$

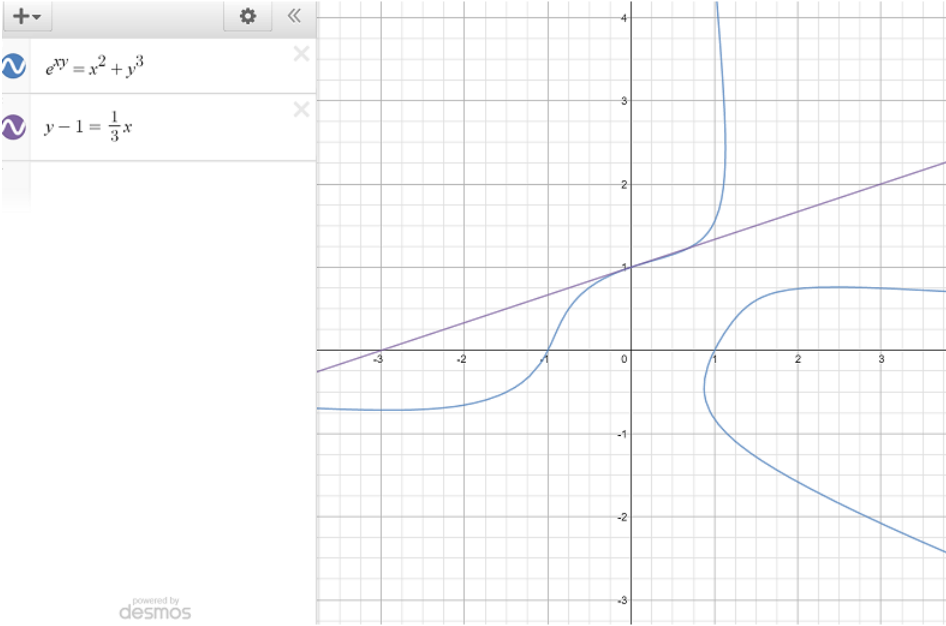
and then find x:

$x = -2y = \frac{-2}{\sqrt[3]{2}}$

$$e^{xy} = x^2 + y^3$$

Find the equation of the tangent line to the curve $e^{xy} = x^2 + y^3$ when $x = 0$.




$e^{xy} = x^2 + y^3$
$y - 1 = \frac{1}{3}x$
powered by desmos
4
3
2
1
0
-1
-2
-3
-3
-2
-1
1
2
3

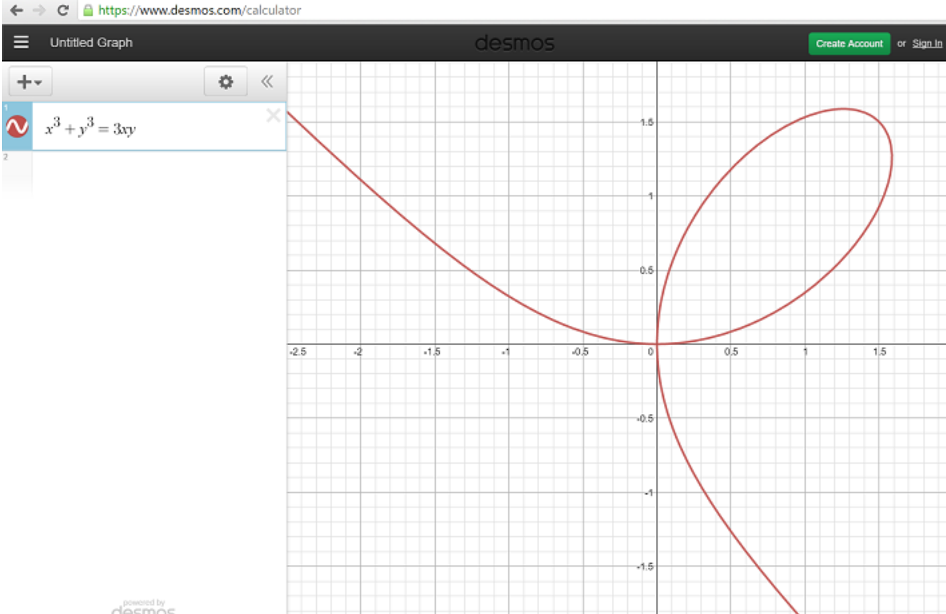
$e^{xy} = x^2 + y^3$

$$e^{\boxed{xy}} \cdot \left[x\frac{dy}{dx} + y(1)\right] = 2x + 3y^2\frac{dy}{dx}$$
$$\frac{dy}{dx}\left[xe^{xy} - 3y^2\right] = 2x - ye^{xy}$$
$$\frac{dx}{dy} = \frac{2x - ye^{xy}}{xe^{xy} - 3y^2}$$

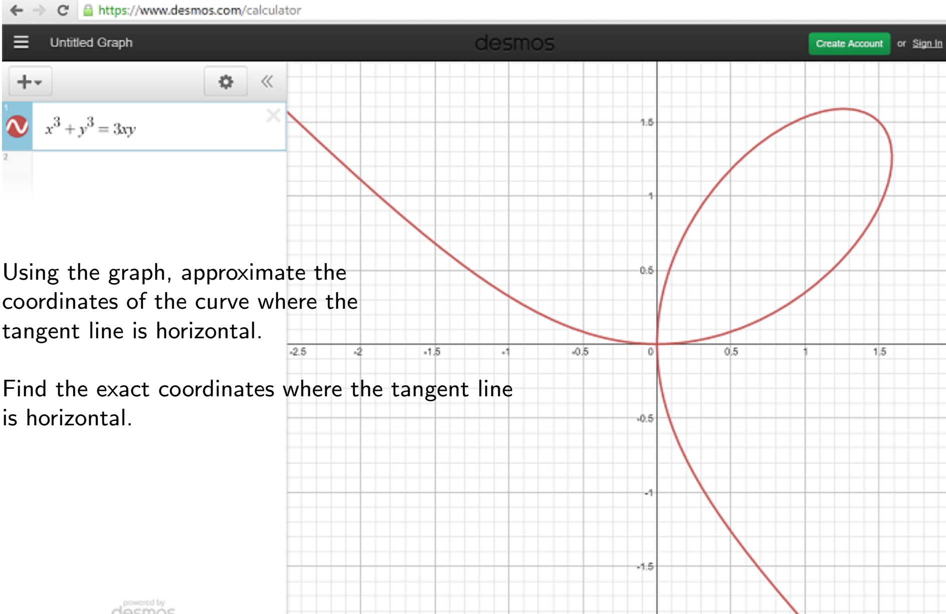
So, when $x = 0$, $\frac{dy}{dx} = \frac{-y}{-3y^2} = \frac{1}{3y}$. So, we need to figure out what y is.
$e^{0\cdot y} = 0^2 + y^3 \Rightarrow 1 = y^3 \Rightarrow y = 1$
so the point is $(0, 1)$ and the slope is $1/3$. Then the tangent line is:

$$(y - 1) = -1/3x$$

Folium of Descartes



Folium of Descartes



Using the graph, approximate the coordinates of the curve where the tangent line is horizontal.

