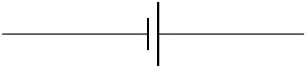
Outline

Week 5: Circuits

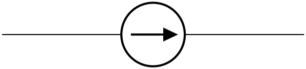
Course Notes: 3.5

Goals: Use linear algebra to determine voltage drops and branch currents.

Components in Resistor Networks



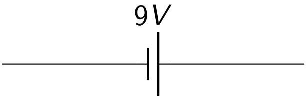
voltage source



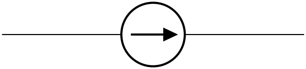
current source

resistor

Components in Resistor Networks



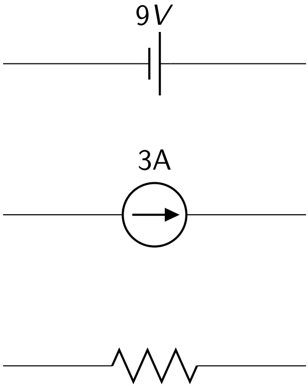
voltage source



current source

resistor

Components in Resistor Networks

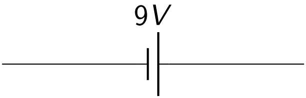


voltage source

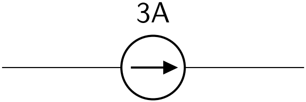
current source (inductor at an instant)

resistor

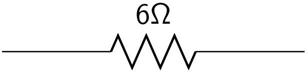
Components in Resistor Networks



voltage source

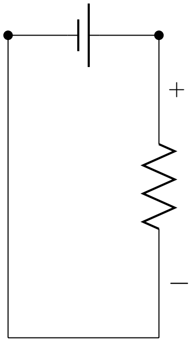


current source (inductor at an instant)

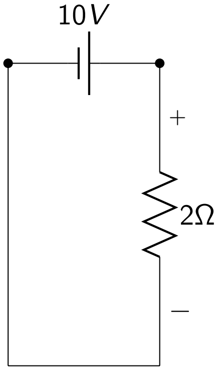


resistor

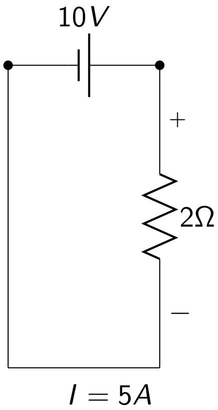
$V = IR$



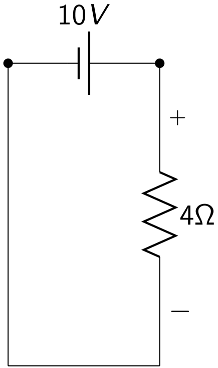
$V = IR$



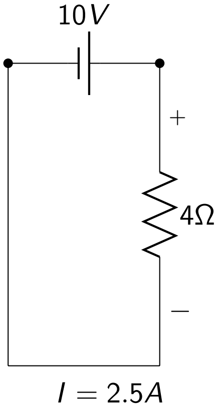
$V = IR$



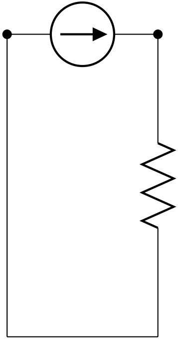
$V = IR$



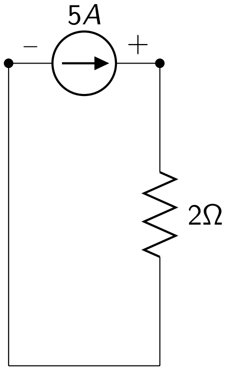
$V = IR$



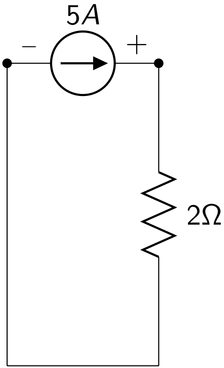
$V = IR$



$V = IR$



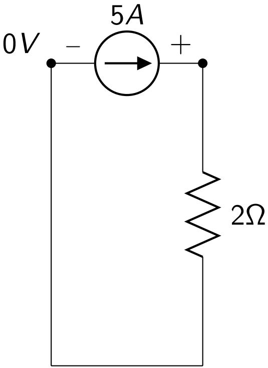
$V = IR$



$V = 10$

(voltage drop of 10 Volts across resistor)

$V = IR$



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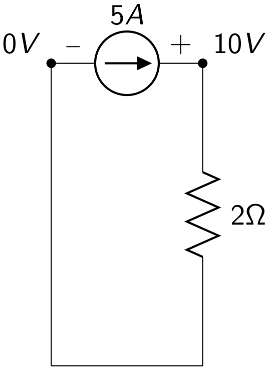

$$V = 10$$

(voltage drop of 10 Volts across resistor)

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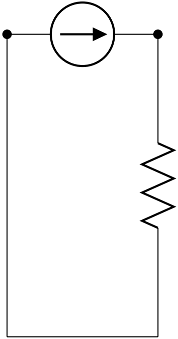
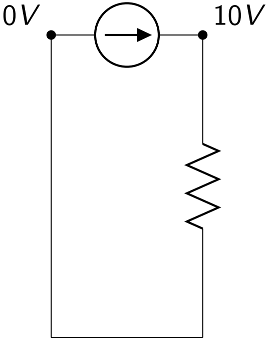


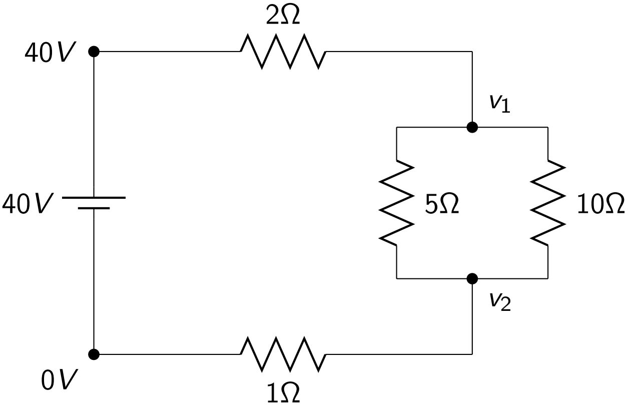
Setup: Given: Resistance of resistors; voltage across voltage sources; current through current sources.

