# Curve Sketching

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https://www.desmos.com/calculator/9funm5gwrt

# Curve Sketching

Good things to check:

- Domain
- Vertical asymptotes: $\lim_{x \to a} f(x) = \pm\infty$
- Intercepts: $x = 0$, $f(x) = 0$
- Horizontal asymptotes and end behavior: $\lim_{x \to \pm\infty} f(x)$

# Curve Sketching

Example: Sketch 2

What does the graph of the following function look like?

$$f(x) = \frac{x-2}{(x+3)^2}$$

Remember: domain, vertical asymptotes, intercepts, and horizontal asymptotes

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https://www.desmos.com/calculator/hyzl5cyq7i

# Curve Sketching

Example: Sketch 3

What does the graph of the following function look like?

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Add complexity: Increasing/decreasing, critical and singular points.

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$

•Domain: all real numbers
•Intercepts: $(0, 0)$ jumps out; we can factor $f(x) = x^2(\frac{1}{2}x^2 - \frac{4}{3}x - 15)$ then use quadratic formula to find y-intercepts at $x = \frac{4\pm\sqrt{286}}{3}$, so $x \approx 7$ and $x \approx -4.3$.
•As $x$ goes to positive or negative infinity, function goes to infinity
•$f'(x) = 2x^3 - 4x^2 - 30x = 2x(x^2 - 2 - 15) = 2x(x-5)(x+3)$ so critical points are $x = 0$, $x = -3$, and $x = 5$. No singular points.

| $x \approx -4.3$ | $x < -3$ | x=-3 | $-3 < x < 0$ | x=0 | $0 < x < 5$ | x=5 | $x > 5$ | $x \approx 7$ |
|---|---|---|---|---|---|---|---|---|
| $f(x) = 0$ | $f' < 0$ | $CP$ | $f' > 0$ | CP | $f' < 0$ | CP | $f' > 0$ | f(x)=0 |
| intercept | decr | l. min | incr | l. max | decr | l. min | incr | intercept |

https://www.desmos.com/calculator/lxdlgmhnsl

Example: Sketch 5

What does the following function look like?

$$f(x) = \frac{1}{3}x^3 + 2x^2 + 4x + 24$$

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What does the following function look like?

$$f(x) = \frac{1}{3}x^3 + 2x^2 + 4x + 24$$

•Domain: all real numbers. No VA. Goes to $\pm\infty$.
•$f(0) = 24$; $f(x) = \frac{1}{3}x^2(x+6) + 4(x+6) = (\frac{1}{3}x^2 + 4)(x+6)$, so only one root: $f(-6) = 0$.
•$f'(x) = x^2 + 4x4 = (x+2)^2$; only one critical point, at $x = -2$, and increasing everywhere else
•So, at the left, comes from negative infinity; levels crosses $x$-axis at $x = -6$; levels out at $x = -2$; crosses $y$-axis at $y = 24$; carries on to infinity
https://www.desmos.com/calculator/xum0mstmiv

Example: Sketch 6

What does the graph of the following function look like?

$$f(x) = e^{\frac{x+1}{x-1}}$$

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•Domain: $x \neq 1$ •VA: something weird happens at $x = 1$. Check out limits: $\displaystyle\lim_{x\to1^-} \frac{x+1}{x-1} = -\infty$ and $\displaystyle\lim_{x\to1^+} \frac{x+1}{x-1} = \infty$, so $\displaystyle\lim_{x\to1^-} f(x) = \lim_{A\to-\infty} e^A = 0$ while $\displaystyle\lim_{x\to1^+} f(x) = \lim_{A\to\infty} e^A = \infty$.

•Horizontal asymptotes: $\displaystyle\lim_{x\to\pm\infty} f(x) = e$

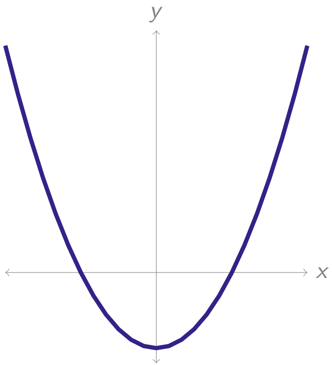
•Intercepts: the function is never zero; $f(0) = \frac{1}{e}$.

•Derivative: $f'(x) = e^{\frac{x+1}{x-1}}\left(\frac{-2}{(x-1)^2}\right)$; so the function is always decreasing (when it's defined!)

•So, on either end, it gets extremely close to $e$; as we move left to right, it dips to $\frac{1}{e}$ at the $y$-axis; gets nearly to the $x$-axis at 1; then has a VA from the right only at 1; then dips back to very close to $e$.

https://www.desmos.com/calculator/x0cccy1ggj

# Concavity



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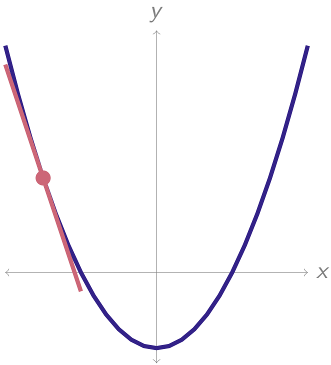

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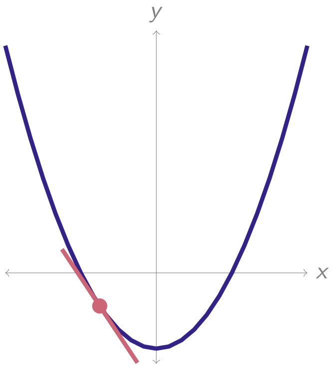

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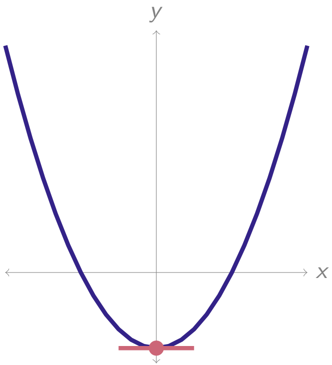

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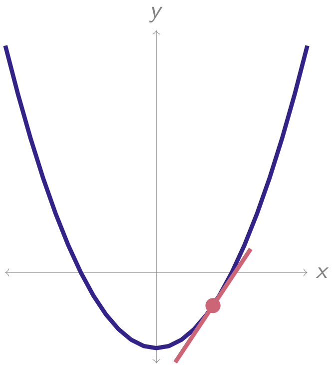

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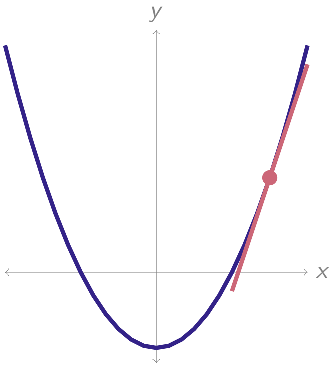

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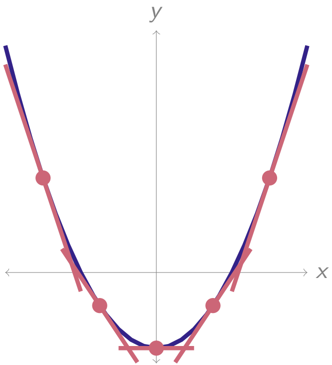

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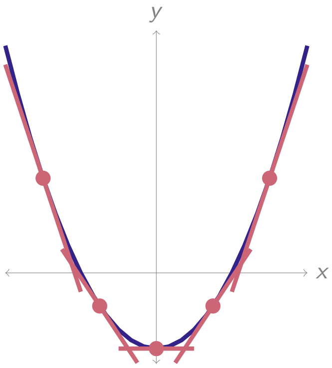