Find the exact coordinates where the tangent line is horizontal.

Folium of Descartes

$$x^3 + y^3 = 3xy$$

Using the graph, approximate the coordinates of the curve where the tangent line is horizontal.

It looks like one arm of the graph is horizontal at $(0, 0)$, and also when $x \approx 1.25$ and $y \approx 1.6$.
To find out for sure, let's take the derivative.

$$x^3 + y^3 = 3xy \Rightarrow \qquad \frac{d}{dx}[x^3 + y^3] = \frac{d}{dx}3xy$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3(x\frac{dy}{dx} + y) \qquad 3y^2\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - 3x^2$$

$$\frac{dy}{dx}(3y^2 - 3x) = 3y - 3x^2 \qquad \frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x} = \frac{y - x^2}{x - y^2}$$

So we can expect the tangent line to be horizontal whenever $y = x^2$, except possibly when also $x = y^2$. If we plug $y = x^2$ into the equation to find out when that happens:

$$x^3 + y^3 = 3xy \qquad x^3 + (x^2)^3 = 3x(x^2)$$

$$x^3 + x^6 = 3x^3 \qquad x^3(x^3 - 2) = 0$$

So, if $x = 0$ (and $y = 0^2 = 0$) or $x = \sqrt[3]{2}$ (and $y = \sqrt[3]{4}$), we might expect to see a horizontal tangent.

$$y^2 = 3x^3 + 9x^2$$

Where is tangent line to this curve horizontal? Where vertical?

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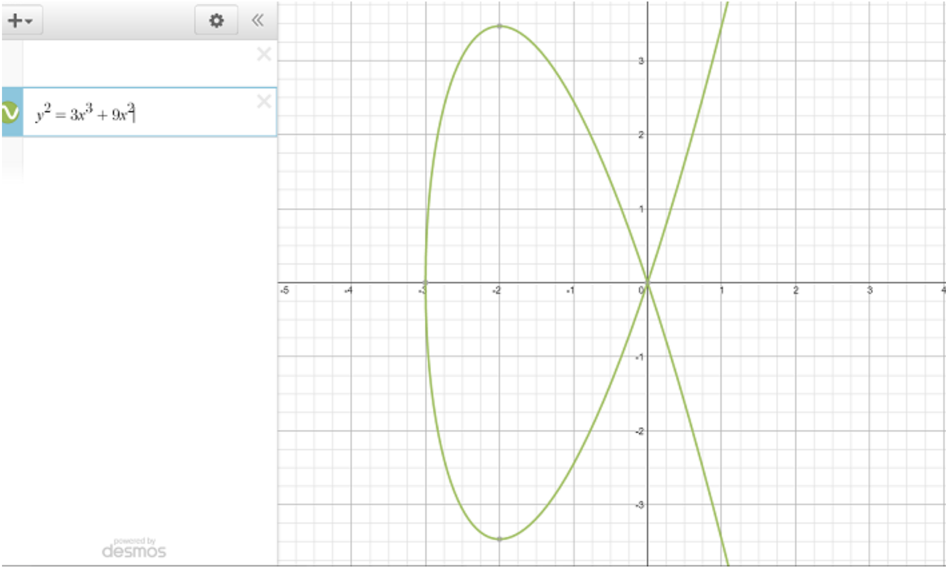
Where is tangent line to this curve horizontal? Where vertical?



$$y^3 = x^2 y - x^4$$

Where might this curve have a vertical tangent line?

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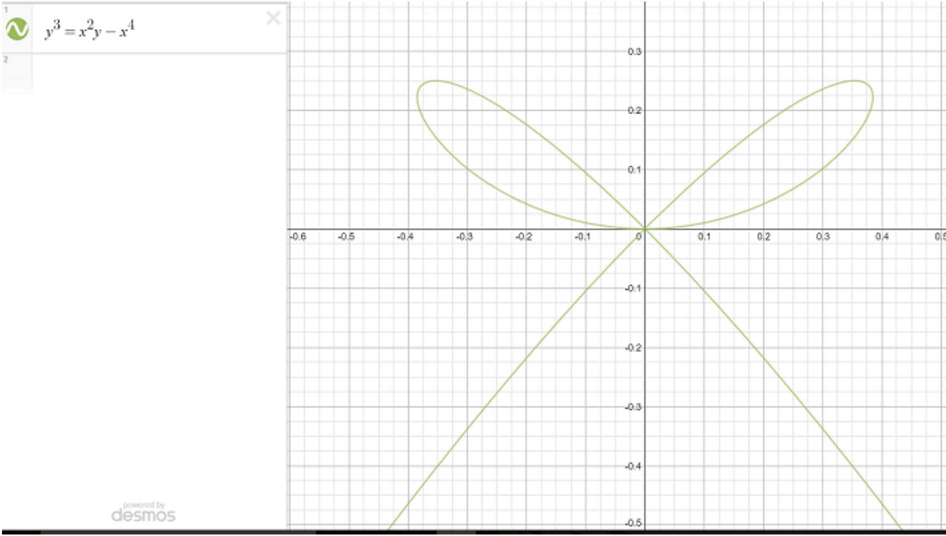


$y^3 = x^2y - x^4$

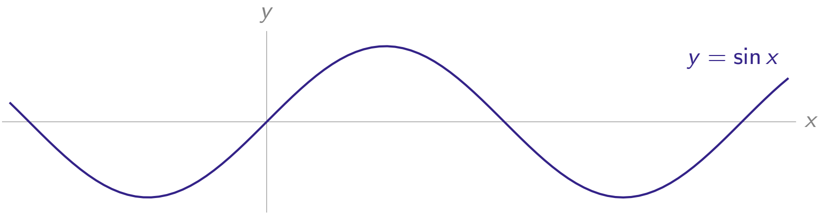
$$\begin{aligned} 3y^2\frac{dy}{dx} &= (x^2)\frac{dy}{dx} + y(2x) - 4x^3 \\ 3y^2\frac{dy}{dx} - x^2\frac{dy}{dx} &= 2xy - 4x^3 \\ (3y^2 - x^2)\frac{dy}{dx} &= 2xy - 4x^3 \\ \frac{dy}{dx} &= \frac{2xy - 4x^3}{3y^2 - x^2} \end{aligned}$$

This derivative doesn't exist when the denominator is zero, which happens when $3y^2 = x^2$. Plugging this into the original equation, that means:

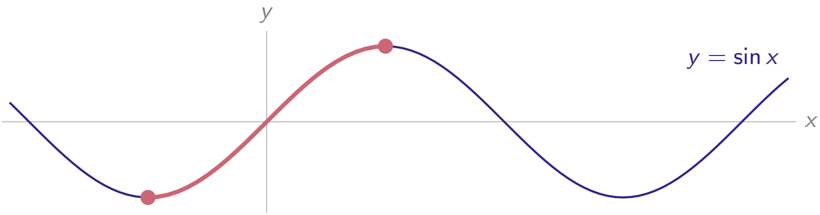
$$y^3 = (3y^2)y - (3y^2)^2 \Rightarrow 0 = y^3(2 - 9y)$$

so $y = 0$ or $y = 2/9$. If $y = 0$ then $x = 0$, and both the top and bottom of the derivative are zero: so we don't know what it looks like. Suppose $y = 2/9$, so $x = \pm 2\sqrt{3}/9$. Then the denominator is zero, and the numerator is some number; so as x and y get close to these numbers, the slope of the tangent line grows. So these are vertical tangent lines.

arcsine



arcsine



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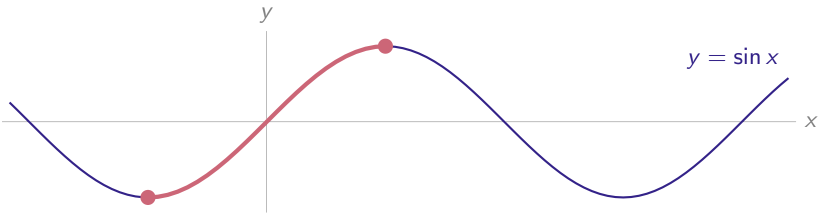
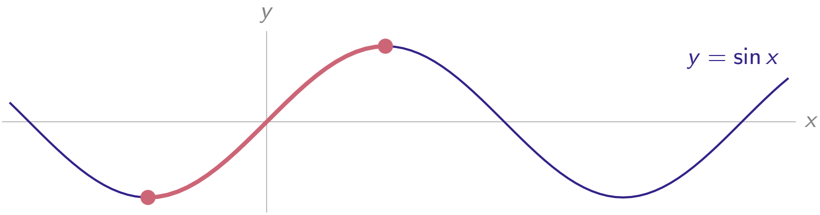


The function $f(x) = \sin(x)$ is invertible over the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and this is the domain we use to define $\arcsin(x)$.

$\arcsin(x)$ gives the number y such that:
(1) $\sin(y) = x$ and
(2) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

arcsine



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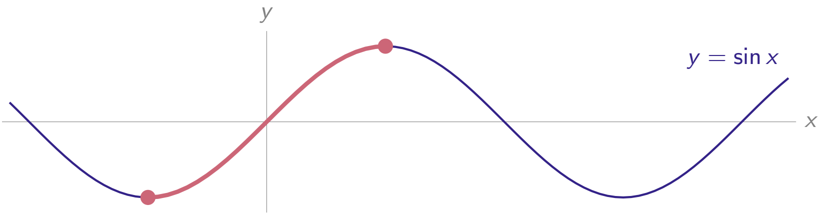

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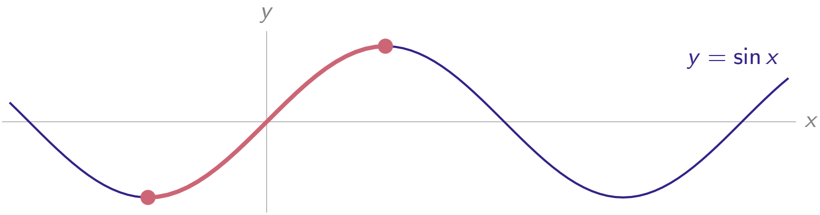

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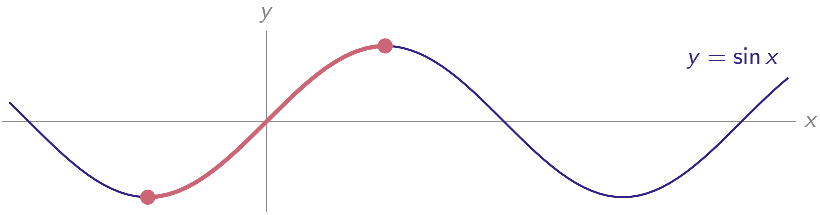
What is $\arcsin(\sin 0)$?
What is $\arcsin\left(\sin \frac{3\pi}{2}\right)$?

arcsine



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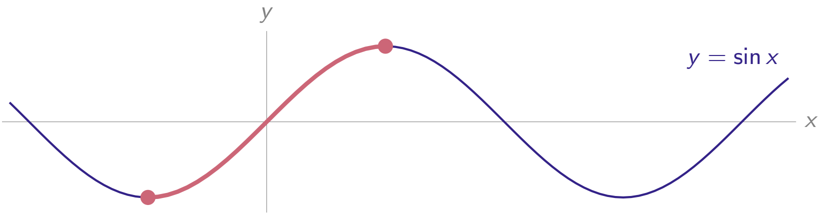

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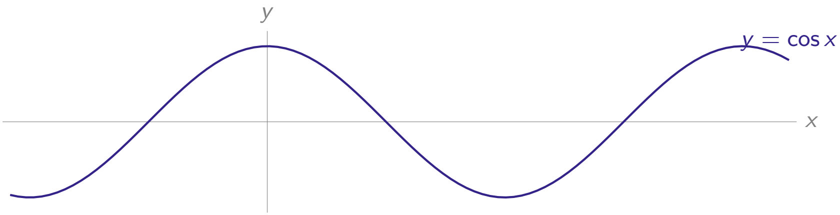
What is $\arcsin\left(\sin \frac{3\pi}{2}\right)$? $-\frac{\pi}{2}$

arccosine



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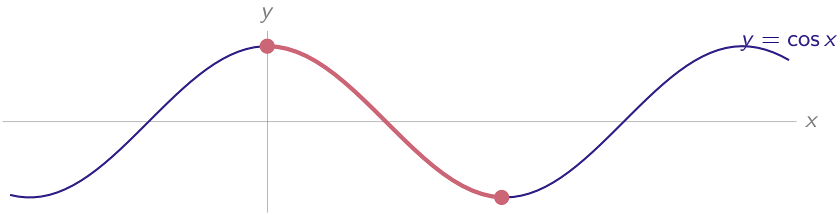

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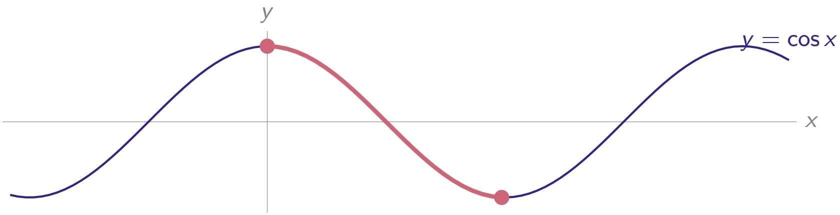

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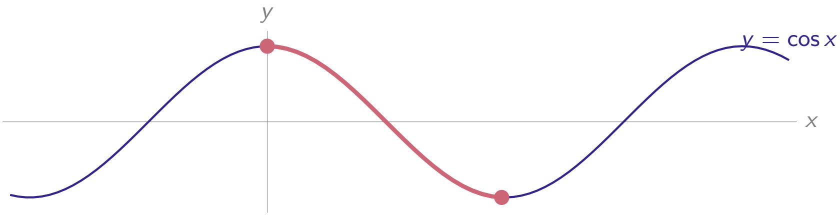