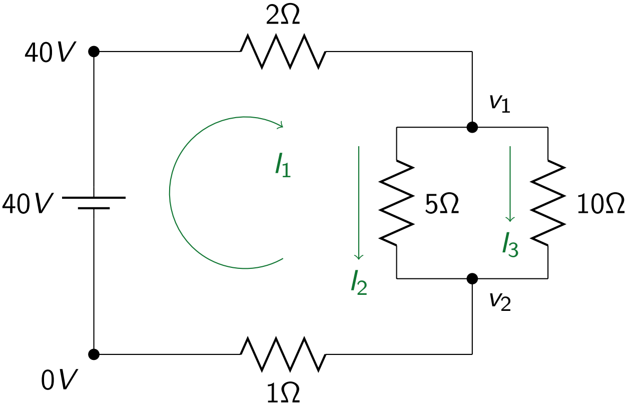
Find: currents through each resistor and each power source; voltage drops across each current source

Kirchhoff's Laws

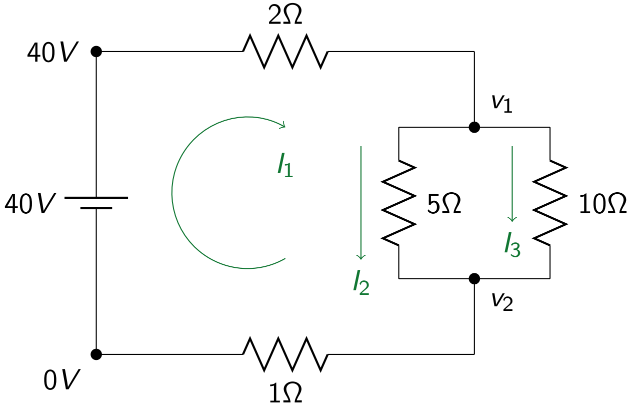
1. The sum of voltage drops around any closed loops in the network must be zero.
2. For any node, current in equals current out



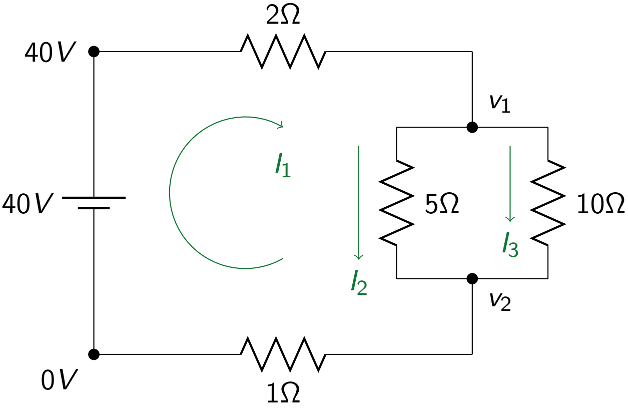

2Ω
40 V
v1
40 V
5Ω
10Ω
v2
0 V
1Ω



2Ω
40 V
v_1
I_1
40 V
5Ω
10Ω
I_3
I_2
v_2
0 V
1Ω



$$I_1 = I_2 + I_3$$
$$40 - v_1 = I_2 \cdot 2$$
$$v_2 - 0 = I_1 \cdot 1$$
$$v_1 - v_2 = 5 I_2$$
$$v_1 - v_2 = 10 I_3$$



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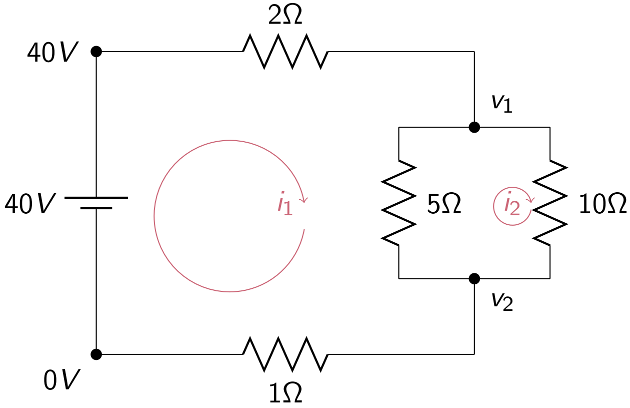
$$v_2 - 0 = I_1 \cdot 1$$
$$v_1 - v_2 = 5 I_2$$
$$v_1 - v_2 = 10 I_3$$

$$I_1 \approx 6.3$$
$$I_2 \approx 4.2$$
$$I_3 \approx 2.1$$
$$v_1 \approx 27.4$$
$$v_2 \approx 6.3$$



$$I_1 = I_2 + I_3$$
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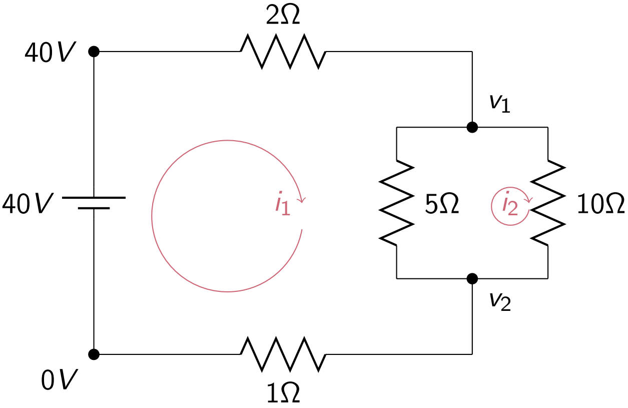