Slopes are increasing

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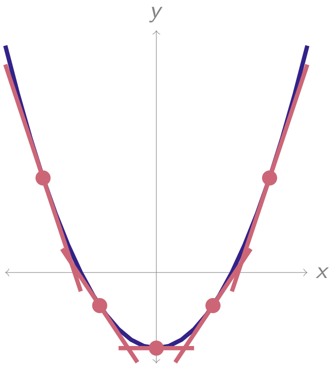


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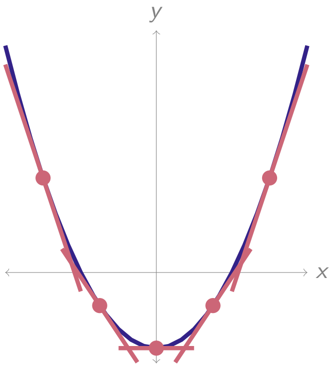


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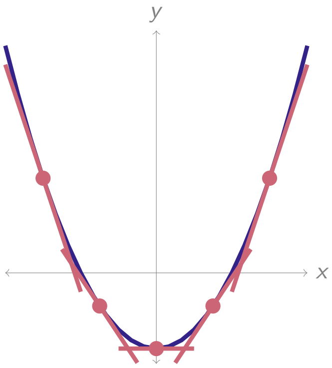


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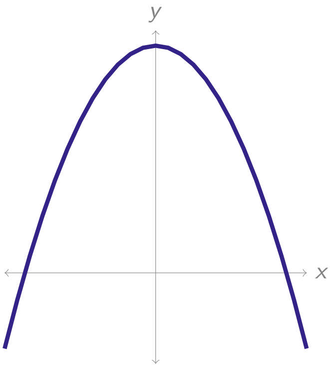

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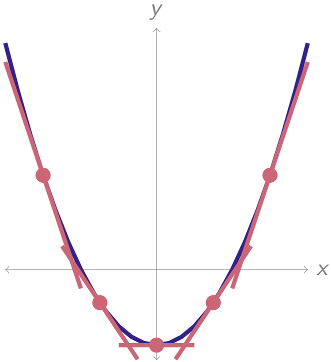


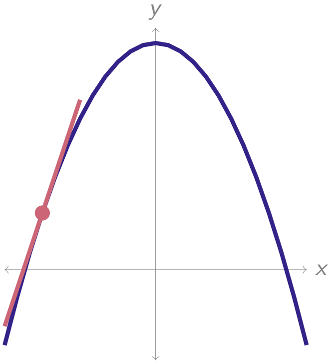


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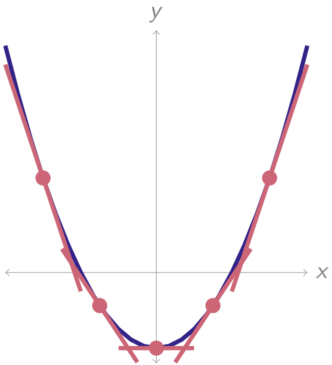


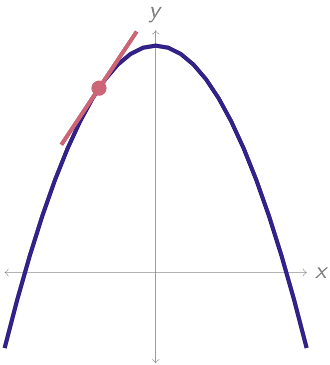


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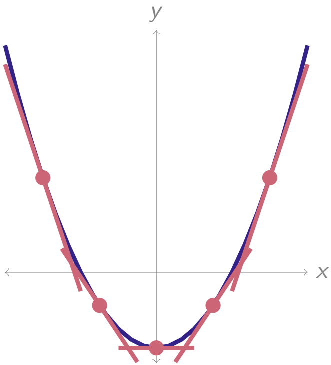


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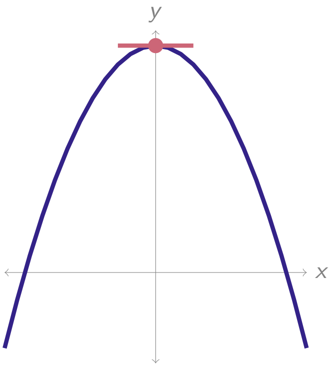

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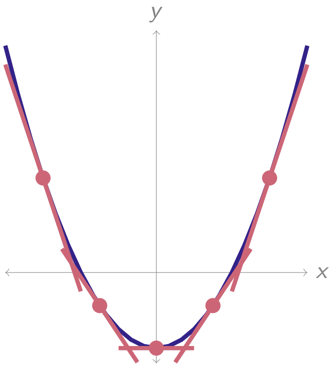


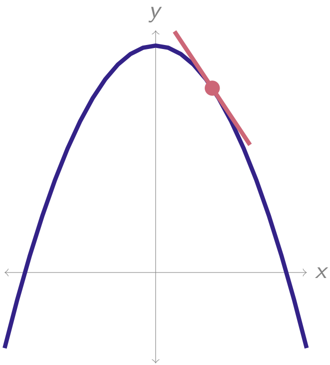


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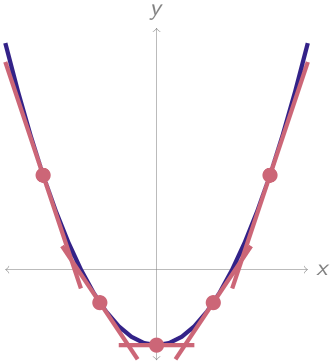


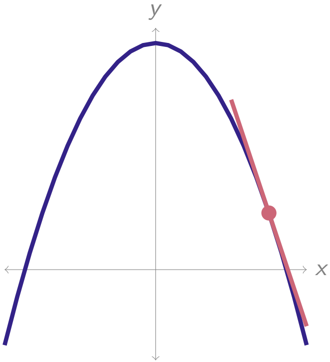


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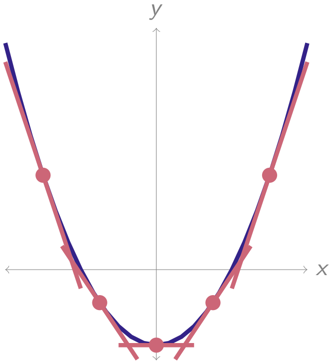


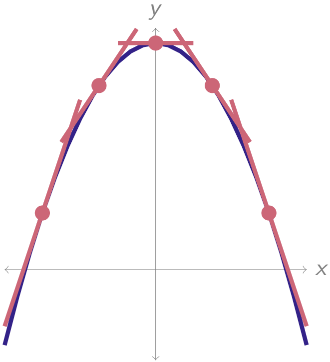


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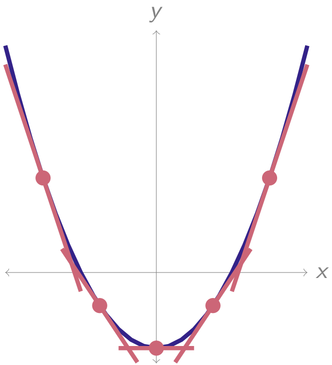


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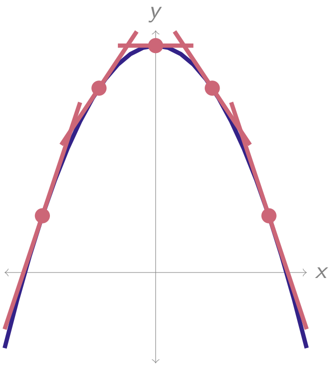


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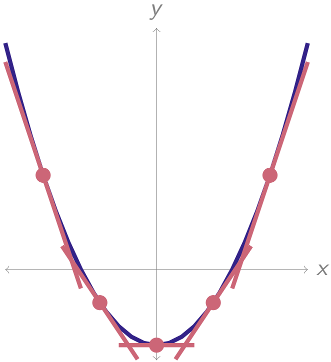


