Outline

Week 9: complex numbers; complex exponential and polar form

Course Notes: 5.1, 5.2, 5.3, 5.4

Goals:
Fluency with arithmetic on complex numbers
Using matrices with complex entries: finding determinants and inverses, solving systems, etc.
Visualizing complex numbers in coordinate systems

Complex Arithmetic

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When we talk about "complex numbers," we allow numbers to have real parts and imaginary parts:

$$2 + 3i \qquad -1 \qquad 2i$$

Complex Arithmetic

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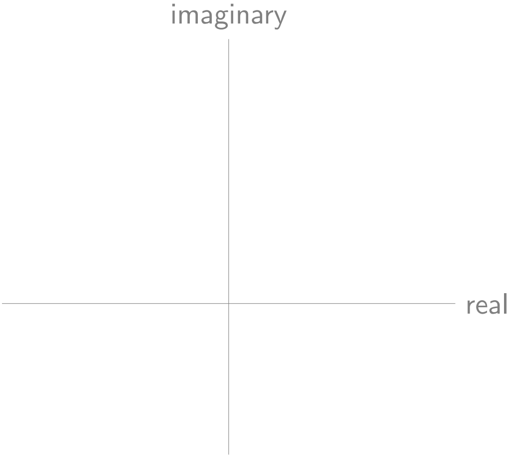

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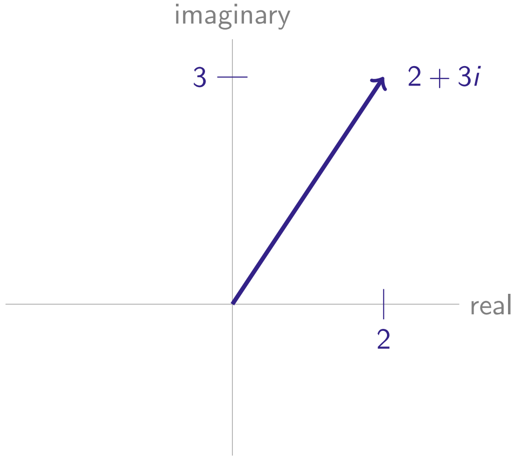

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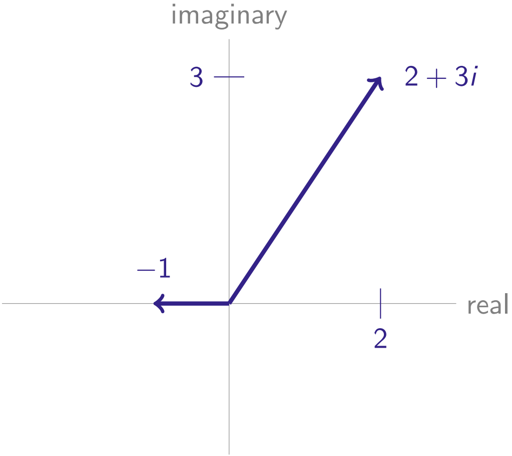

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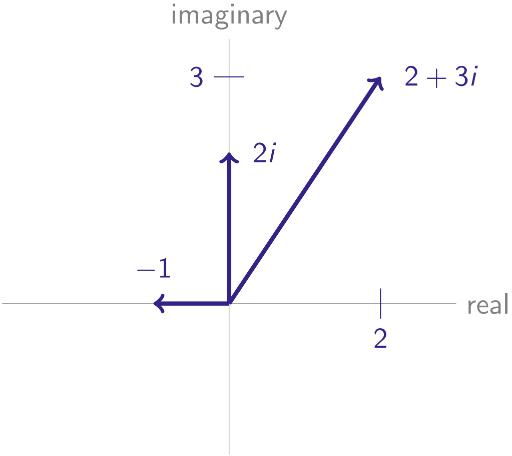
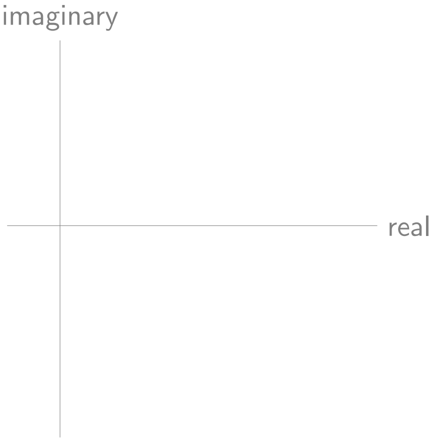


Complex Arithmetic

Addition happens component-wise, just like with vectors or polynomials.