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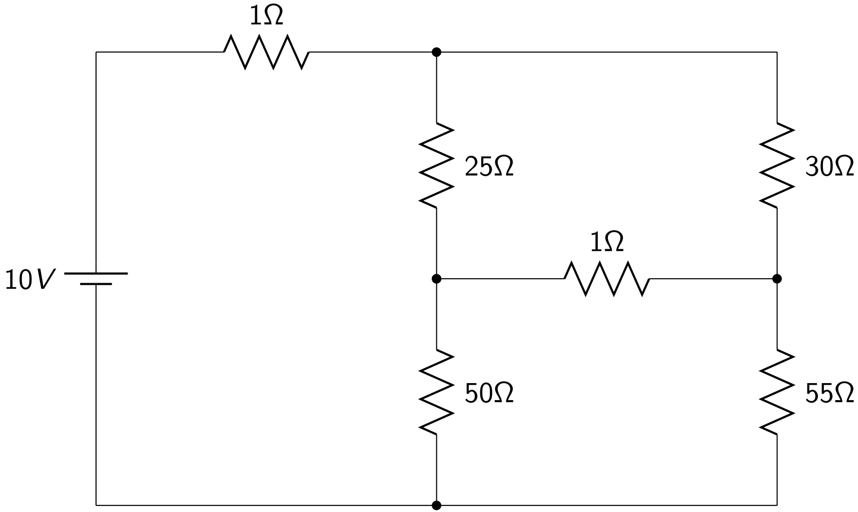
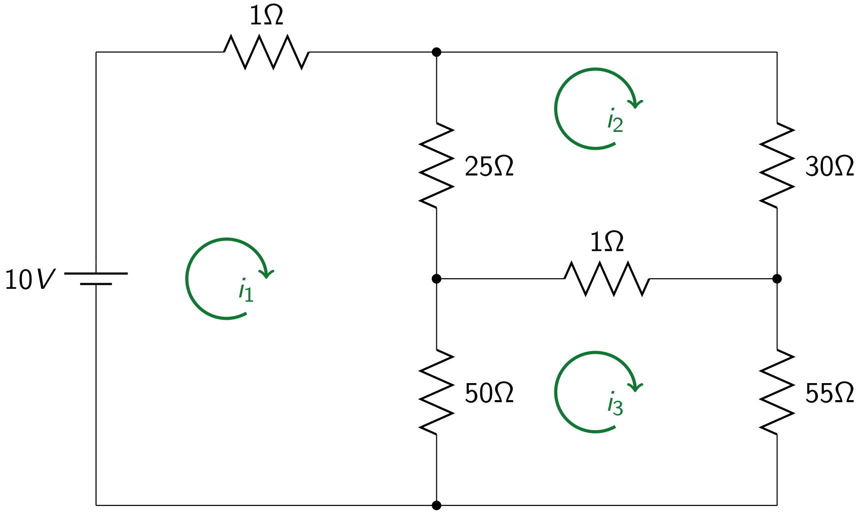
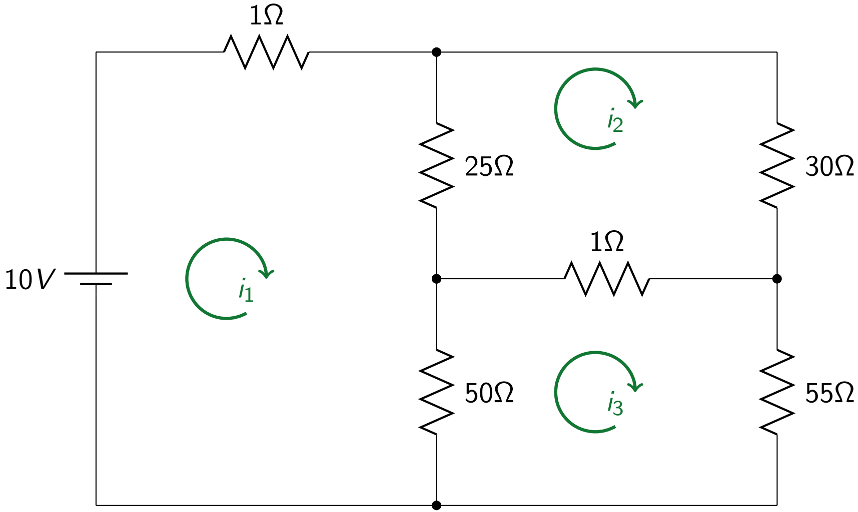
$$v_1 \approx 27.4$$
$$v_2 \approx 6.3$$

