Outline

Today: Introduction; vectors

Course Notes: 2.1-2.2

Course Outline: Week 1

Goals: Intro course, explain expectations. Possibly talk about vectors: algebraic and geometric properties.

Next Time: Vectors; Geometric Aspects of Vectors

Course Notes: 2.2-2.3

Welcome to Math 152: Linear Systems!

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- Linear equations: degree-1 polynomials.

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- Linear equations: degree-1 polynomials.
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- Linear systems: many equations, many unknowns
- Computers are our friends

Online Resources

General Course Website:
http://www.math.ubc.ca/~solymosi/2016_152/m152_common_solymosi.html
All the important info is here. Grading, course notes, schedule, resources.

Section Website:
http://www.math.ubc.ca/~elyse/152.html
Very sparse.

Connect:
http://connect.ubc.ca
WebWork; computer labs

Vectors

What is a Vector?

Vectors are used to describe quantities with a magnitude and a direction.

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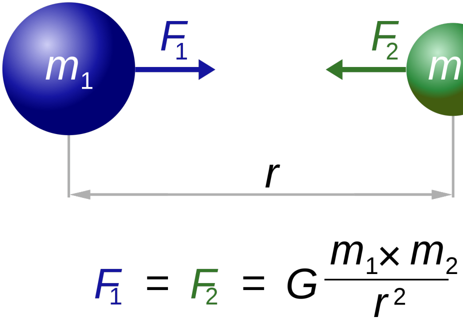

Dennis Nilsson, CC, unedited, `https://en.wikipedia.org/wiki/Kepler_orbit`

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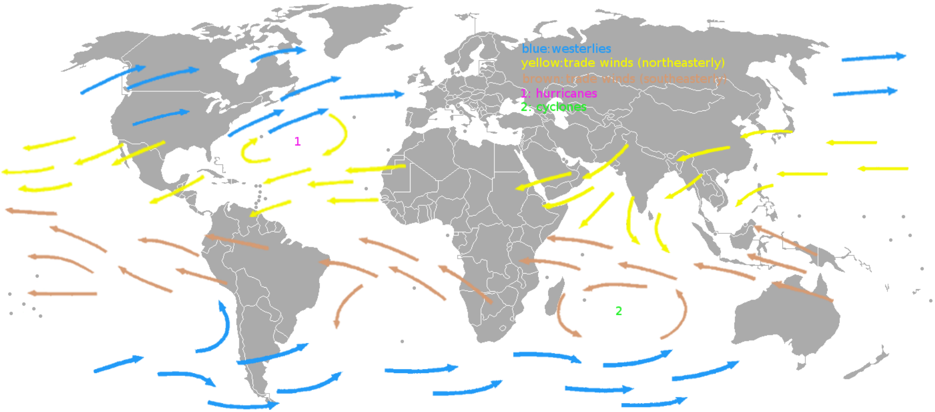

KVDP, public domain, unedited, `https://commons.wikimedia.org/wiki/File:Map_prevailing_winds_on_earth.png`

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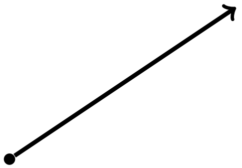

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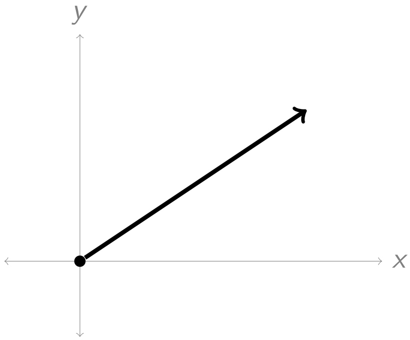

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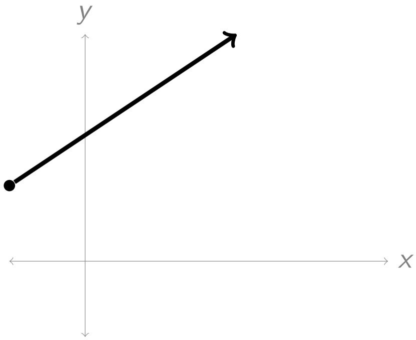

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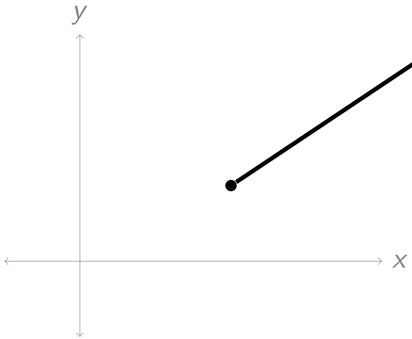

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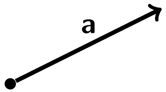
Vector Operations: Multiply by a Number

Scalar Multiplication

Multiplying a vector $\mathbf{a}$ by a scalar s results in a vector with length $|s|$ times the length of $\mathbf{a}$. The new vector $s\mathbf{a}$ points in the same direction if s is positive, and in the opposite direction if s is negative.