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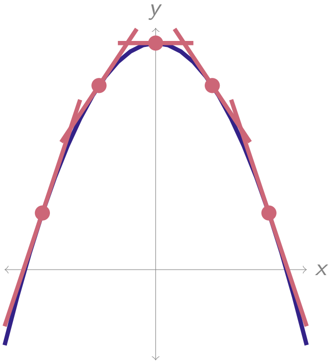


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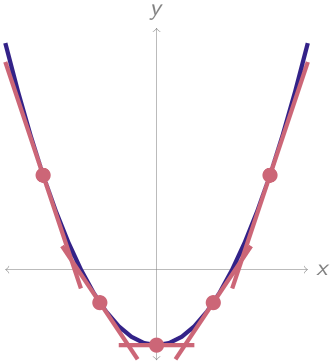


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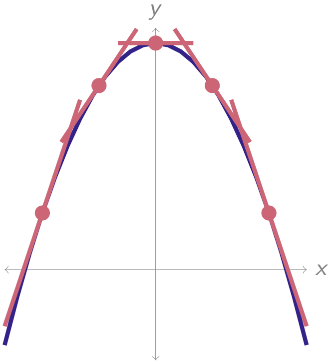


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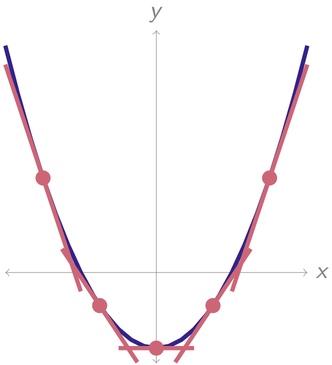


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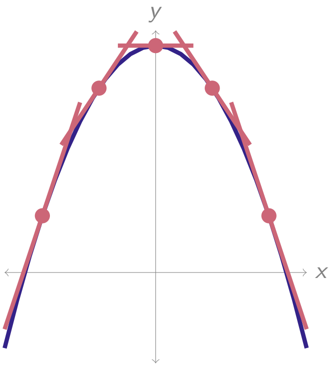
"concave up"

tangent line below curve

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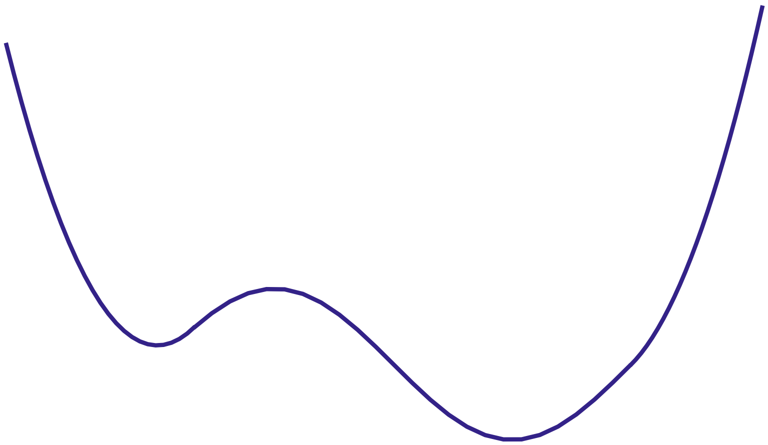
# Mnemonic

# Concavity



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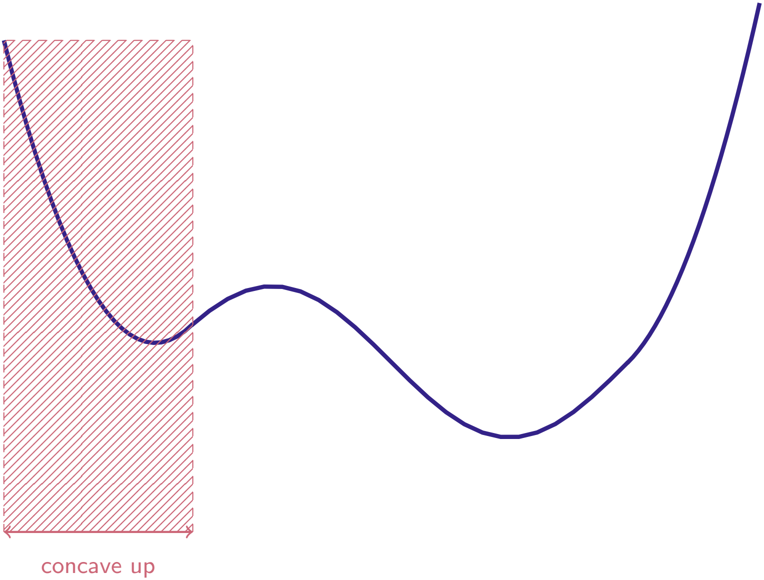

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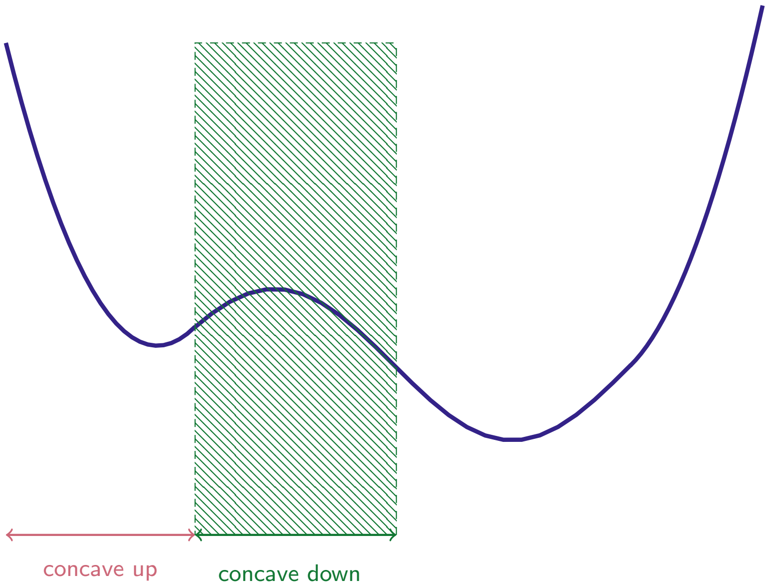

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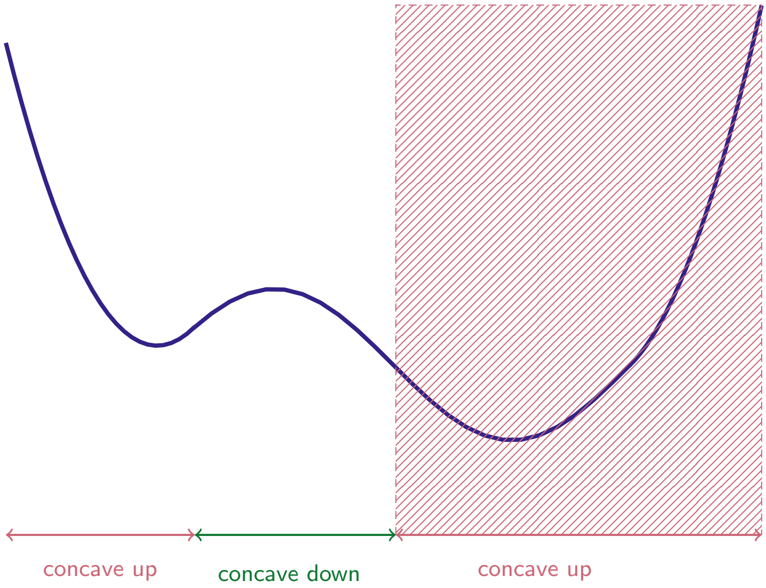