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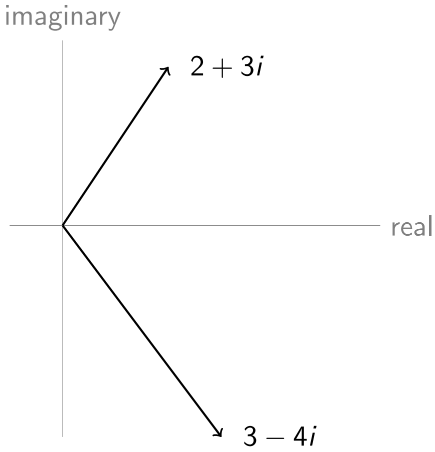
$(2 + 3i) + (3 - 4i) =$



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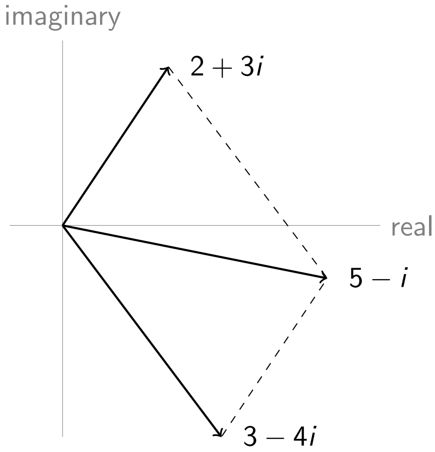
$(2 + 3i) + (3 - 4i) = 5 - i$



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A: $(-4+3i)+(1-i)$

B: $i(2+3i)$

C: $(i+1)(i-1)$

D: $(2i+3)(i+4)$

I: 0

II: -1

III: -2

IV: 2i+12

V: -3+2i

VI: 3+2i

VII: 10+11i

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Complex Conjugate

The **complex conjugate** of $(x + yi)$ is:

$$\overline{x + yi} = x - yi$$

the reflection of the vector over the real (x) axis.

Complex Arithmetic

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- $\overline{z}$
- $|z|$
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- $z - \overline{z}$
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- $z\overline{z} - |z|^2$
- $\overline{zw} - (\overline{z})(\overline{w})$

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$$\frac{z}{w} =$$

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Compute:

- $\frac{2+3i}{3+4i}$
- $\frac{1+3i}{1-3i}$
- $\frac{2}{1+i}$
- $\frac{5}{i}$

Complex Arithmetic

$$\frac{z}{w} = \frac{z\overline{w}}{|w|^2}$$

Compute:

- $\frac{2+3i}{3+4i} = \frac{18}{25} + \frac{1}{25}i$
- $\frac{1+3i}{1-3i}$
- $\frac{2}{1+i}$
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Gaussian Elimination

Give a parametric equation for all solutions to the homogeneous system:

$$\begin{array}{rcrcrcl} ix_1 & + & x_2 & + & 2x_3 & = & 0 \\ & & ix_2 & + & 3x_3 & = & 0 \\ 2ix_1 & + & (2-i)x_2 & + & x_3 & = & 0 \end{array}$$

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Find the inverse of the matrix $\begin{bmatrix} i & 1 \\ 2 & 3i \end{bmatrix}$ $\quad \begin{bmatrix} \frac{-3}{5}i & \frac{1}{5} \\ \frac{2}{5} & -\frac{1}{5}i \end{bmatrix}$

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$\frac{d}{dx}[e^{ix}] = \frac{d}{dx}[\cos x + i \sin x]$
$= -\sin x + i \cos x = i^2 \sin x + i \cos x = i(\cos x + i \sin x) = ie^{ix}$

Does that even make sense?