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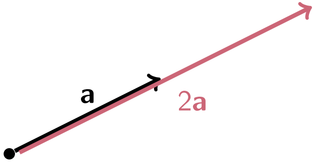

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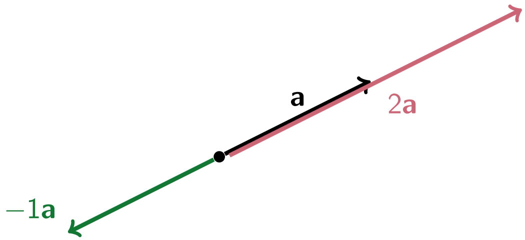

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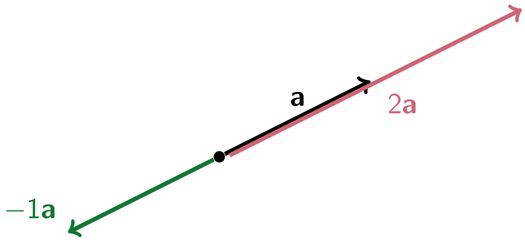
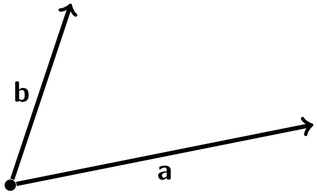


If the length of $\mathbf{a}$ is 1 unit, then the length of $2\mathbf{a}$ is 2. What is the length of $-1\mathbf{a}$: is it 1, or -1?

Vector Operations: Adding Vectors

Vector Addition

To add vectors **a** and **b** , we can slide the tail of **a** to sit at the head of **b** , and take $\mathbf{a} + \mathbf{b}$ to be the vector with tail where the tail of **b** is, and head where the head of **a** is.
This is equivalent to making a parallelogram out of **a** and **b** (with the same tail) and taking the diagonal (again with the same tail) to be the vector $\mathbf{a} + \mathbf{b}$.



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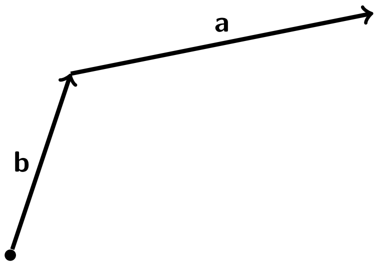

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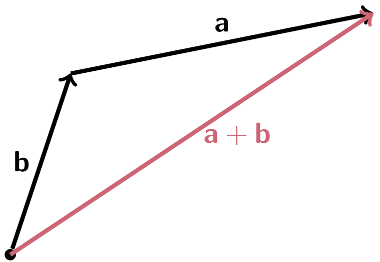

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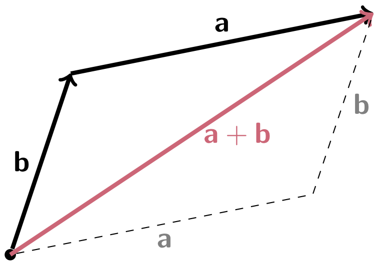

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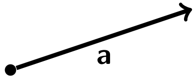

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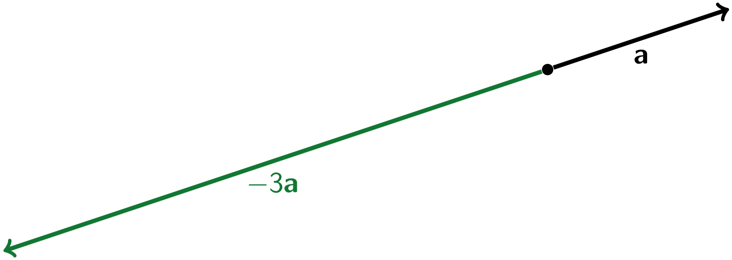

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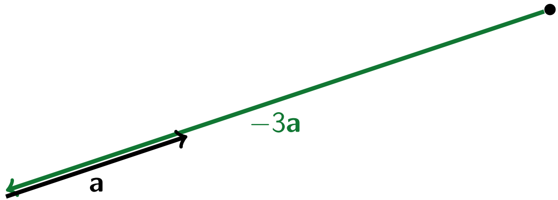