$$1i_1 - 40 + 2i_1 + 5(i_1 - i_2) = 0$$
$$10i_2 + 5(i_2 - i_1) = 0$$

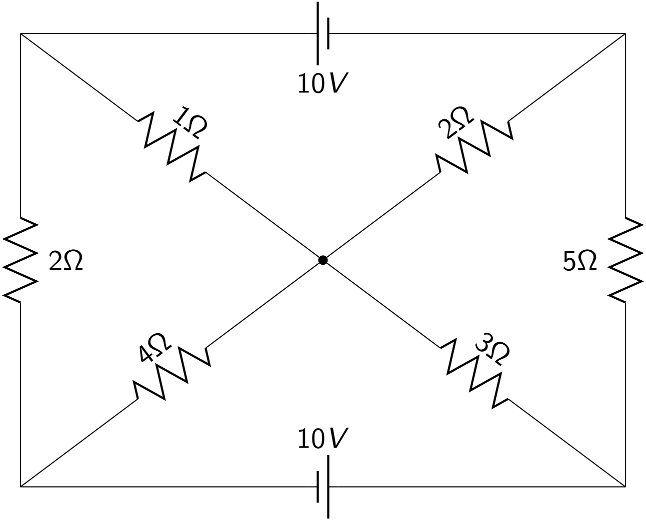


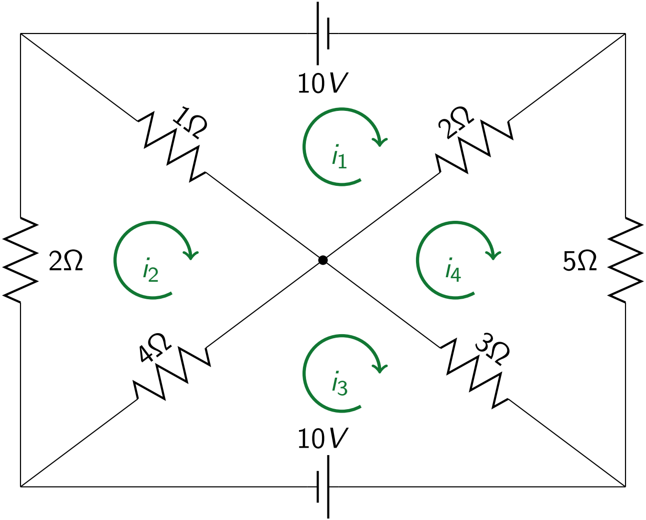
1Ω
25Ω
30Ω
1Ω
10V
50Ω
55Ω

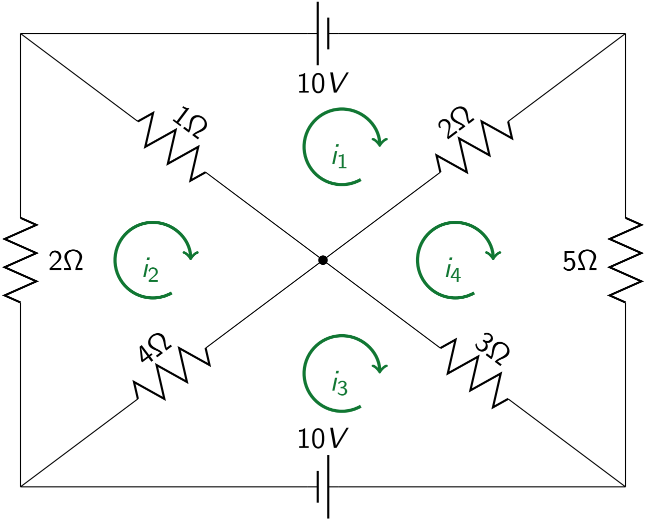

1Ω
25Ω
30Ω
1Ω
10V
i_1
i_2
i_3
50Ω
55Ω



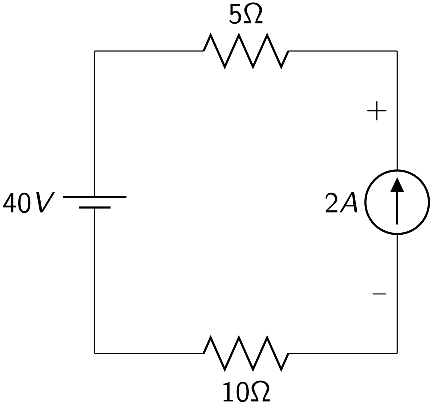
$$i_1 \approx 0.2249, \quad i_2 \approx 0.1114, \quad i_3 \approx 0.1166$$