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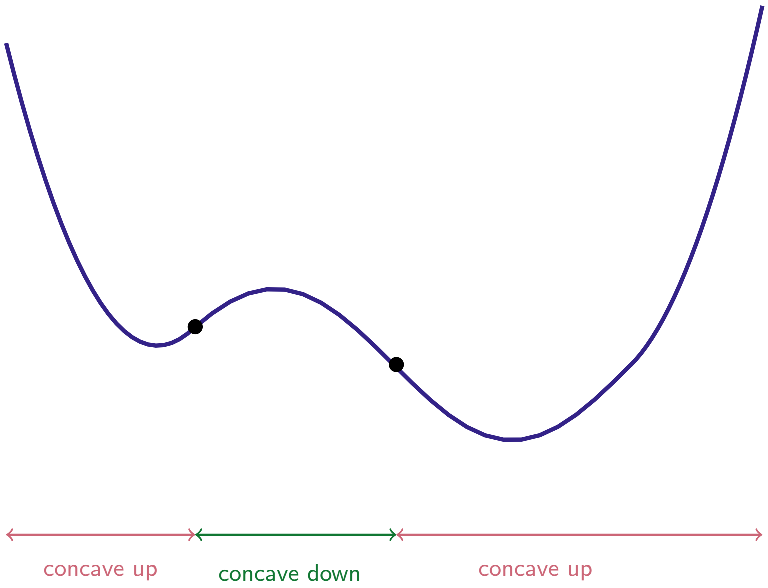

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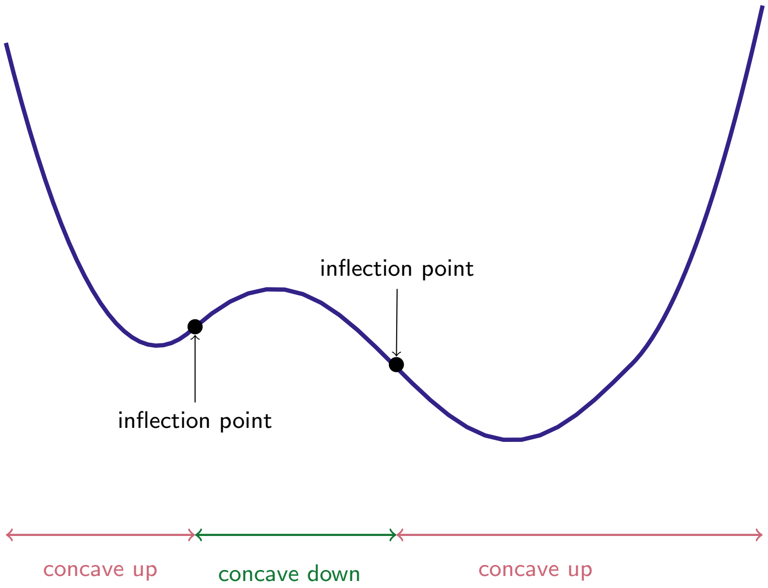
inflection point
inflection point
concave up
concave down
concave up

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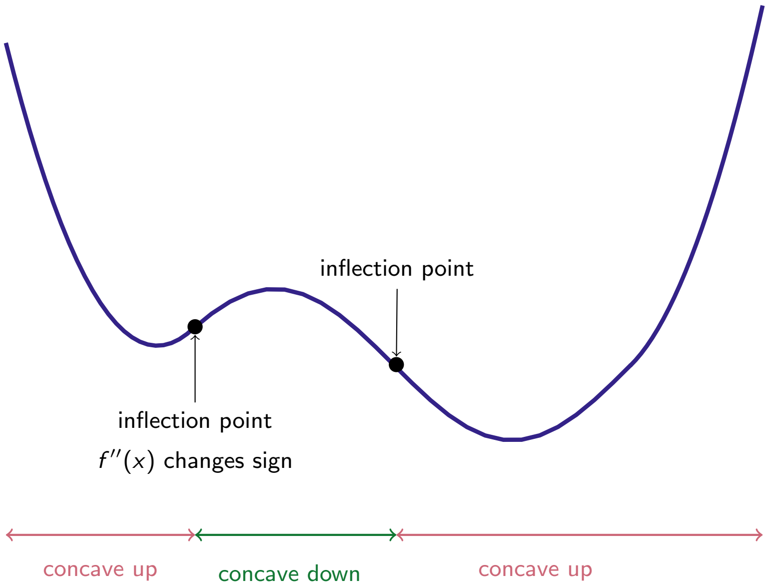
inflection point
inflection point
$f''(x)$ changes sign
concave up
concave down
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# Poll Questions

Describe the concavity of the function $f(x) = e^x$.

A. concave up

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C. concave up for $x < 0$; concave down for $x > 0$

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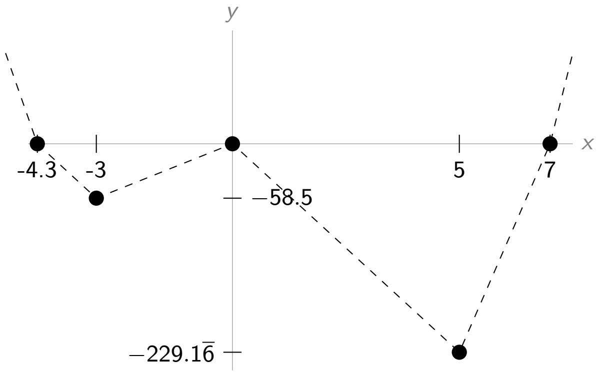
# From Last Time

Example: Sketch 6.5

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$



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Example: Sketch 6.5

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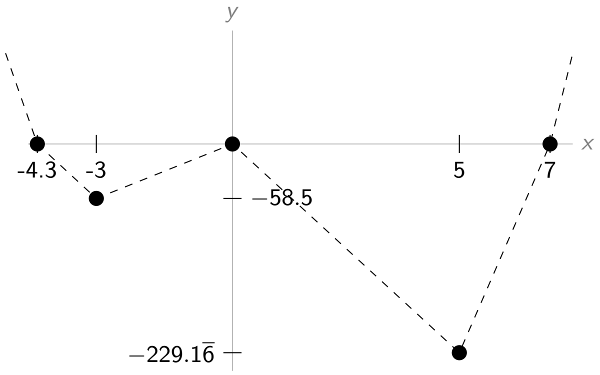
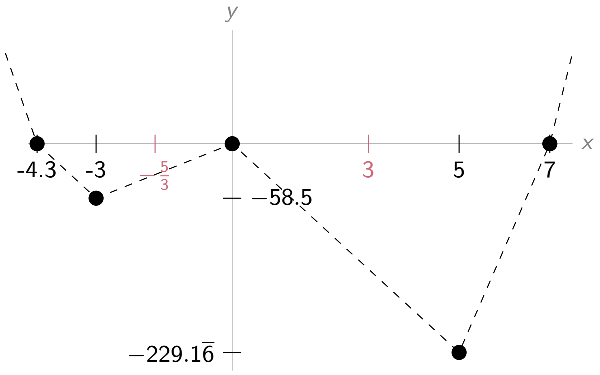



$f''(x) = 6x^2 - 8x - 30 = 2(x-3)(3x+5)$

# From Last Time

Example: Sketch 6.5

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$

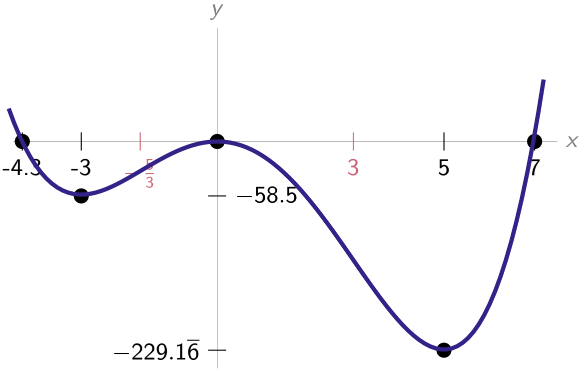



$f''(x) = 6x^2 - 8x - 30 = 2(x-3)(3x+5)$

# From Last Time

Example: Sketch 6.5

$$f(x) = \frac{1}{2}x^4 - \frac{4}{3}x^3 - 15x^2$$




$f''(x) = 6x^2 - 8x - 30 = 2(x-3)(3x+5)$

Example: Sketch 7

Sketch:

$$f(x) = x^5 - 15x^3$$

Example: Sketch 7

Sketch:

$$f(x) = x^5 - 15x^3$$

Symmetry!