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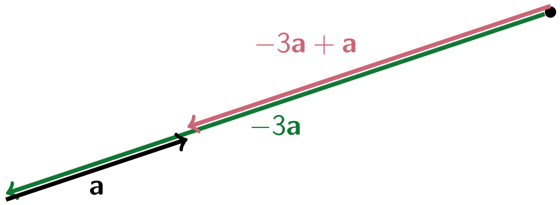

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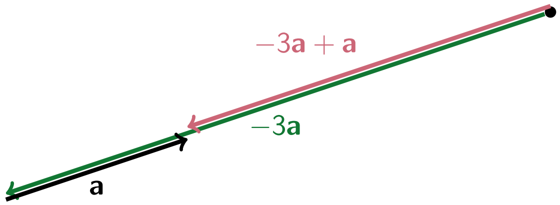

As we might expect, $\mathbf{a} - 3\mathbf{a} = -2\mathbf{a}$.

Limits of Sketching

Suppose a ship is sailing in the ocean. The current is pushing the ship at 5 knots per hour due east, while the wind is pushing this ship 3 knots per hour northwest. Rowers onboard are providing a force equal to 2 knots per hour east-southeast. What direction is the ship moving, and how fast?

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Time for coordinates.

See also: `https://en.wikipedia.org/wiki/Wind_triangle`

Coordinates and Vectors

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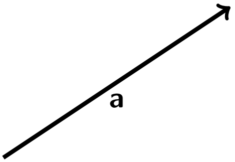

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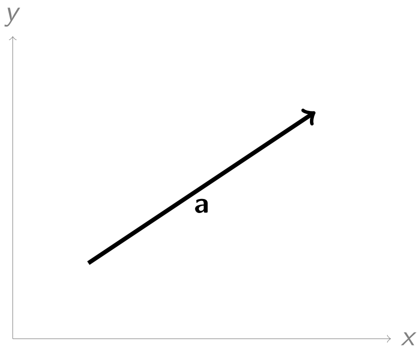

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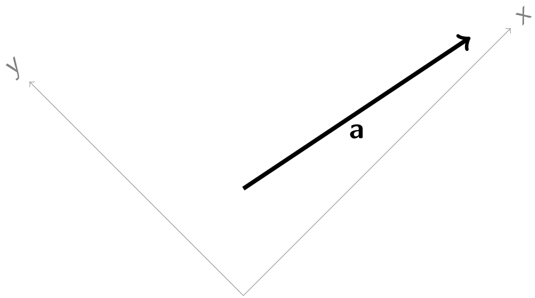

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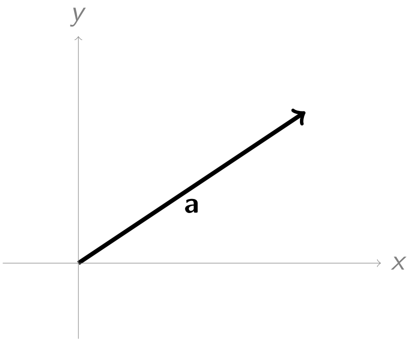

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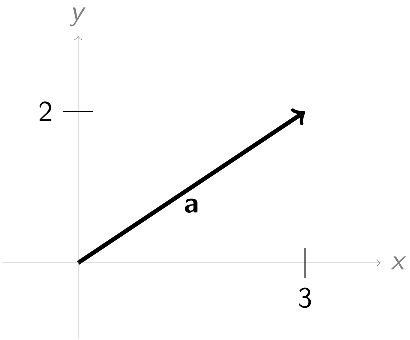

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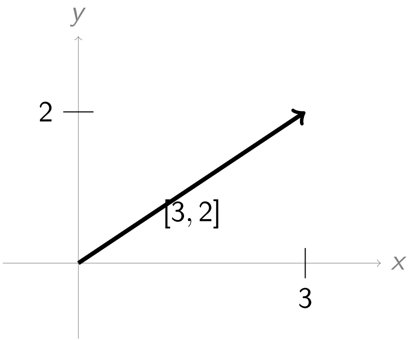

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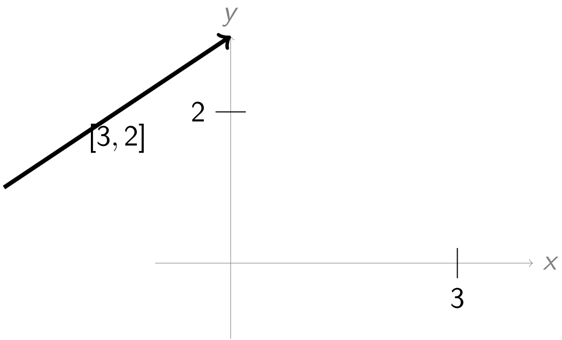