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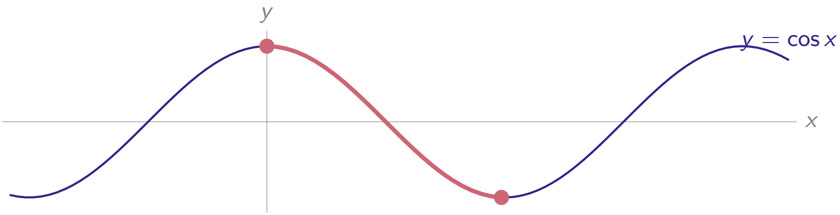

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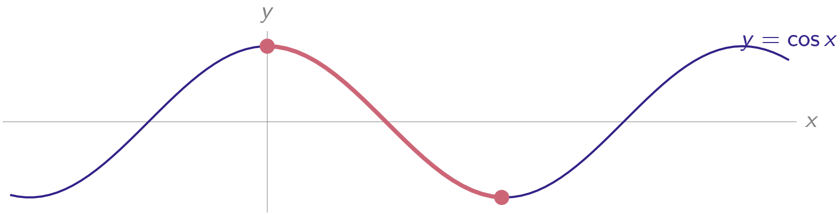


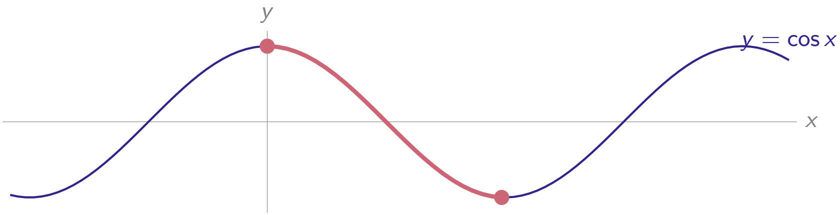
The function $f(x) = \cos(x)$ is invertible over the domain $[0, \pi]$, and this is the domain we use to define $\arccos(x)$.

$\arccos(x)$ gives the number y such that:
(1) $\cos(y) = x$ and $\leftarrow\leftarrow\leftarrow$ inverse
(2) $0 \leq y \leq \pi$ $\leftarrow\leftarrow\leftarrow$ function

$\arccos\left(\cos\frac{5\pi}{4}\right) =$

$\tan\left(\arccos x\right) =$

arccosine



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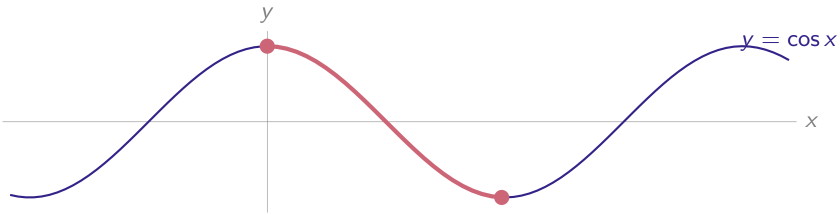
$\arccos\left(\cos\frac{5\pi}{4}\right) = \arccos\left(\cos\frac{3\pi}{4}\right) = \frac{3\pi}{4}$

$\tan(\arccos x) =$

arccosine



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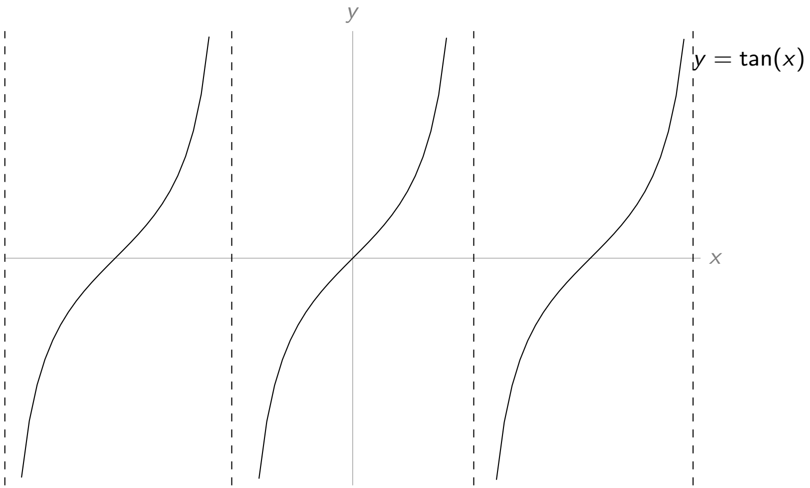
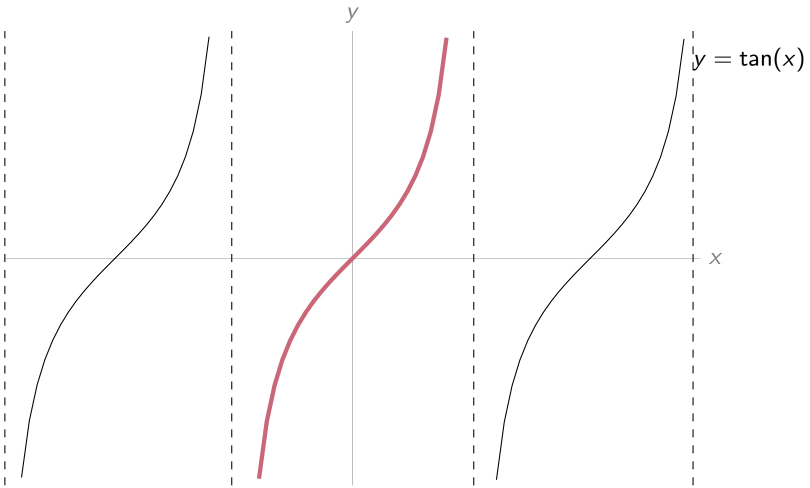
$\arccos\left(\cos \frac{5\pi}{4}\right) = \arccos\left(\cos \frac{3\pi}{4}\right) = \frac{3\pi}{4}$