Example: Sketch 7

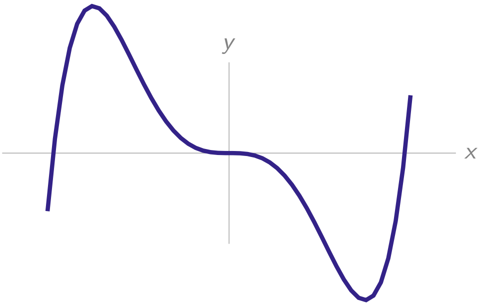
Sketch:

$$f(x) = x^5 - 15x^3$$

Symmetry!

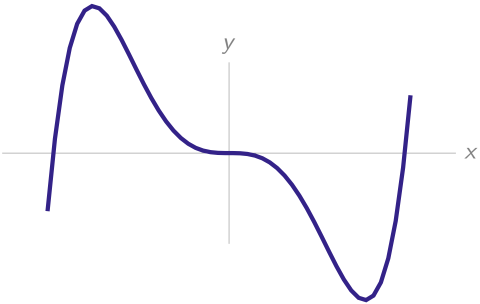
•Defined and differentiable for all real numbers.
•Roots: $x = 0$, $x = \pm\sqrt{15} \approx 4$
•Goes to $\pm\infty$ as $x$ goes to $\pm\infty$
•CP: $x = 0$, $x = \pm 3$. Increasing on $(-\infty, -3)$, decreasing $(-3, 0)$ and $(0, 3)$, decreasing $(3, \infty)$
•So, local max at $x = -3$ and local min at $x = 3$
•$f''(x) = 0$ for $x = 0$ and $x = \pm\frac{3}{\sqrt{2}} \approx \pm 2$. All of these are inflection points; concave down $(-\infty, -\frac{3}{\sqrt{2}})$, concave up $(\frac{3}{\sqrt{2}}, 0)$, concave down $(0, \frac{3}{\sqrt{2}})$, and concave up $(\frac{3}{\sqrt{2}}, \infty)$.
•$f(3) = -162$, $f(-3) = -162$, $f(-3/\sqrt{2}) \approx 100$, $f(3/\sqrt{2}) \approx -100$
https://www.desmos.com/calculator/uoii6nmgr8

# Even and Odd Functions




$$f(x) = x^5 - 15x^3$$

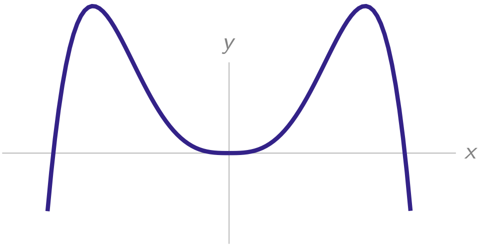
# Even and Odd Functions



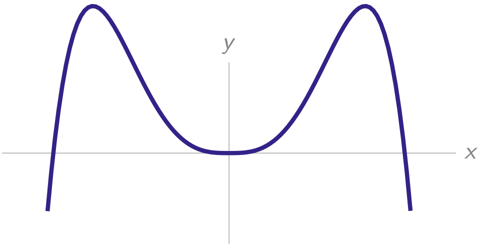

$$f(x) = x^5 - 15x^3$$

odd function

# Even and Odd Functions



# Even and Odd Functions



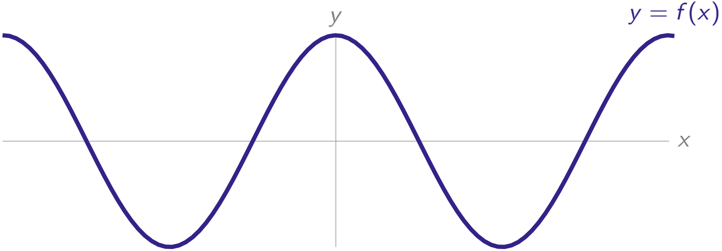

even function

# Even Functions

## Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$




even function

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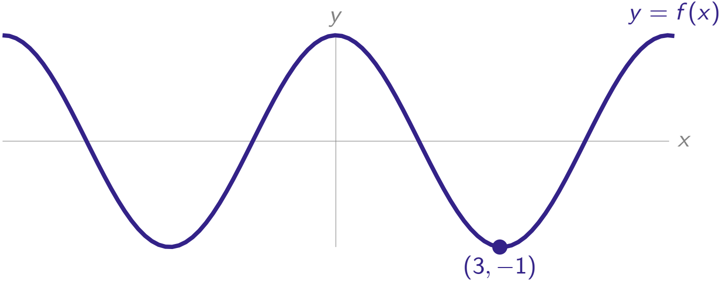



even function

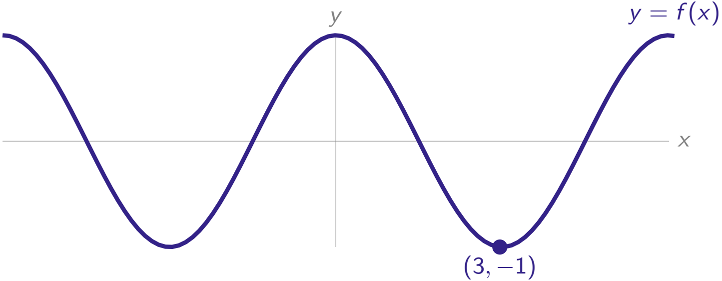
Suppose $f(3) = -1$.

# Even Functions

## Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$




even function

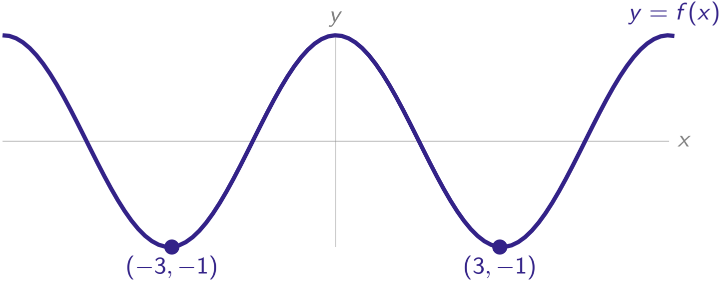
Suppose $f(3) = -1$.Then $f(-3) =$

# Even Functions

## Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$




even function

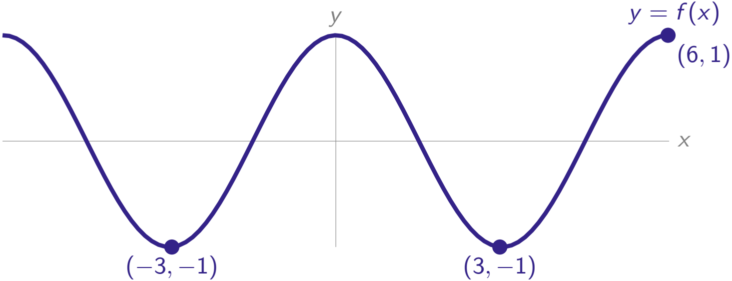
Suppose $f(3) = -1$.Then $f(-3) = -1$ also.