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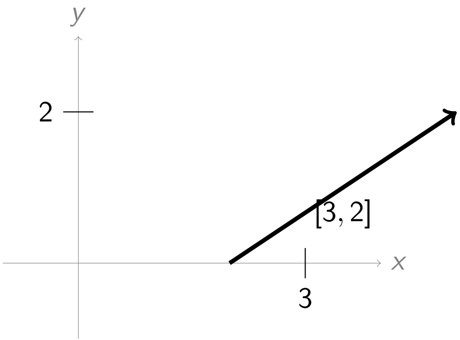

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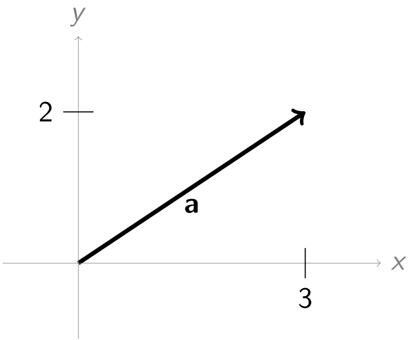

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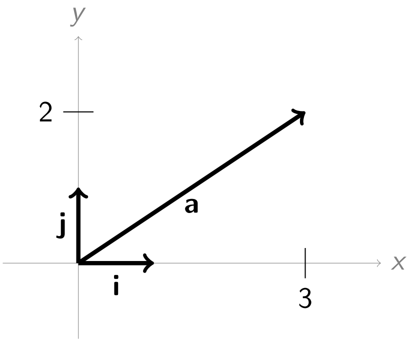

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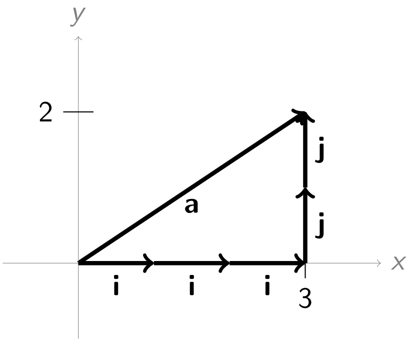

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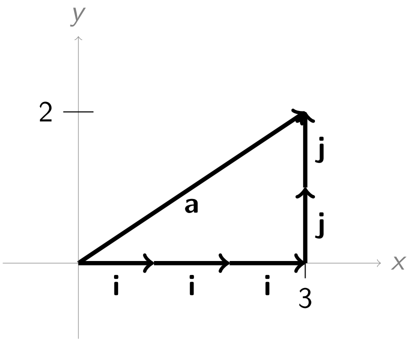
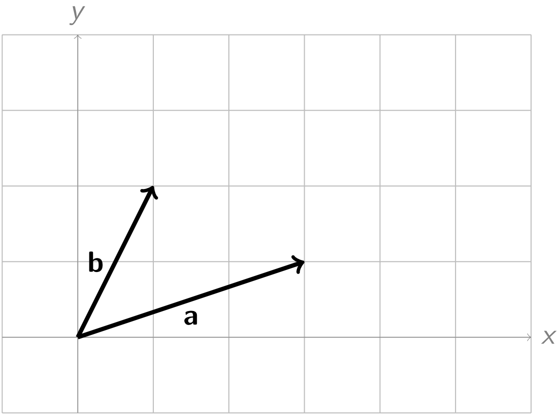


$3\mathbf{i} + 2\mathbf{j} = \mathbf{a}$

i and **j** are *unit vectors*, and we can write any vector in $\mathbb{R}^2$ as a linear combination of them.

Vector Operations on Coordinates



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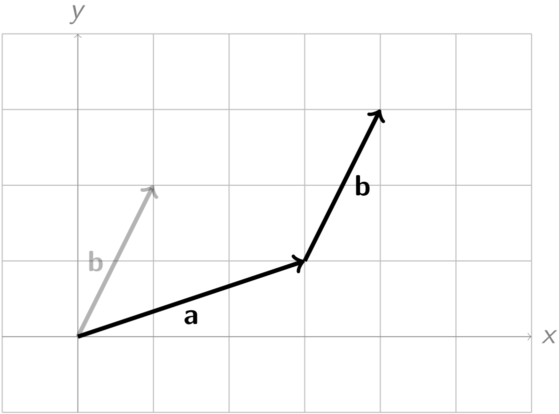