$$\tan(\arccos x) = \frac{\sqrt{1-x^2}}{x}$$

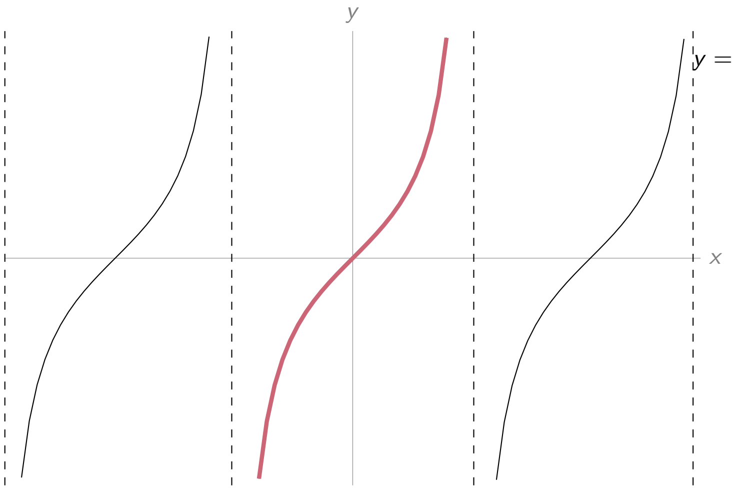
arctangent



arctangent



arctangent



$\arctan(x) = y$ means: (1) $\tan(y) = x$ and (2) $\pi/2 < y < \pi/2$

arcsecant, arcsine, and arccotangent

$\text{arcsec}(x) = y$

$\text{arccsc}(x) = y$

$\text{arccot}(x) = y$

arcsecant, arcsine, and arccotangent

$\text{arcsec}(x) = y$
$\sec(y) = x$

$\text{arccsc}(x) = y$

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arcsecant, arcsine, and arccotangent

$\text{arcsec}(x) = y$
$\sec(y) = x$
$\frac{1}{\cos(y)} = x$

$\text{arccsc}(x) = y$

$\text{arccot}(x) = y$

arcsecant, arcsine, and arccotangent

$\text{arcsec}(x) = y$
$\sec(y) = x$
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$\cos(y) = \frac{1}{x}$

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arcsecant, arcsine, and arccotangent

$\text{arcsec}(x) = y$
$\sec(y) = x$
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$$\boxed{y = \arccos\left(\frac{1}{x}\right)}$$

$\text{arccsc}(x) = y$

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arcsecant, arcsine, and arccotangent

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Domain of $\arccos(x)$ is $-1 \leq x \leq 1$, so domain of $\text{arcsec}(x)$ is $(-\infty, -1] \cup [1, \infty)$.

$\text{arccsc}(x) = y$

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arcsecant, arcsine, and arccotangent

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$\text{arccsc}(x) = y$
$\csc(y) = x$
$\frac{1}{\sin(y)} = x$
$\sin(y) = \frac{1}{x}$
$\boxed{y = \arcsin\left(\frac{1}{x}\right)}$

Domain of $\arcsin(x)$ is $-1 \leq x \leq 1$, so domain of $\text{arccsc}(x)$ is $(-\infty, -1] \cup [1, \infty)$.

$\text{arccot}(x) = y$
$\cot(y) = x$
$\frac{1}{\tan(y)} = x$
$\tan(y) = \frac{1}{x}$
$\boxed{y = \arctan\left(\frac{1}{x}\right)}$

Domain of $\arctan(x)$ is all real numbers, so domain of $\text{arccot}(x)$ is $(-\infty, 0) \cup (0, \infty)$.

Derivative of $\arctan(x)$

$$y = \arctan x$$

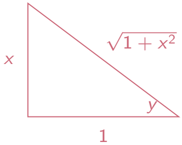
Find $\frac{dy}{dx}$.

Derivative of $\arctan(x)$

$$y = \arctan x$$

Find $\frac{dy}{dx}$.

$$\begin{aligned} y &= \arctan x \\ x &= \tan y \\ \frac{d}{dx}[x] &= \frac{d}{dx}[\tan y] \\ 1 &= \sec^2 y \cdot \frac{dy}{dx} \\ \frac{dy}{dx} &= cos^2 y \\ \frac{dy}{dx} &= \left(\frac{adj}{hyp}\right)^2 = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 = \frac{1}{|1+x^2|} = \frac{1}{1+x^2} \end{aligned}$$



Derivative of $\arccos(x)$

$$y = \arccos x$$

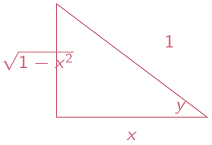
Find $\frac{dy}{dx}$.

Derivative of $\arccos(x)$

$$y = \arccos x$$

Find $\frac{dy}{dx}$.

$$\begin{aligned}
y &= \arccos x \\
x &= \cos y \\
\frac{d}{dx}[x] &= \frac{d}{dx}[\cos y] \\
1 &= -\sin y \cdot \frac{dy}{dx} \\
\frac{dy}{dx} &= \frac{-1}{\sin y} \\
\frac{dy}{dx} &= \frac{-hyp}{opp} = \frac{-1}{\sqrt{1-x^2}}
\end{aligned}$$



Derivative of arcsin(x)

$$y = \arcsin x$$

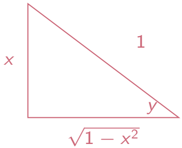
Find $\frac{dy}{dx}$.

Derivative of arcsin(x)

$$y = \arcsin x$$

Find $\frac{dy}{dx}$.

$$\begin{aligned} y &= \arcsin x \\ x &= \sin y \\ \frac{d}{dx}\{x\} &= \frac{d}{dx}\{\sin y\} \\ 1 &= \cos y \cdot \frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{1}{\cos y} \\ \frac{dy}{dx} &= \frac{hyp}{adj} = \frac{1}{\sqrt{1-x^2}} \end{aligned}$$



Derivatives of other inverse functions

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

Derivatives of other inverse functions

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$$\frac{d}{dx}\{\text{arccsc}(x)\} = \frac{d}{dx}\left\{\arcsin\left(\frac{1}{x}\right)\right\} = \frac{d}{dx}\left\{\arcsin\left(x^{-1}\right)\right\}$$

Derivatives of other inverse functions

To differentiate arcsecant, arccosecant, and arccotangent, you can use the chain rule!

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$$\begin{aligned}\frac{d}{dx}\left\{\arcsin\left(\boxed{x^{-1}}\right)\right\} &= \frac{1}{\sqrt{1-\left(\boxed{x^{-1}}\right)^2}} \cdot \boxed{\left(-x^{-2}\right)} \\ &= \frac{-1}{x^2\sqrt{1-x^{-2}}} = \frac{-1}{\sqrt{x^4}\sqrt{1-x^{-2}}} \\ &= \frac{-1}{\sqrt{x^2}\sqrt{x^2}\sqrt{1-x^{-2}}} = \frac{-1}{\sqrt{x^2}\sqrt{x^2-1}} = \frac{-1}{|x|\sqrt{x^2-1}}\end{aligned}$$

Derivatives of Inverse Trig Functions

Derivatives of Inverse Trig Functions

Memorize	Figure Out
$\frac{d}{dx}\{\arcsin x\} = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\{\text{arccsc} x\} = \frac{d}{dx}\left\{\arcsin\left(x^{-1}\right)\right\}$
$\frac{d}{dx}\{\arccos x\} = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\{\text{arcsec} x\} = \frac{d}{dx}\left\{\arccos\left(x^{-1}\right)\right\}$
$\frac{d}{dx}\{\arctan x\} = \frac{1}{1+x^2}$	$\frac{d}{dx}\{\text{arccot} x\} = \frac{d}{dx}\left\{\arctan\left(x^{-1}\right)\right\}$