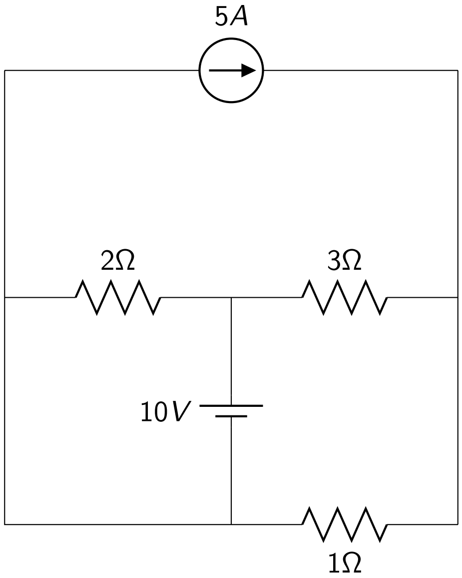

10V
1Ω
2Ω
2Ω
5Ω
4Ω
3Ω
10V


10 V
1Ω
2Ω
i_1
2Ω
i_2
i_4
5Ω
4Ω
3Ω
i_3
10 V

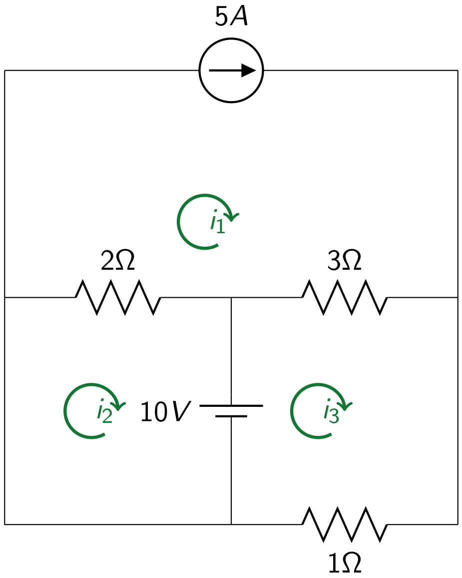


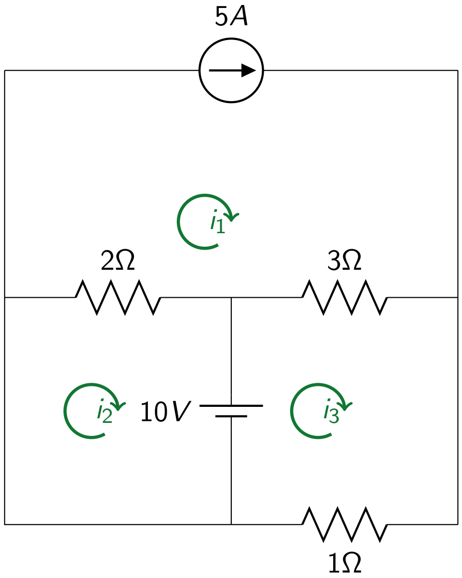
$$i_1 = -3.3036 \quad i_2 = 0.4464 \quad i_3 = 1.6071 \quad i_4 = -0.1786$$


5Ω
+
40V
2A
−
10Ω

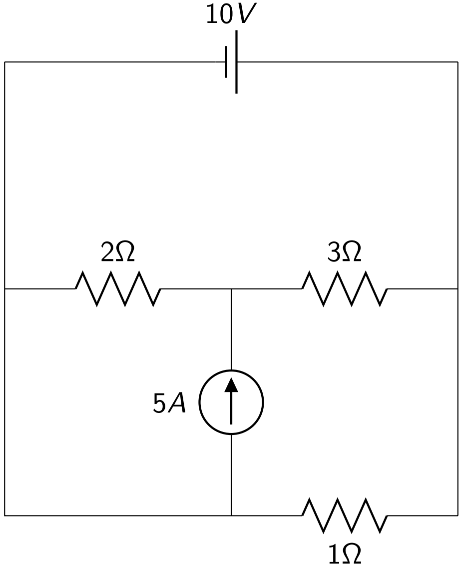


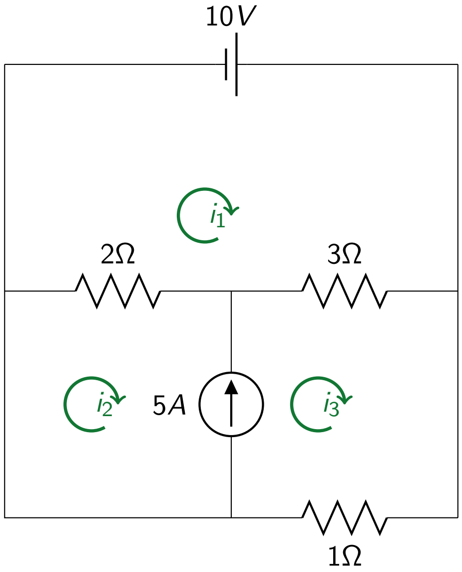
5A
2Ω
3Ω
10V
1Ω

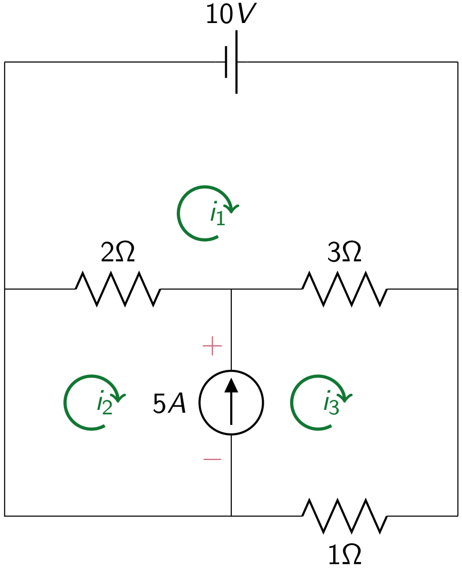

5A
i_1
2Ω
3Ω
i_2
10V
i_3
1Ω



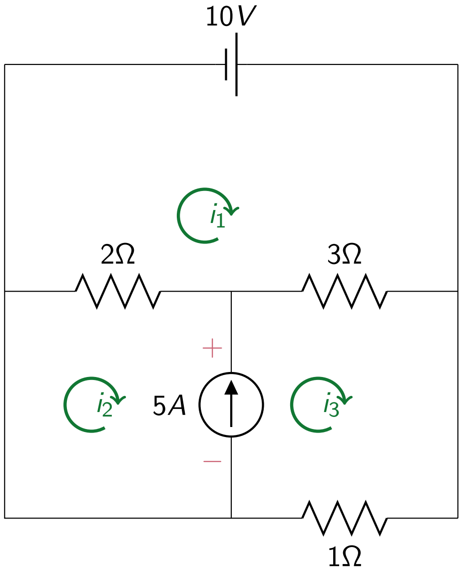
$$i_1 = 5, \quad i_2 = 0, \quad i_3 = \frac{25}{4}$$