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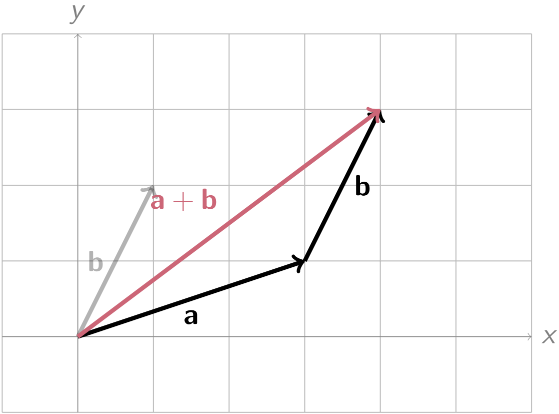

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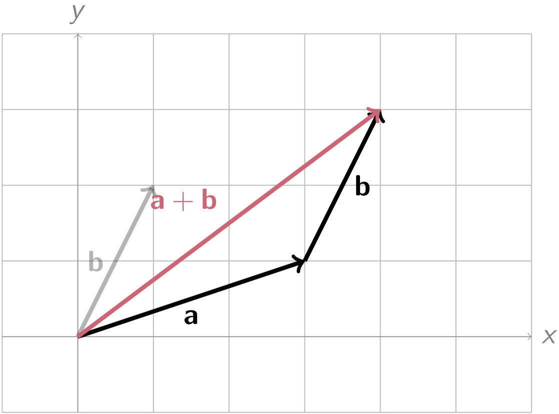

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

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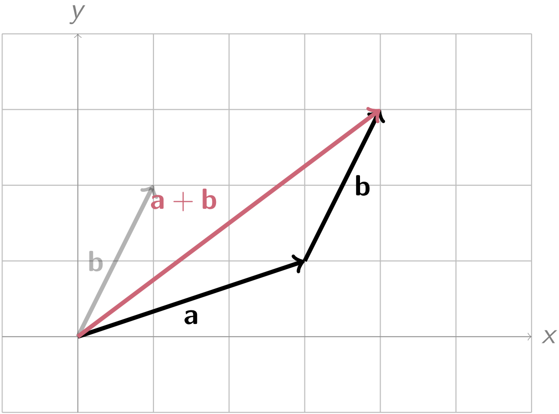

$$\begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 7 \end{bmatrix}$$

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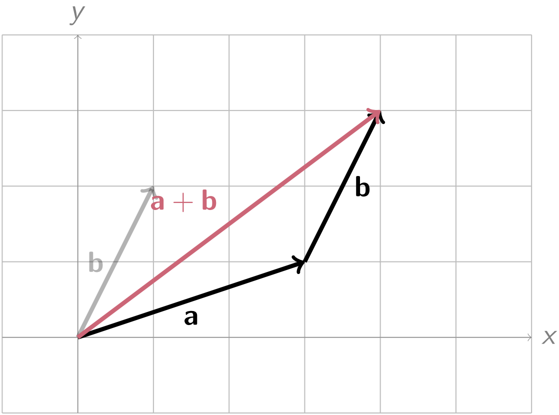

$$\begin{bmatrix} 3 \\ 1 \\ 7 \\ 10 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 7 \\ 30 \end{bmatrix}$$

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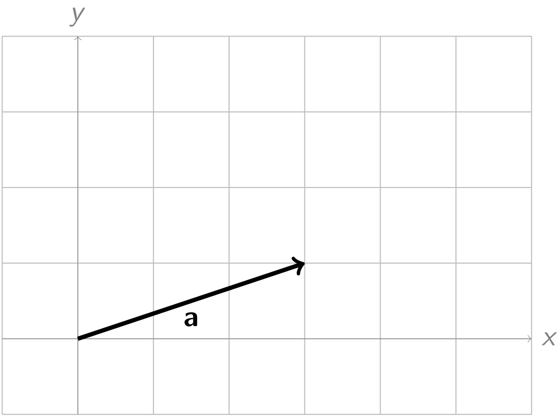

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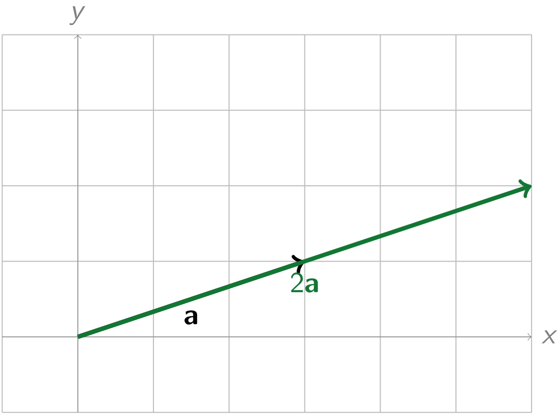