Derivatives of Inverse Trig Functions

Derivatives of Inverse Trig Functions

Memorize

$$\frac{d}{dx}\{\arcsin x\} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\{\arccos x\} = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\{\arctan x\} = \frac{1}{1+x^2}$$

Figure Out

$$\frac{d}{dx}\{\mathrm{arccsc} x\} = \frac{d}{dx}\left\{\arcsin\left(x^{-1}\right)\right\}$$

$$\frac{d}{dx}\{\mathrm{arcsec} x\} = \frac{d}{dx}\left\{\arccos\left(x^{-1}\right)\right\}$$

$$\frac{d}{dx}\{\mathrm{arccot} x\} = \frac{d}{dx}\left\{\arctan\left(x^{-1}\right)\right\}$$

Evaluate:

$$\lim_{x\to\infty} \arctan x$$

$$\lim_{x\to\infty} \left(\frac{d}{dx}\{\arctan x\}\right)$$

$$\lim_{x\to -1^+} \arcsin x$$

$$\lim_{x\to -1^+} \left(\frac{d}{dx}\{\arcsin x\}\right)$$

Derivatives of Inverse Trig Functions

Derivatives of Inverse Trig Functions

Memorize	Figure Out
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$\frac{d}{dx}\{\arccos x\} = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\{\operatorname{arcsec} x\} = \frac{d}{dx}\left\{\arccos\left(x^{-1}\right)\right\}$
$\frac{d}{dx}\{\arctan x\} = \frac{1}{1+x^2}$	$\frac{d}{dx}\{\operatorname{arccot} x\} = \frac{d}{dx}\left\{\arctan\left(x^{-1}\right)\right\}$

Evaluate:

$$\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$$

$$\lim_{x\to\infty} \left(\frac{d}{dx}\{\arctan x\}\right)$$

$$\lim_{x\to -1^+} \arcsin x$$

$$\lim_{x\to -1^+} \left(\frac{d}{dx}\{\arcsin x\}\right)$$

Derivatives of Inverse Trig Functions

Derivatives of Inverse Trig Functions

Memorize

$$\frac{d}{dx}\{\arcsin x\} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\{\arccos x\} = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\{\arctan x\} = \frac{1}{1+x^2}$$

Figure Out

$$\frac{d}{dx}\{\mathrm{arccsc} x\} = \frac{d}{dx}\left\{\arcsin\left(x^{-1}\right)\right\}$$

$$\frac{d}{dx}\{\mathrm{arcsec} x\} = \frac{d}{dx}\left\{\arccos\left(x^{-1}\right)\right\}$$

$$\frac{d}{dx}\{\mathrm{arccot} x\} = \frac{d}{dx}\left\{\arctan\left(x^{-1}\right)\right\}$$

Evaluate:

$$\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$$

$$\lim_{x\to\infty} \left(\frac{d}{dx}\{\arctan x\}\right) = 0$$

$$\lim_{x\to -1^+} \arcsin x$$

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Derivatives of Inverse Trig Functions

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Memorize	Figure Out
$\frac{d}{dx}\{\arcsin x\} = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\{\text{arccsc}x\} = \frac{d}{dx}\left\{\arcsin\left(x^{-1}\right)\right\}$
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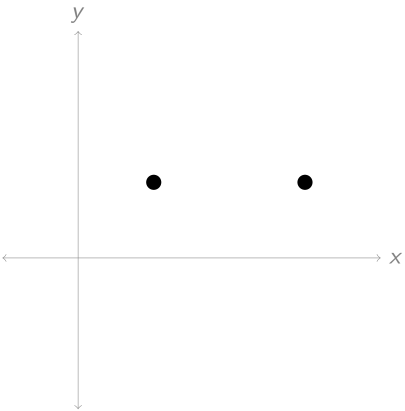
$$\lim_{x\to -1^+} \left(\frac{d}{dx}\{\arcsin x\}\right) = \infty$$

Rolle's Theorem



Rolle's Theorem

Let a and b be real numbers, with $a < b$. And let f be a function with the properties:

-
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- and $f(a) = f(b)$.

Then there exists a number c with $a < c < b$ such that

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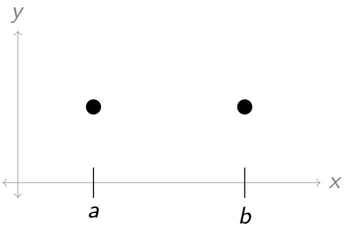
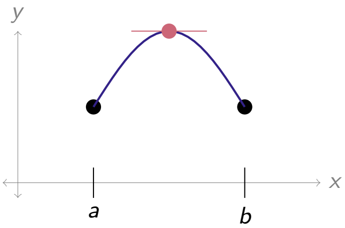


Suppose $a < b$ and $f(a) = f(b)$, $f(x)$ is continuous over $[a, b]$, and $f(x)$ is differentiable over (a, b).

How many different values of x between a and b have $f'(x) = 0$?

A. 0 or 1
B. 1
C. 0, 1, or more
D. 1 or more
E. I'm not sure

Rolle's Theorem



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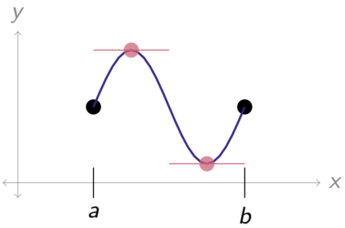

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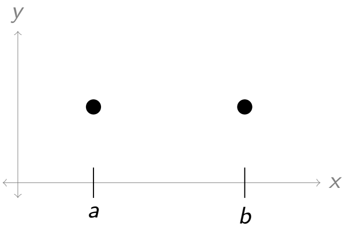

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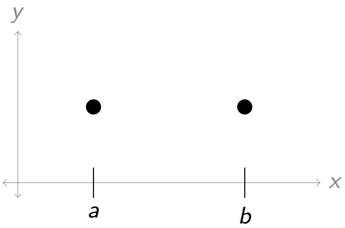

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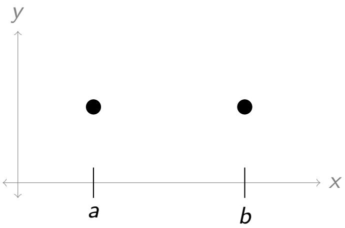

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Can f have an infinte number of points where $f'(x) = 0$ between a and b?

A. Sure! :D
B. No way! >:-[
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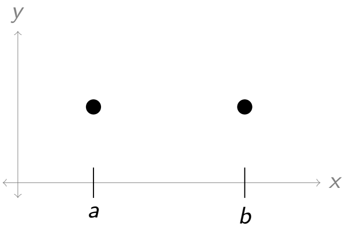
Rolle's Theorem



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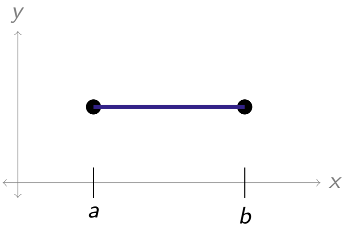

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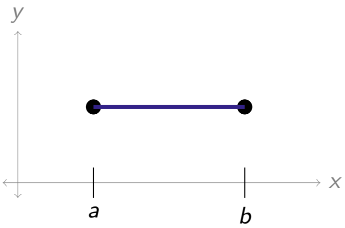

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Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f(x)$ has precisely seven roots. How roots does $f'(x)$ have?