10V
2Ω
3Ω
5A
1Ω


10 V
i1
2Ω
3Ω
i2
5A
i3
1Ω

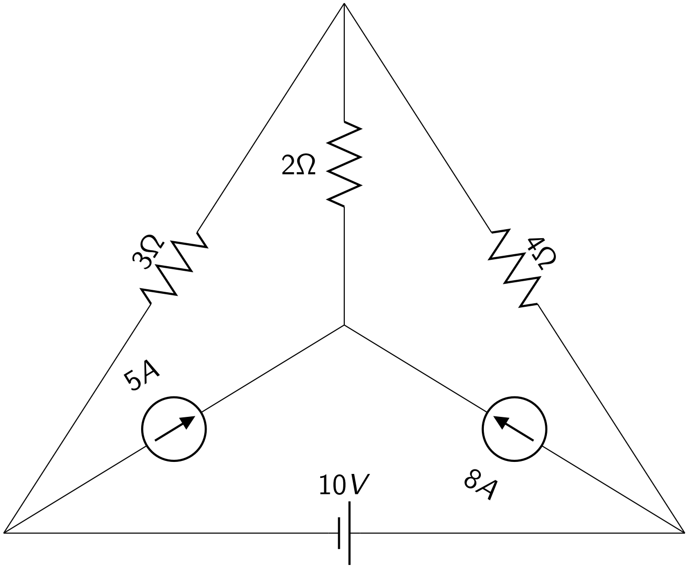


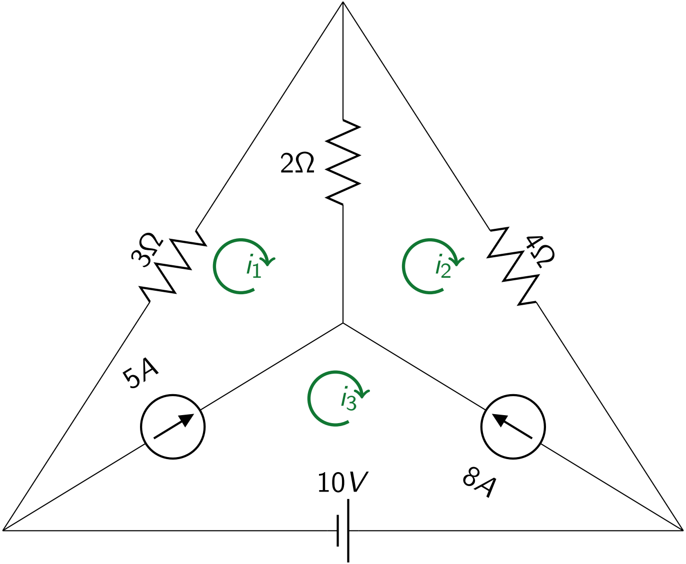
Let E be the voltage drop across the current source.



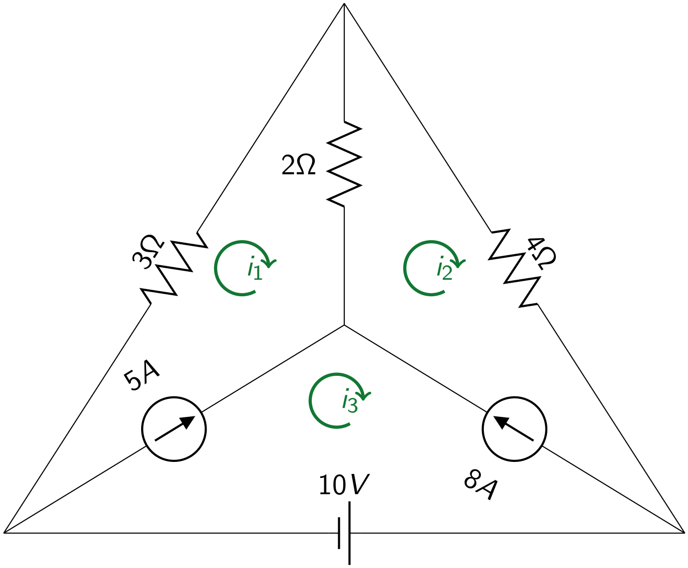
Let E be the voltage drop across the current source.