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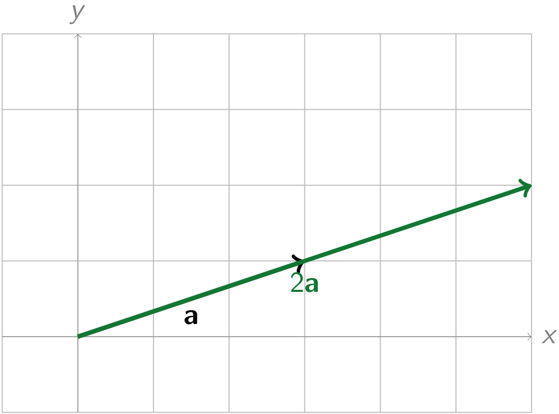

$$2\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}6\\2\end{bmatrix}$$

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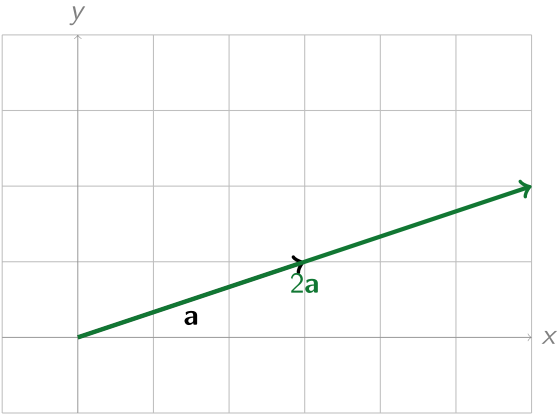

$$\frac{1}{3}\begin{bmatrix}3\\1\\6\\9\end{bmatrix} = \begin{bmatrix}1\\1/3\\2\\3\end{bmatrix}$$

Properties of Vector Addition and Scalar Multiplication

(Notes: 2.2.3)

Let $\mathbf{0}$ be the zero vector: this is the vector whose components are all zero. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be vectors, and let s and t be scalars. The following facts about vector addition, and multiplication of vectors by scalars, are true:

1. $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$
2. $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$
3. $\mathbf{a}+\mathbf{0}=\mathbf{a}$
4. $\mathbf{a}+(-\mathbf{a})=\mathbf{0}$
5. $s(\mathbf{a}+\mathbf{b})=s\mathbf{a}+s\mathbf{b}$
6. $(s+t)\mathbf{a}=s\mathbf{a}+t\mathbf{a}$
7. $(st)\mathbf{a}=s(t\mathbf{a})$
8. $1\mathbf{a}=\mathbf{a}$

MatLab interlude

Vectors versus Coordinates

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Example: Let $\mathbf{a}$ be a fixed, nonzero vector. Describe and sketch the sets of points in two dimensions:

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Example: Let $\mathbf{a}$ and $\mathbf{b}$ be fixed, nonzero vectors. Describe and sketch the sets of points in two dimensions:

$$\{s\mathbf{a} + t\mathbf{b} : s, t \in \mathbb{R}\}$$

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See `http://thejuniverse.org/PUBLIC/LinearAlgebra/LOLA/spans/two.html`

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$$\{s\mathbf{a} + t\mathbf{b} : s, t \in \mathbb{R}\}$$

Vectors and Coordinates

Let **a** and **b** be fixed, nonzero vectors.

- Give an expression for the midpoint of the line segment halfway between **a** and **b** .

- Give an expression for the point that is one-third of the way along the line segment between **a** and **b** .

- What is the geometric interpretation of the following set of points:

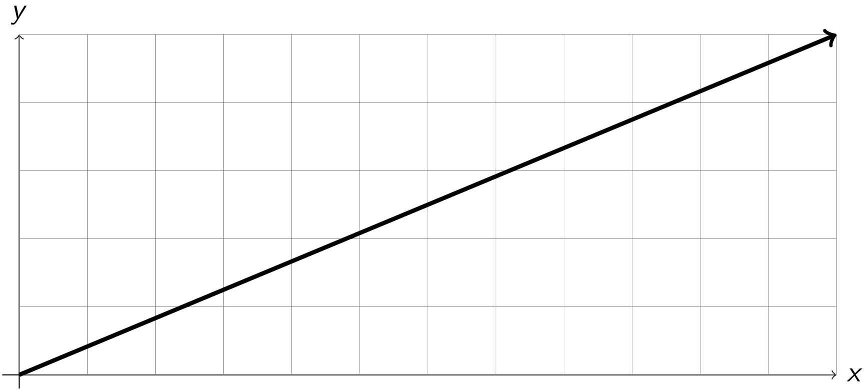
$$\{s\mathbf{a} + (1-s)\mathbf{b} : 0 \leq s \leq 1\}$$