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$\quad = \cos x \cos y - \sin x \sin y + i[\sin x \cos y + \cos x \sin y]$
$\quad = (\cos x + i \sin y)(\cos y + i \sin x) = e^{ix} e^{iy}$

Does that even make sense?

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$e^{x+y} = e^x e^y$;
$e^{ix+iy} = e^i(x + y) = \cos(x + y) + i \sin(x + y)$
$= \cos x \cos y - \sin x \sin y + i[\sin x \cos y + \cos x \sin y]$
$= (\cos x + i \sin y)(\cos y + i \sin x) = e^{ix} e^{iy}$

Simplify:
e^{2+i}
$e^{2+\frac{\pi}{4}i}$
$e^{\pi i} + 1$

Complex exponentiation: $e^{ix} = \cos x + i \sin x$

True or False?

(1) $e^i = \cos 1 + i \sin 1$

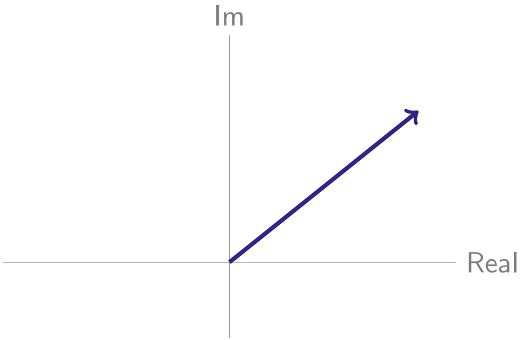
(2) $e^x = \cos x$

(3) $e^{ix} = e^{i(x+2\pi)}$

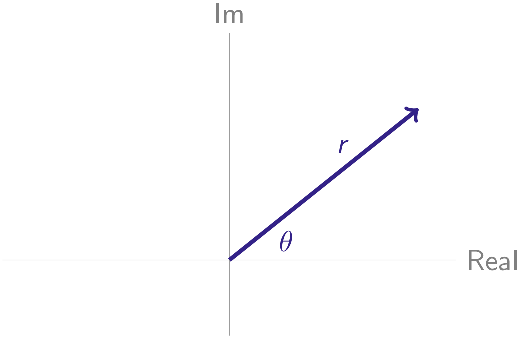
(4) $e^{ix} = -e^{i(x+\pi)}$

(5) $e^{ix} + e^{-ix}$ is a real number

Coordinates Revisited



Coordinates Revisited



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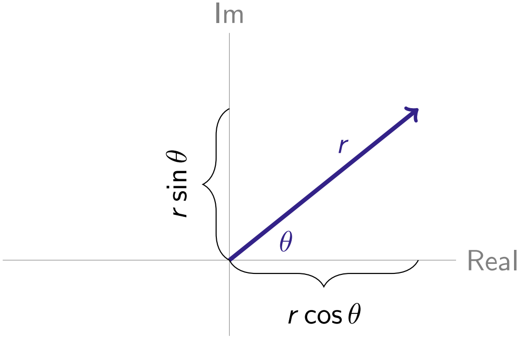

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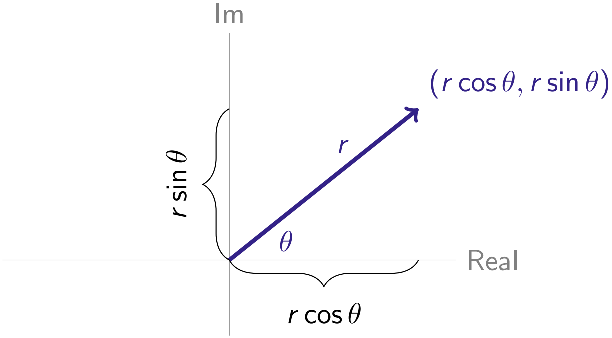