A. precisely six

B. precisely seven

C. at most seven

D. at least six

E. I don't know

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Then there exists a number c with $a < c < b$ such that

$$f'(c) = 0.$$

Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f'(x)$ is also continuous and differentiable for all real numbers, and $f(x)$ has precisely seven roots. How many roots does $f''(x)$ have?

A. precisely six

B. precisely five

C. at most five

D. at least five

E. I don't know

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Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f(x)$ has precisely three places where $f'(x) = 0$. How many roots does $f(x)$ have?

A. at most three

B. at most four

C. at least three

D. at least four

E. I don't know

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Suppose $f(x)$ is continuous and differentiable for all real numbers, and $f'(x) = 0$ for precisely three values of x. How many distinct values x exist with $f(x) = 17$?

A. at most three

B. at most four

C. at least three

D. at least four

E. I don't know

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$f'(x) = 3x^2 + 1$, and this is always positive, so it's never zero. Therefore, by Rolle's Theorem, $f(x)$ does not have two roots; so it has at most one.

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Logical Structure:

- If A is true, then B is true.
- B is false.
- Therefore, A is false.

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Prove that the function $f(x) = x^3 + x - 1$ has at most one real root.

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Logical Structure:

- If A is true, then B is true.
- B is false.
- Therefore, A is false.

- If $f(x)$ has two (or more) roots, then $f'(x)$ has a root.
- $f'(x)$ does not have a root.
- Therefore, $f(x)$ does not have two (or more) roots.

Show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

Show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

Again we use the structure:

- *If $f(x)$ has three roots, then $f'(x)$ has two roots.*
- *$f'(x)$ does not have two roots.*
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Since $f(x)$ is continuous and differentiable over all real numbers, the conditions of Rolle's Theorem are satisfied.

$f'(x) = x^2 + 6x + 9 = (x+3)^2$, which only has ONE root.

Show that the function $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x - 3$ has at most two real roots.

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$f'(x) = x^3 + 1$, which only has ONE root.

Therefore, $f'(x)$ does not have two roots, so $f(x)$ does not have three roots.

Show that the function $f(x) = \frac{1}{4}x^4 + x + 9$ has at most two real roots.

Again we use the structure:

- *If $f(x)$ has three roots, then $f'(x)$ has two roots.*
- *$f'(x)$ does not have two roots.*
- *Therefore, $f(x)$ does not have three roots.*

So, all we need to do is make sure the conditions of Rolle's Theorem are satisfied, and show that $f'(x)$ does not have three roots.

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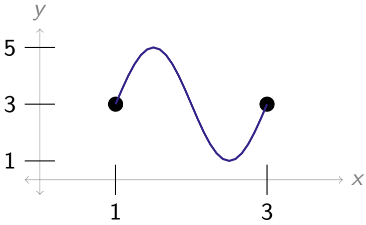
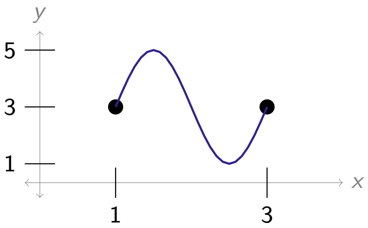
Therefore, $f'(x)$ does not have two roots, so $f(x)$ does not have three roots.

So, $f(x)$ has at most two roots.

Average Rate of Change



Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 1$ to $x = 3$?

A. 0

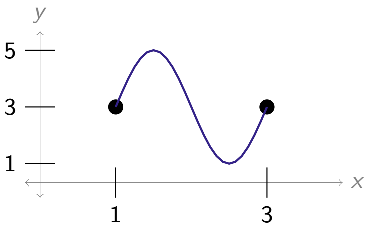
B. 1

C. 2

D. 4

E. I'm not sure

Average Rate of Change



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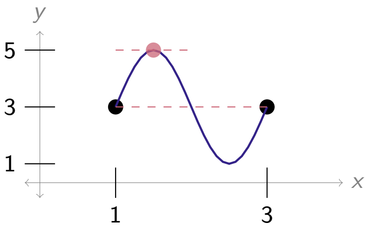

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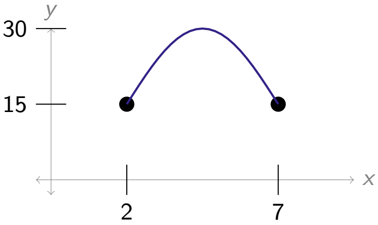
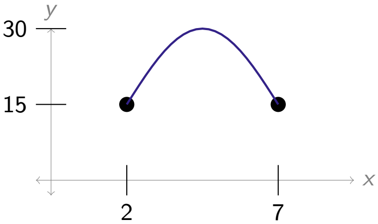


Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 2$ to $x = 7$?

A. 0

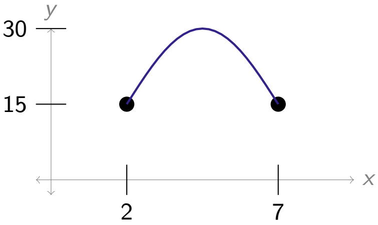
B. 3

C. 5

D. 15

E. I'm not sure

Average Rate of Change



What is the average rate of change of $f(x)$ from $x = 2$ to $x = 7$?

A. 0

$\frac{\Delta y}{\Delta x} = \frac{15-15}{7-2} = \frac{0}{5} = 0$

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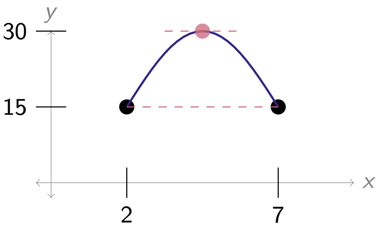
Average Rate of Change



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Rolle's Theorem and Average Rate of Change

Suppose $f(x)$ is continuous on the interval $[a, b]$, differentiable on the interval (a, b), and $f(a) = f(b)$. Then there exists a number c between a and b so that

$$f'(c) = 0 = \frac{f(b) - f(a)}{b - a}.$$

So there exists a point where the derivative is the same as the average rate of change.

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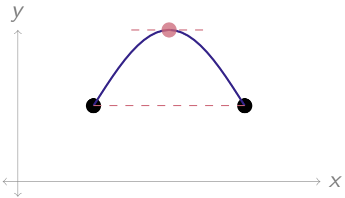
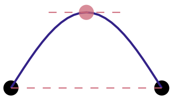
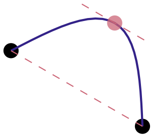
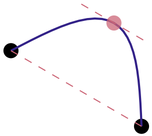
So there exists a point where the derivative is the same as the average rate of change. For example, think of throwing a ball straight up, and catching it.