$$i_1 = 10, \quad i_2 = 5, \quad i_3 = 10, \quad E = 10$$

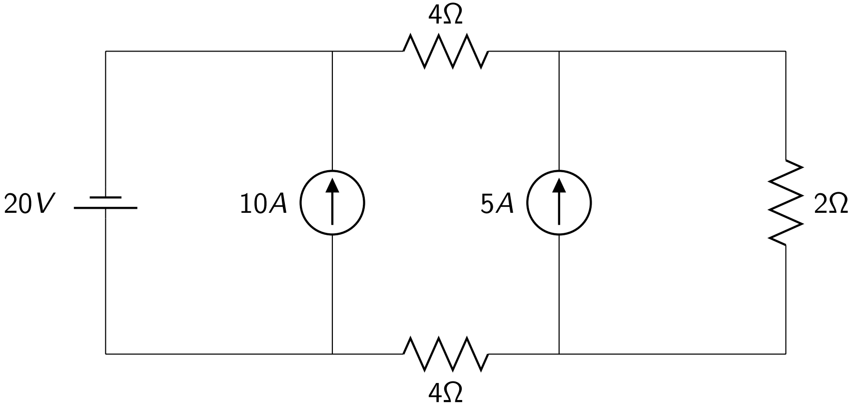

2Ω
3Ω
4Ω
5A
8A
10V

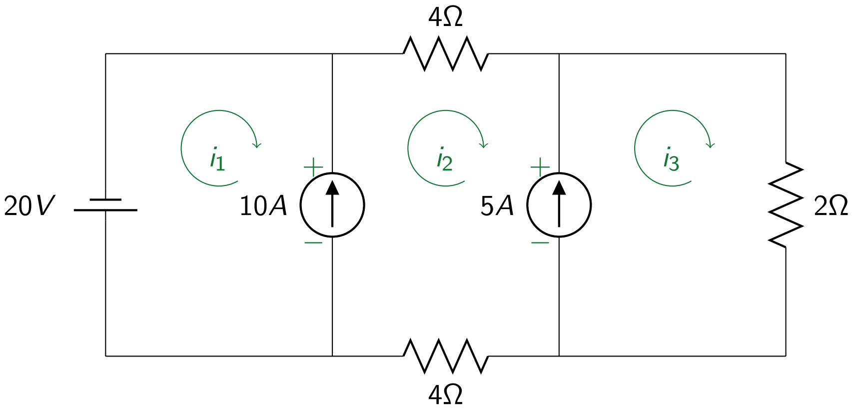


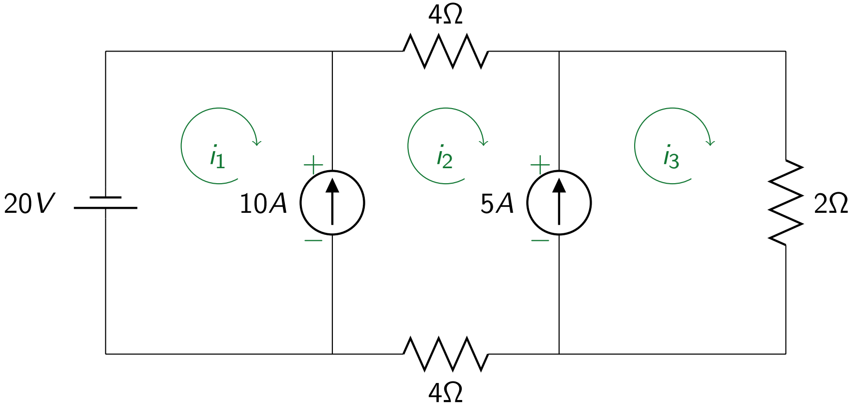
2Ω
3Ω
4Ω
i1
i2
i3
5A
8A
10V



$i_1 = -8.8571, \quad i_2 = 4.1429, \quad i_3 = -3.8571, \quad E_1 = 52.5744, \quad E_2 = 42.5714$

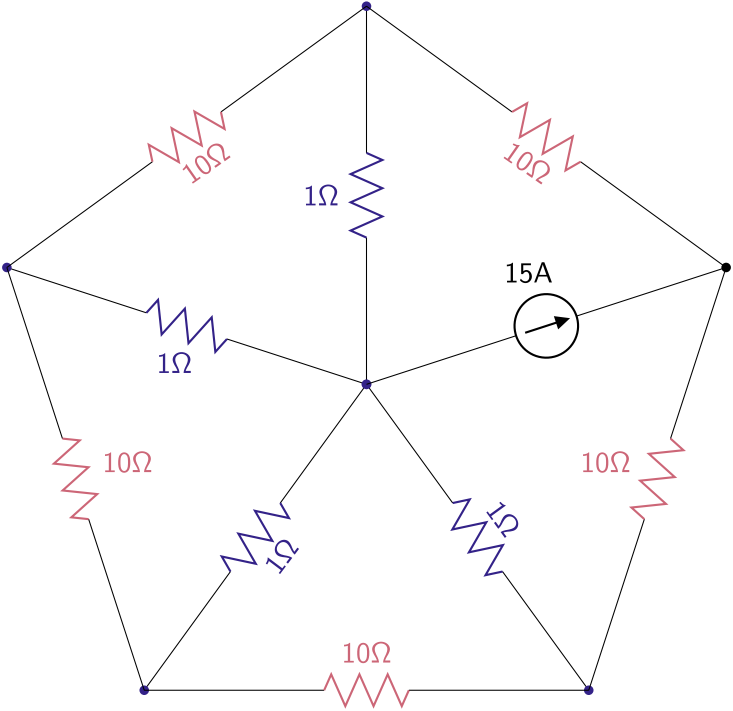

4Ω
20V
10A
5A
2Ω
4Ω

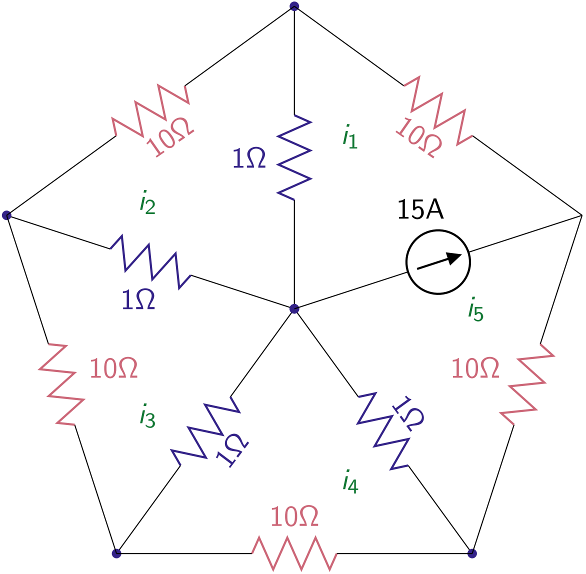

4Ω
20V
i_1
10A
i_2
5A
i_3
2Ω
4Ω



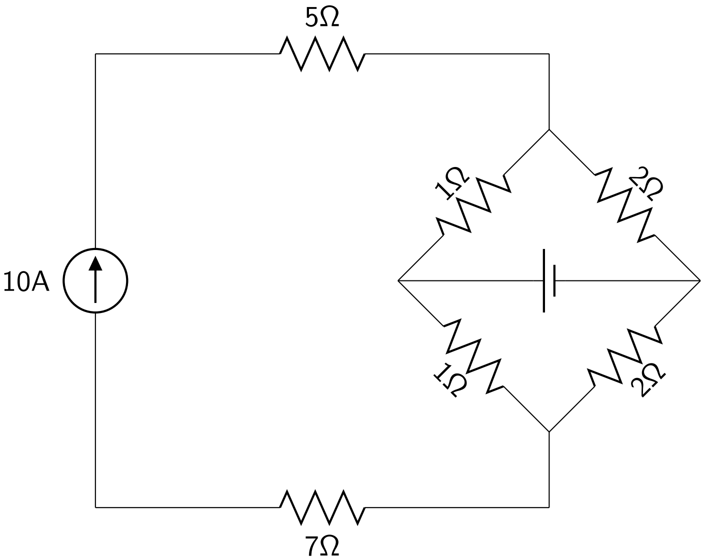
$$i_1 = -13A, \quad i_2 = -3A, \quad i_3 = 2A, \quad E_1 = -20V, \quad E_2 = 4V$$