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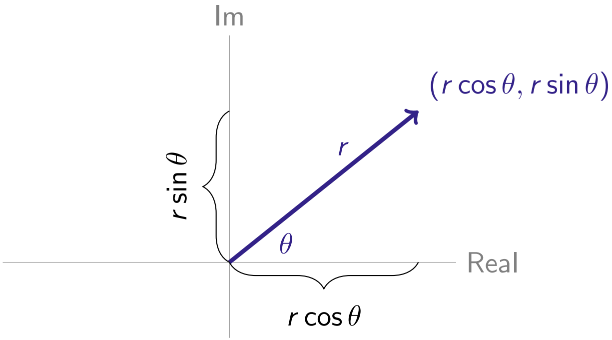
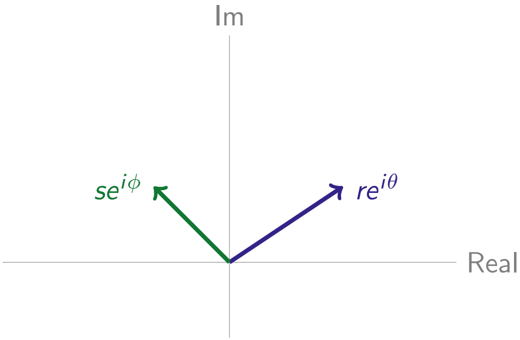
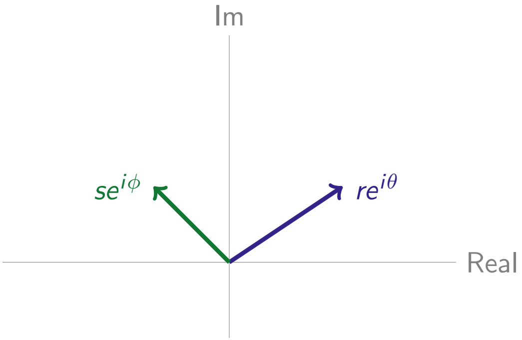


Complex number : $r(\cos\theta + i\sin\theta) = re^{i\theta}$

Coordinates Revisited

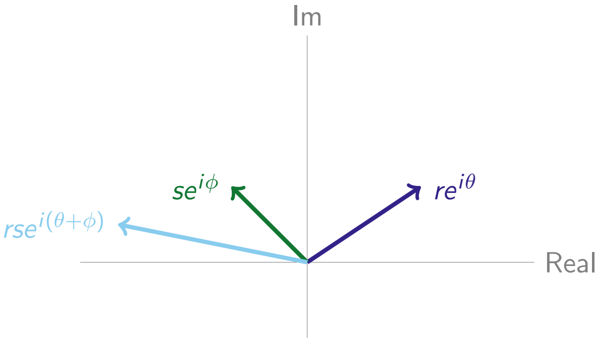


Coordinates Revisited



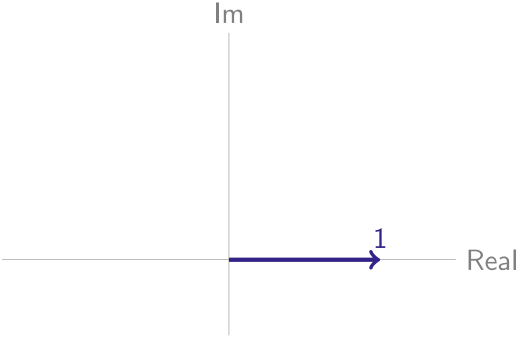
$$re^{i\theta} \cdot se^{i\phi} = (rs)e^{i(\theta+\phi)}$$