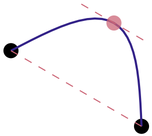

y
x







Mean Value Theorem



Mean Value Theorem

Let $f(x)$ be continuous on the interval $[a, b]$ and differentiable on (a, b). Then there is a number c between a and b such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

That is: there is some point c between a and b where the instantaneous rate of change of the function is equal to the average rate of change of the function on the interval $[a, b]$.

Rolle's Theorem

Let $f(x)$ be continuous on the interval $[a, b]$, differentiable on (a, b), and let $f(a) = f(b)$. Then there is a number c between a and b such that:

$$f'(c) = 0 = \frac{f(b) - f(a)}{b - a}$$

Suppose you are driving along a long, straight highway with no shortcuts. The speed limit is 100 kph. A police officer notices your car going 90 kph, and uploads your plate and the time they saw you to their database. 150 km down this same straight road, 75 minutes later, another police officer notices your car going 85kph, and uploads your plates to the database. Then they pull you over, and give you a speeding ticket. Why were they justified?

link, Wikimedia commons, creative commons

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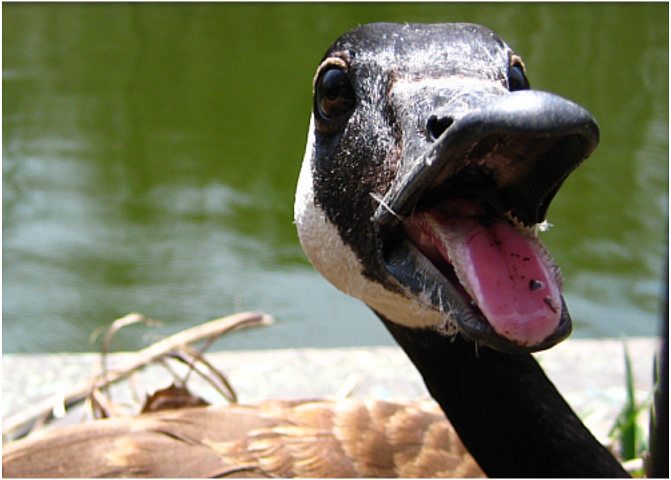
You travelled 150 km in 75 minutes. Since a moving car has a position that is continuous and differentiable, the MVT tells us that at some point, your instantaneous velocity was $\frac{150}{75}$ kilometers per minute, which works out to $\frac{150 \cdot 60}{75} = 125$ kph. So even though you weren't speeding when the officers saw you, you were definitely speeding some time in between.

Alternately, if you were going at most 100kph, then you would travel 150 kilometers in at least 90 minutes.

According to this website, Canada geese may fly 1500 miles in a single day under favorable conditions. It also says their top speed is around 70mph. Does this seem like a typo? (If it contradicts the Mean Value Theorem, it's probably a typo.)

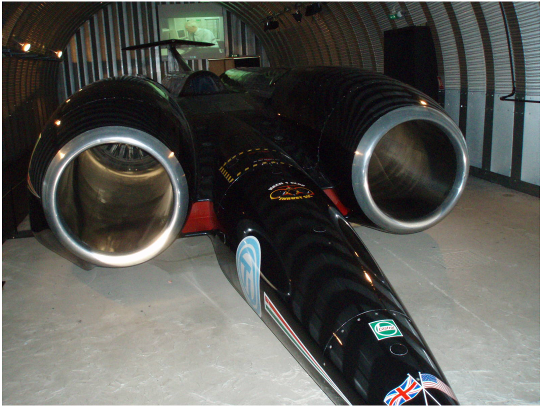


Credit: This Incredible World, link, unedited, creative commons license

According to this website, Canada geese may fly 1500 miles in a single day under favorable conditions. It also says their top speed is around 70mph. Does this seem like a typo? (If it contradicts the Mean Value Theorem, it's probably a typo.)

We can assume that the position of a goose is continuous and differentiable. Then the MVT tells us that a goose that travels 1500 miles in a day (24 hours) achieves, at some instant, a speed of $\frac{1500}{24}$ mph. Since $\frac{1500}{24} = 62.5$, these two facts seem compatible (and amazing!).

The record for fastest wheel-driven land speed is around 700 kph. [1] However, non-wheel driven cars (such as those powered by jet engines) have achieved higher speeds. [2] Suppose a driver of a jet-powered car starts a 10km race at 12:00, and finishes at 12:01. Did they beat 700kph?



[1] George Poteet, `https://en.wikipedia.org/wiki/Wheel-driven_land_speed_record`

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Suppose a driver of a jet-powered car starts a 10km race at 12:00, and finishes at 12:01. Did they beat 700kph?

Maybe, but not necessarily. We are guaranteed by the MVT that at some point they reached the following speed: $\frac{10}{(1/60)} = 600$ kph.

[1] George Poteet, `https://en.wikipedia.org/wiki/Wheel-driven_land_speed_record`
[2] record-holder ThrustSSC shown

Suppose you want to download a file that is 3000 MB (slightly under 3GB). Your internet provider guarantees you that your download speeds will always be between 1 MBPS (MB per second) and 5 MBPS (because you bought the cheap plan). Using the Mean Value Theorem, give an upper and lower bound for how long the download can take (assuming your providers aren't lying, and your device is performing adequately).

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We assume the download is continuous and differentiable, so we can use the MVT. Let T be the time (in seconds) the download takes. The MVT tells us that at some point, our speed was exactly $\frac{3000}{T}$, so it must be true that

$$1 \leq \frac{3000}{T} \leq 5$$

So, $\frac{3000}{5} \leq T \leq 3000$. That is, T is between 600 and 3000 seconds, or between 10 and 50 minutes.

Suppose $1 \leq f'(t) \leq 5$ for all values of t, and $f(0) = 0$. What are the possible solutions to $f(t) = 3000$?

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Notice: since the derivative exists for all real numbers, $f(x)$ is differentiable and continuous for all real numbers!

Since f is continuous and differentiable, we can use the MVT.

$$\frac{f(t) - f(0)}{t - 0} = \frac{3000}{t} = f'(c)$$

for some value c between 0 and t.
So,

$$1 \leq \frac{3000}{t} \leq 5$$

hence

$$600 \leq t \leq 3000$$

Corollaries to the MVT

Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = 0$ for all x in (a, b), then

If $f'(x) = g'(x)$, then

If $f'(x) > 0$ for all x in (a, b), then

If $f'(x) < 0$ for all x in (a, b), then

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Let $a < b$ be numbers in the domain of $f(x)$ and $g(x)$, which are continuous over $[a, b]$ and differentiable over (a, b).

If $f'(x) = 0$ for all x in (a, b), then $f(x)$ is constant. That is, $f(a') = f(b')$ for all a', b' in $[a, b]$

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