- What is the geometric interpretation of the following set of points:

$$\{(1-s)\mathbf{a} + s\mathbf{b} : 0 \leq s \leq 1\}$$

Geometric Aspects of Vectors

How long is the vector $\begin{bmatrix} 12 \\ 5 \end{bmatrix}$?



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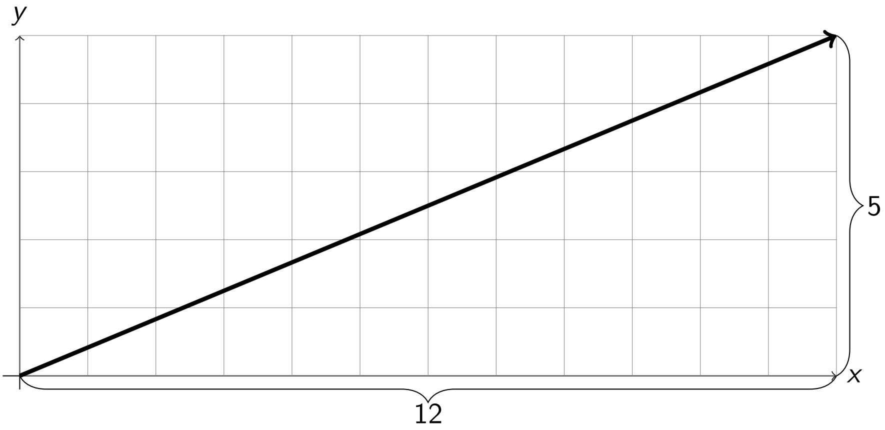

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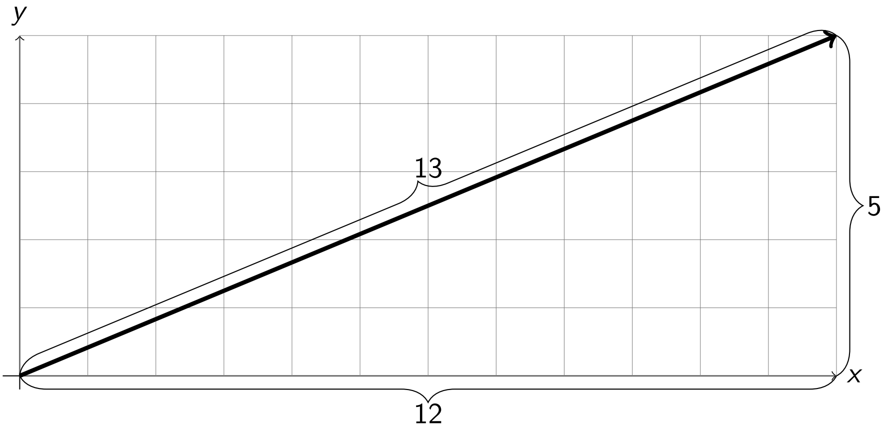

Geometric Aspects of Vectors

How long is the vector $\begin{bmatrix} 12 \\ 5 \end{bmatrix}$?

The length of $\begin{bmatrix} 12 \\ 5 \end{bmatrix}$ is denoted $\left\| \begin{bmatrix} 12 \\ 5 \end{bmatrix} \right\|$, and calculated

$$\left\| \begin{bmatrix} 12 \\ 5 \end{bmatrix} \right\| = \sqrt{12^2 + 5^2} = 13.$$

We also call this quantity the norm of the vector.

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The length of $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ is denoted $\|\mathbf{a}\|$, and calculated

$$\|\mathbf{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Quick Concept Test

Let $\mathbf{a}$ be a vector, and let s be a scalar. For each of the following expressions, decide whether it is a vector or a scalar.

A. $\|\mathbf{a}\|$

B. $s\mathbf{a}$

C. $s\|\mathbf{a}\|$

D. $\|s\mathbf{a}\|$

E. $s + \mathbf{a}$

F. $s + \|\mathbf{a}\|$

Unit Vectors

As we noted before, a unit vector is a vector of length one.

What is the unit vector in the direction of the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$?