Coordinates Revisited

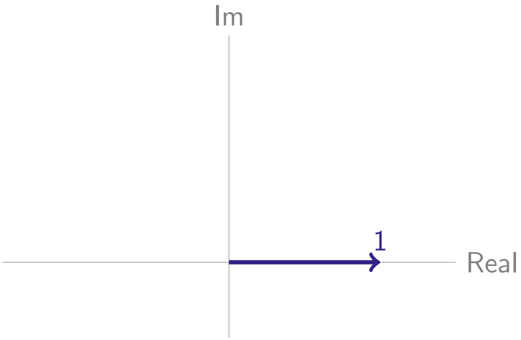


$$re^{i\theta} \cdot se^{i\phi} = (rs)e^{i(\theta+\phi)}$$

Roots of Unity

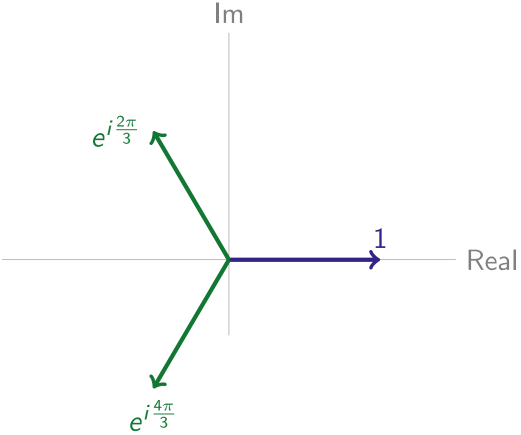


Roots of Unity



$$\left(re^{i\theta}\right)^3 = 1$$

Roots of Unity



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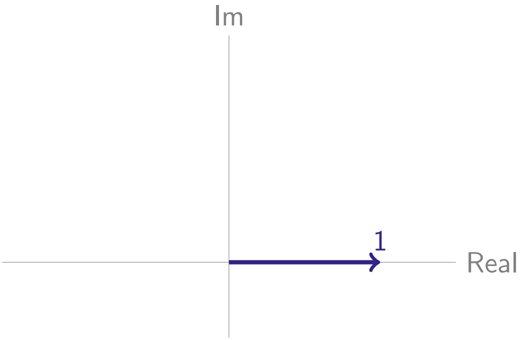
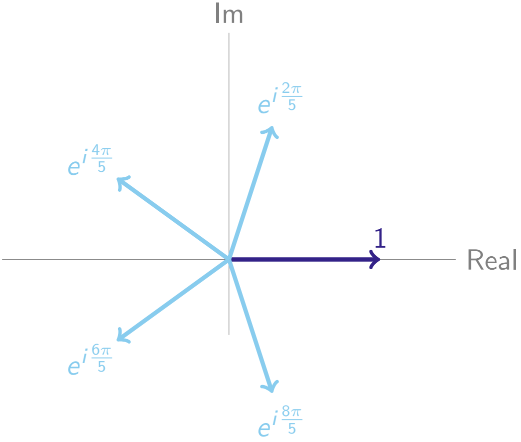
Roots of Unity



$$\left(re^{i\theta}\right)^5 = 1$$

Roots of Unity



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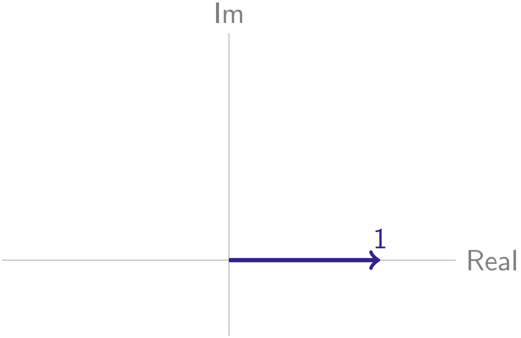
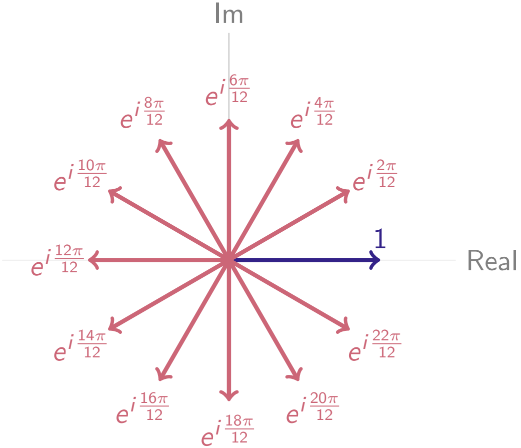
Roots of Unity



$$\left(re^{i\theta}\right)^{12} = 1$$

Roots of Unity



$$\left(re^{i\theta}\right)^{12} = 1$$

Roots

Find all complex numbers z such that $z^3 = 8$.

Find all complex numbers z such that $z^3 = 27e^{\frac{i\pi}{2}}$.

Find all complex numbers z such that $z^4 = 81e^{2i}$.