# Even Functions

## Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$




even function

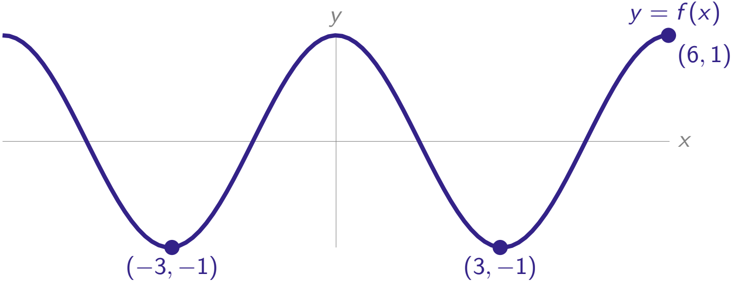
Suppose $f(3) = -1$.Then $f(-3) = -1$ also.
Suppose $f(6) = 1$.

# Even Functions

## Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$




even function

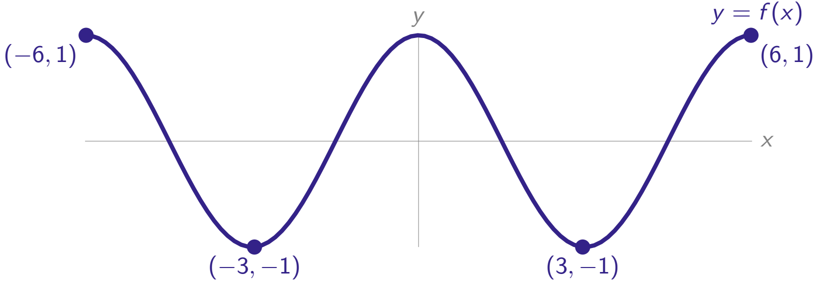
Suppose $f(3) = -1$.Then $f(-3) = -1$ also.
Suppose $f(6) = 1$.Then $f(-6) =$

# Even Functions

## Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$




even function

Suppose $f(3) = -1$.Then $f(-3) =-1$ also.
Suppose $f(6) = 1$.Then $f(-6) =1$ also.

# Even Functions

## Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$

Examples:

# Even Functions

### Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$

Examples:
$f(x) = x^2$

# Even Functions

## Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$

Examples:
$f(x) = x^2$
$f(x) = x^4$

# Even Functions

### Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$

Examples:
$f(x) = x^2$
$f(x) = x^4$
$f(x) = \cos(x)$

# Even Functions

## Even Function

A function $f(x)$ is even if, for all $x$ in its domain,

$$f(-x) = f(x)$$

Examples:

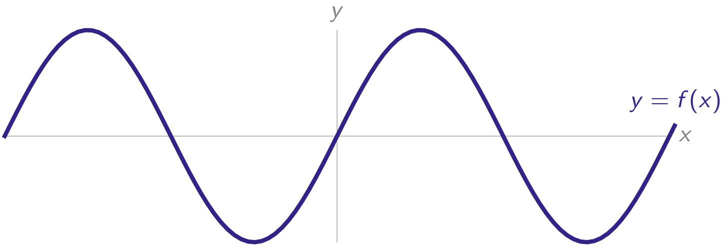
$f(x) = x^2$

$f(x) = x^4$

$f(x) = \cos(x)$

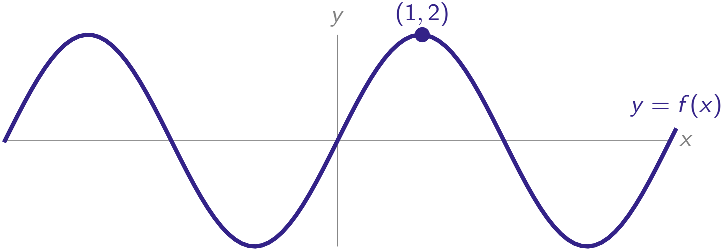
$$f(x) = \frac{x^4 + \cos(x)}{x^{16} + 7}$$

# Odd Functions




odd function

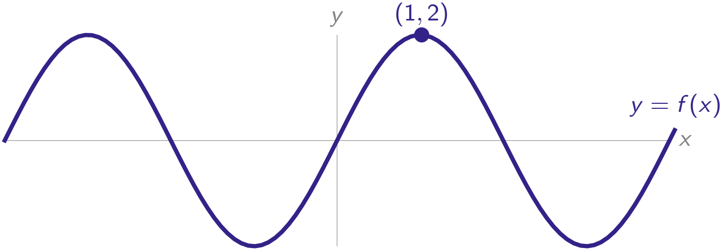
# Odd Functions




odd function

Suppose $f(1) = 2$.

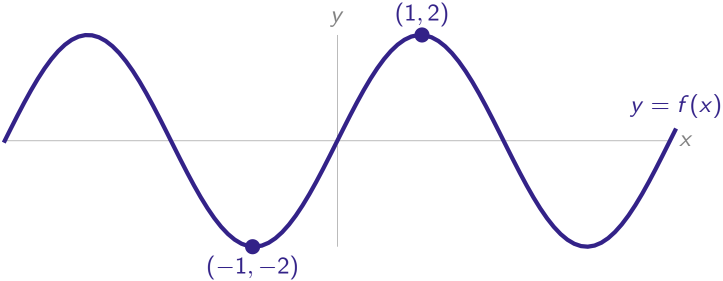
# Odd Functions




odd function

Suppose $f(1) = 2$.Then $f(-1) =$

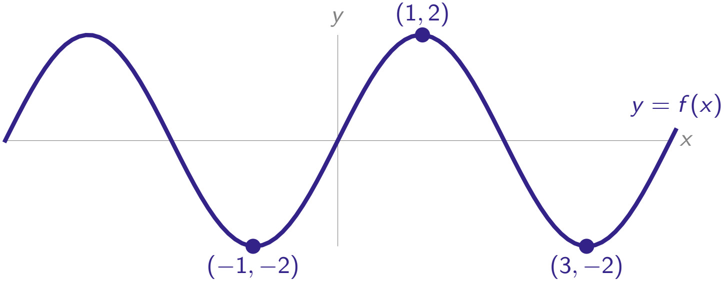
# Odd Functions




odd function

Suppose $f(1) = 2$.Then $f(-1) = -2$.

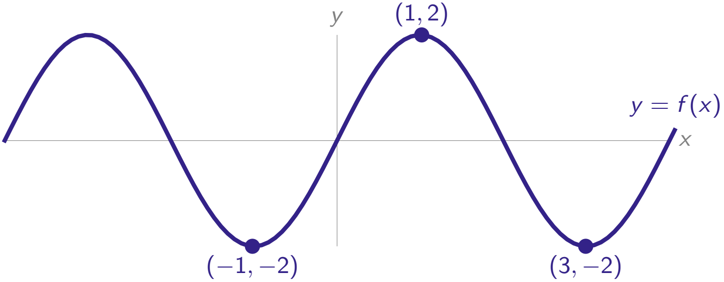
# Odd Functions




odd function

Suppose $f(1) = 2$.Then $f(-1) =-2$.
Suppose $f(3) = -2$.