Current across voltage source: 13A, top to bottom

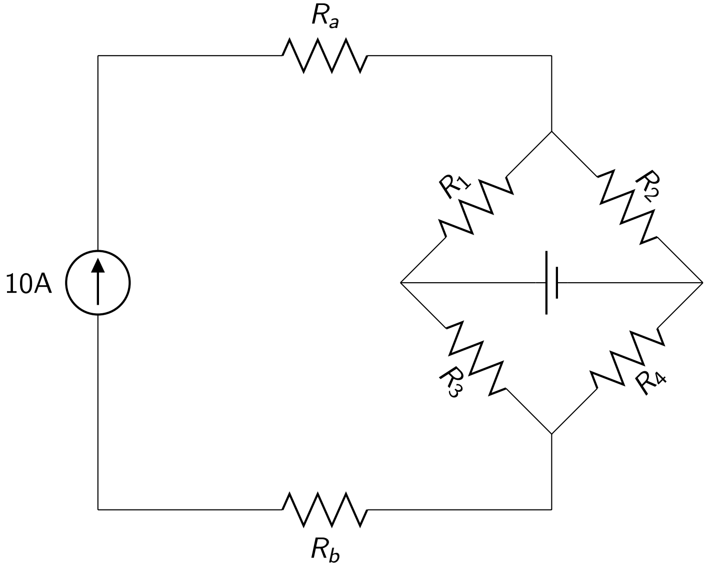

10Ω
10Ω
1Ω
1Ω
15A
10Ω
10Ω
1Ω
1Ω
10Ω



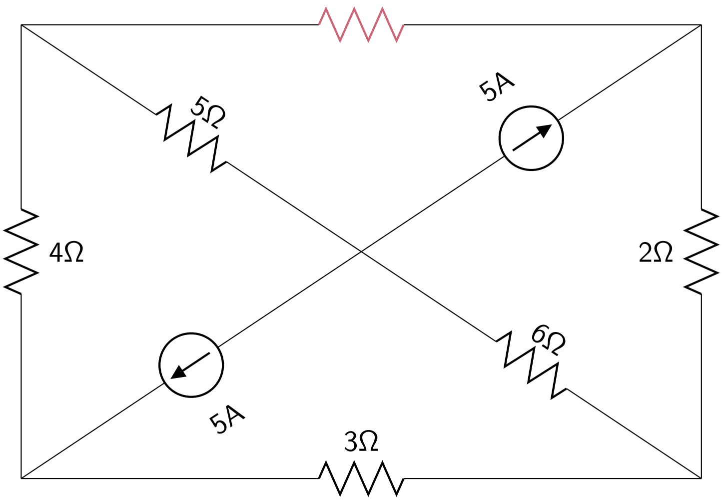
clockwise: $i_1 = -7.5, \quad i_2 = -0.625, \quad i_3 = 0, \quad i_4 = 0.625, \quad i_5 = 7.5$



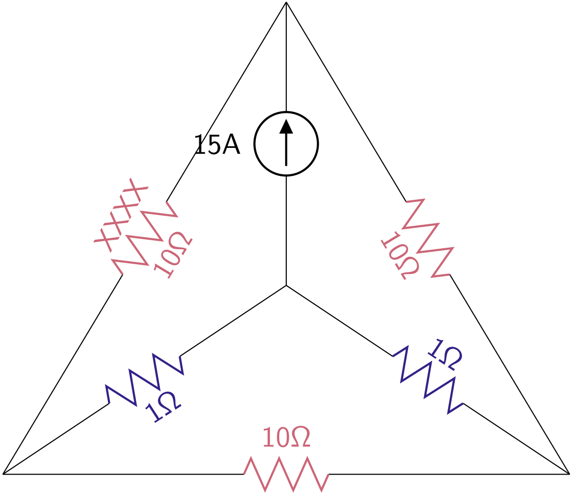
What voltage should the voltage source have, in order for there to be no current across it?



What voltage should the voltage source have, in order for there to be no current across it?

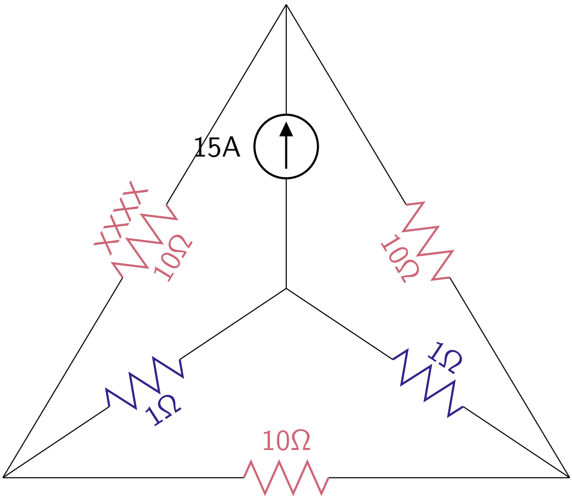


What resistance should the top resistor have, if you want each wire touching the centre to have current 5A?



Replace ONE resistor (with a different resistor or a different component) so that the current through the marked resistor is zero.

(OK fine, one way is to remove the marked resistor itself. Try something else :))



Find all ways to change the resistances of the non-marked resistors so that the current flowing through the marked resistor is zero. Justify your answer with algebra.