Dot Product

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Given vectors $\mathbf{a} = [a_1, \dots, a_k]$ and $\mathbf{b} = [b_1, \dots, b_k]$, we define the dot product $\mathbf{a} \cdot \mathbf{b} := a_1 a_2 + \cdots + a_k a_2$. Note $\mathbf{a} \cdot \mathbf{b}$ is a number, not a vector.

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Example: $\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = -4 + 0 + 15 = 11$

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Note: $\mathbf{a} \cdot \mathbf{a} = \|\mathbf{a}\|^2$.

Properties of the Dot Product

Notes: p20

For nonzero vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, zero vector $\mathbf{0}$, and scalar s:

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3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
4. $s(\mathbf{a} \cdot \mathbf{b}) = (s\mathbf{a}) \cdot \mathbf{b}$
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5. $\mathbf{0} \cdot \mathbf{a} = 0$
6. $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta$, where θ is the angle between **a** and **b**
7. $\mathbf{a} \cdot \mathbf{b} = 0$ if and only if $\mathbf{a} = 0$, $\mathbf{b} = 0$, or **a** and **b** are perpendicular

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Example: are $\mathbf{a}$ and $\mathbf{b}$ perpendicular?

- $\mathbf{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

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$\mathbf{a} = \begin{bmatrix} 2, -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -1, 2 \end{bmatrix}$

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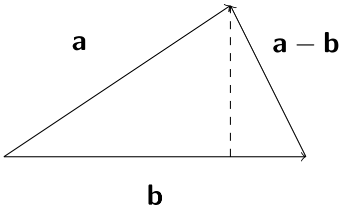


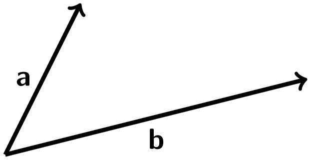
Angle between Two Vectors

Recall $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta$, where θ is the angle between **a** and **b** .

What is the angle between vectors $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$?

Projections

We apply a force to an object in the direction of **a** , but we're only concerned with the object's movement in the direction of vector **b** .



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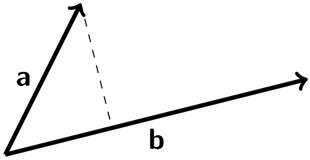

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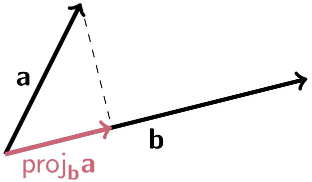

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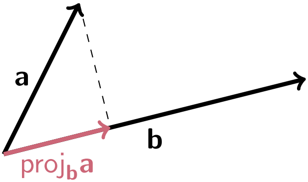


$$\text{proj}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|^2}\right)\mathbf{b}$$

A man pulls a truck up a hill for some reason. If we take level ground as our coordinate axis, the hill is in the direction of the vector $\begin{bmatrix} 10 \\ 2 \end{bmatrix}$, and the man applies force represented by the vector $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$. What vector represents the force acting on the truck in the direction it is moving?

Announcements

New IIC: a new common webpage will be up, possibly on Connect.

We are working on Connect issues.

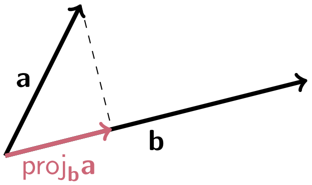
Assignemnts 0 and 1 should be up on webwork.

Midterm dates.

Projections

Recall:

$$proj_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{b}\|^2}\right)\mathbf{b}$$



A man pulls a truck up a hill for some reason. He pulls with a force of 1000 pounds, and pulls at an angle of 20 degrees to the hill. What force is exerted in the direction of the hill? That is, what is the magnitude of the component of the force that is in the direction of the truck's motion?

What is the projection of the vector $\begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$ onto the vector $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$?

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A. I solved this by computing $\left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|^2} \right) \mathbf{b}$

B. I solved this by drawing a picture

C. I solved this by noticing that the y component is precisely the component of the vector in the direction of $\mathbf{j}$

D. I solved this another way

What is the projection of the vector $\begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$ onto the vector $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$?

What is the projection of the vector $\begin{bmatrix} 8 \\ 2 \\ 5 \end{bmatrix}$ onto the vector $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$?

A: still $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ B: probably not $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ any more C: I'm not sure

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What is the projection of the vector $\begin{bmatrix} 8 \\ 2 \\ 5 \end{bmatrix}$ onto the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$?

What is the projection of the vector $\mathbf{a}$ onto itself?