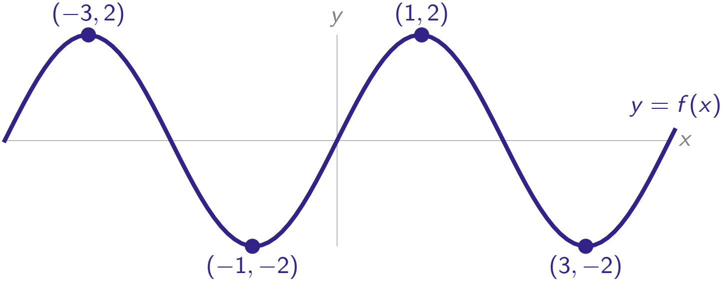
# Odd Functions




odd function

Suppose $f(1) = 2$.Then $f(-1) =-2$.
Suppose $f(3) = -2$.Then $f(-3) =$

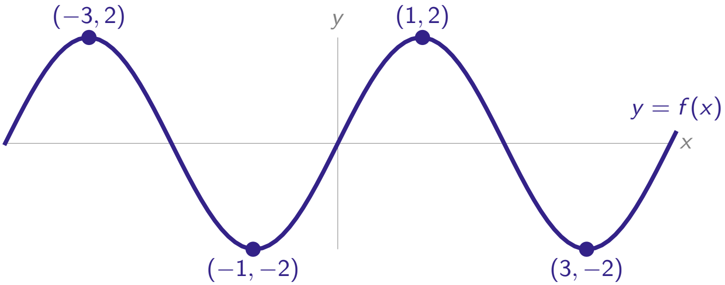
# Odd Functions




odd function

Suppose $f(1) = 2$.Then $f(-1) =-2$.
Suppose $f(3) = -2$.Then $f(-3) =2$.

# Odd Functions



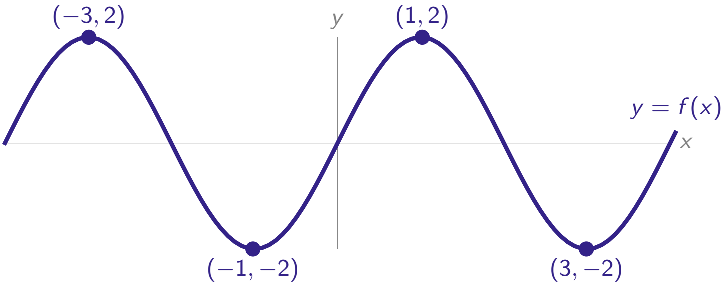

odd function

Suppose $f(1) = 2$.Then $f(-1) =-2$.
Suppose $f(3) = -2$.Then $f(-3) =2$.

## Even Function

A function $f(x)$ is odd if, for all $x$ in its domain,

# Odd Functions




odd function

Suppose $f(1) = 2$.Then $f(-1) =-2$.
Suppose $f(3) = -2$.Then $f(-3) =2$.

## Even Function

A function $f(x)$ is odd if, for all $x$ in its domain,

$$f(-x) = -f(x)$$

# Even Functions

## Even Function

A function $f(x)$ is odd if, for all $x$ in its domain,

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Examples:

# Even Functions

### Even Function

A function $f(x)$ is odd if, for all $x$ in its domain,

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Examples:

$f(x) = x$

# Even Functions

### Even Function

A function $f(x)$ is odd if, for all $x$ in its domain,

$$f(-x) = -f(x)$$

Examples:
$f(x) = x$
$f(x) = x^3$

# Even Functions

## Even Function

A function $f(x)$ is odd if, for all $x$ in its domain,

$$f(-x) = -f(x)$$

Examples:

$f(x) = x$
$f(x) = x^3$
$f(x) = \sin(x)$

# Even Functions

## Even Function

A function $f(x)$ is odd if, for all $x$ in its domain,

$$f(-x) = -f(x)$$

Examples:

$f(x) = x$

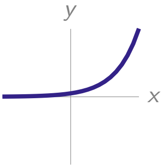
$f(x) = x^3$

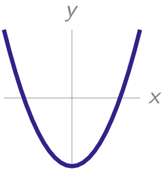
$f(x) = \sin(x)$

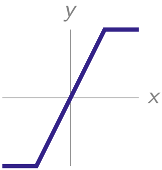
$f(x) = \dfrac{x(1+x^2)}{x^2+5}$

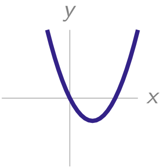
# Poll Tiiime

Pick out the odd function.

A:


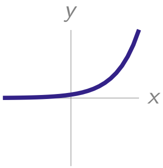

B:


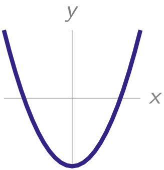

C:


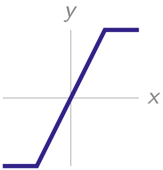

D:


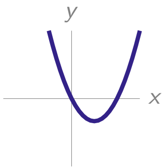
# Poll Tiiime

Pick out the odd function.

A: 

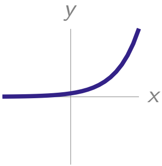

B: 

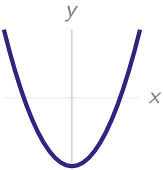

C: 

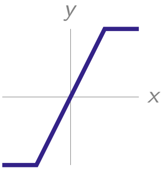

D: 

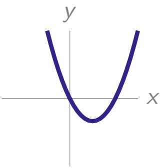
# Poll Tiiime

Pick out the even function.

A: 

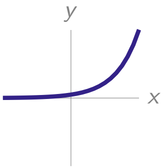

B: 

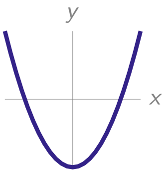

C: 

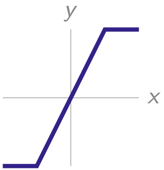

D: 

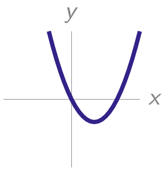
# Poll Tiiime

Pick out the even function.

A: 


B: 


C: 


D: 

# Even more Poll tiiiiime

Suppose $f(x)$ is an odd function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

A. $f(0) = f(-0)$

B. $f(0) = -f(0)$

C. $f(0) = 0$

D. all of the above are true

E. none of the above are necessarily true

# Even more Poll tiiiiime

Suppose $f(x)$ is an odd function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

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# Even more Poll tiiiiime

Suppose $f(x)$ is an odd function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

A. $f(0) = f(-0) <$— true but uninteresting, for all functions

B. $f(0) = -f(0)$

C. $f(0) = 0$

D. all of the above are true

E. none of the above are necessarily true

# Even more Poll tiiiiime

Suppose $f(x)$ is an odd function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

A. $f(0) = f(-0)$ <— true but uninteresting, for all functions

B. $f(0) = -f(0)$ <— only possible for $f(0) = 0$

C. $f(0) = 0$

D. all of the above are true

E. none of the above are necessarily true

# Even more Poll tiiiiime

Suppose $f(x)$ is an odd function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

A. $f(0) = f(-0)$ $<$— true but uninteresting, for all functions

B. $f(0) = -f(0)$ $<$— only possible for $f(0) = 0$

C. $f(0) = 0$ $<$— this is equivalent to the choice above

D. all of the above are true

E. none of the above are necessarily true

# Even more and more Poll tiiiiime

Suppose $f(x)$ is an even function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

A. $f(0) = f(-0)$

B. $f(0) = -f(0)$

C. $f(0) = 0$

D. all of the above are true

E. none of the above are necessarily true

# Even more and more Poll tiiiiime

Suppose $f(x)$ is an even function, continuous, defined for all real numbers. What is $f(0)$? Pick the best answer.

A. $f(0) = f(-0)$

B. $f(0) = -f(0)$

C. $f(0) = 0$

D. all of the above are true

E. none of the above are necessarily true

# OK OK... last one

Suppose $f(x)$ is an even function, differentiable for all real numbers. What can we say about $f'(x)$?

A. $f'(x)$ is also even

B. $f'(x)$ is odd

C. $f'(x)$ is constant

D. all of the above are true

E. none of the above are necessarily true

# OK OK... last one

Suppose $f(x)$ is an even function, differentiable for all real numbers. What can we say about $f'(x)$?

A. $f'(x)$ is also even
B. $f'(x)$ is odd
C. $f'(x)$ is constant
D. all of the above are true
E. none of the above are necessarily true

# Periodicity

### Periodic

A function is periodic with period $P$ if

$$f(x) = f(x + P)$$

whenever $x$ and $x + P$ are in the domain of $f$, and $P$ is the smallest such (positive) number

Examples: $\sin(x)$, $\cos(x)$ both have period $2\pi$; $\tan(x)$ has period $\pi$.

Example: Sketch 8

$$f(x) = \sin(\sin x)$$

(ignore concavity)

Example: Sketch 8

$$f(x) = \sin(\sin x)$$

(ignore concavity)

Example: Sketch 9

$$g(x) = \sin(2\pi \sin x)$$

# Let's Graph

Example: Sketch 10

$$f(x) = (x^2 - 64)^{1/3}$$

# Let's Graph

Example: Sketch 10

$$f(x) = (x^2 - 64)^{1/3}$$

$$f'(x) = \frac{2x}{3(x^2 - 64)^{2/3}};$$

$$f''(x) = \frac{-2(\frac{1}{3}x^2 + 64)}{3(x^2 - 64)^{5/3}}$$

# Let's Graph

Example: Sketch 11

$$f(x) = \frac{x^2 + x}{(x+1)(x^2+1)^2}$$

# Let's Graph

Example: Sketch 11

$$f(x) = \frac{x^2 + x}{(x+1)(x^2+1)^2}$$

Note for $x \neq -1$, $f(x) = \dfrac{x(x+1)}{(x+1)(x^2+1)^2} = \dfrac{x}{(x^2+1)^2}$

# Let's Graph

Example: Sketch 11

$$f(x) = \frac{x^2 + x}{(x+1)(x^2+1)^2}$$

Note for $x \neq -1$, $f(x) = \dfrac{x(x+1)}{(x+1)(x^2+1)^2} = \dfrac{x}{(x^2+1)^2}$

Example: Sketch 12

$$g(x) := \frac{x}{(x^2+1)^2}$$

# Let's Graph

Example: Sketch 11

$$f(x) = \frac{x^2 + x}{(x+1)(x^2+1)^2}$$

Note for $x \neq -1$, $f(x) = \dfrac{x(x+1)}{(x+1)(x^2+1)^2} = \dfrac{x}{(x^2+1)^2}$

Example: Sketch 12

$$g(x) := \frac{x}{(x^2+1)^2}$$

$g'(x) = \dfrac{1-3x^2}{(x^2+1)^3}$; $g''(x) = \dfrac{12x(x^2-1)}{(x^2+1)^4}$

# Let's Graph

Example: Sketch 13

$$f(x) = x(x-1)^{2/3}$$

# Match the Function to its Graph

A. $f(x) = \dfrac{x-1}{(x+1)(x+2)}$

B. $f(x) = \dfrac{(x-1)^2}{(x+1)(x+2)}$

C. $f(x) = \dfrac{x-1}{(x+1)^2(x+2)}$

D. $f(x) = \dfrac{(x-1)^2}{(x+1)^2(x+2)}$

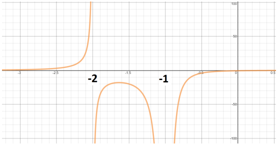
# Match the Function to its Graph

A. $f(x) = \dfrac{x-1}{(x+1)(x+2)}$

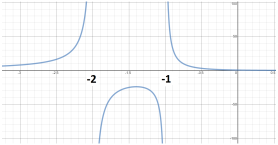
B. $f(x) = \dfrac{(x-1)^2}{(x+1)(x+2)}$

C. $f(x) = \dfrac{x-1}{(x+1)^2(x+2)}$

D. $f(x) = \dfrac{(x-1)^2}{(x+1)^2(x+2)}$




ORANGE

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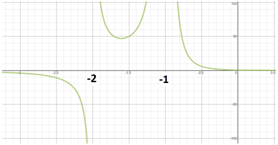


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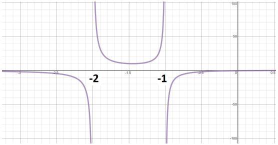


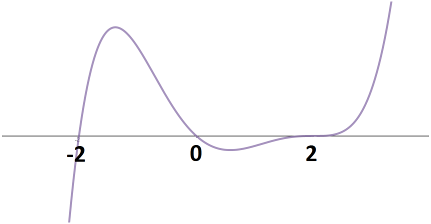

PURPLE

# Match the Function to its Graph

A. $f(x) = x^3(x+2)(x-2) = x^5 - 4x^3$
B. $f(x) = x(x+2)^3(x-2) = x^5 + 4x^4 - 16x^2 - 16x$
C. $f(x) = x(x+2)(x-2)^3 = x^5 - 4x^4 + 16x^2 - 16x$

# Match the Function to its Graph

A. $f(x) = x^3(x+2)(x-2) = x^5 - 4x^3$
B. $f(x) = x(x+2)^3(x-2) = x^5 + 4x^4 - 16x^2 - 16x$
C. $f(x) = x(x+2)(x-2)^3 = x^5 - 4x^4 + 16x^2 - 16x$

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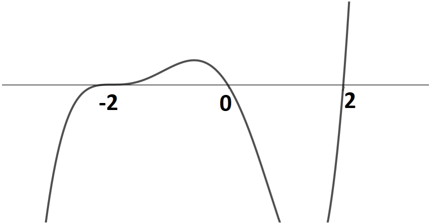


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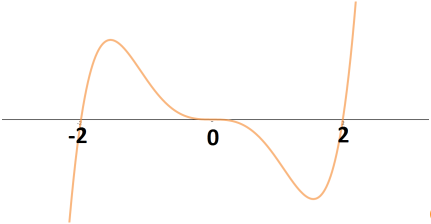
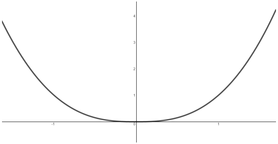



ORANGE

# Match the Function to its Graph

A. $f(x) = |x|^e$ B. $f(x) = e^{|x|}$ C. $f(x) = e^{x^2}$ D. $f(x) = e^{x^4 - x}$



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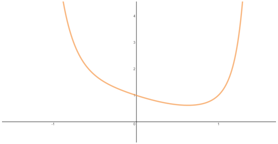

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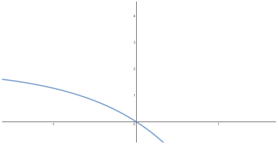
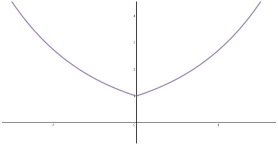


BLUE 1



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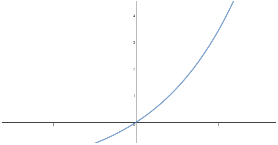
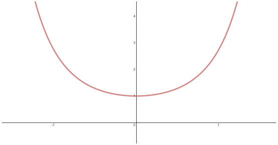


BLUE 2



RED

# Match the Function to its Graph

A. $f(x) = x^5 + 15x^3$ B. $f(x) = x^5 - 15x^3$ C. $f(x) = x^5 - 15x^2$
D. $f(x) = x^3 - 15x$ E. $f(x) = x^7 - 15x^4$

# Match the Function to its Graph

A. $f(x) = x^5 + 15x^3$ B. $f(x) = x^5 - 15x^3$ C. $f(x) = x^5 - 15x^2$
D. $f(x) = x^3 - 15x$ E. $f(x) = x^7 - 15x^4$