Saturation Number of Ramsey-Minimal Families

Mike Ferrara [1] Jaehoon Kim [2] Elyse Yeager [2]

[1]University of Colorado-Denver

[2]University of Illinois at Urbana-Champaign

yeager2@illinois.edu

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Given a forbidden graph H, a graph G is H-**saturated** if H is not a subgraph of G, but for every $e \in \overline{G}$, H is a subgraph of $G + e$.

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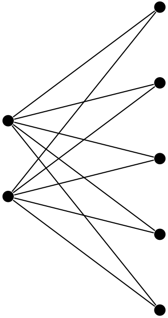

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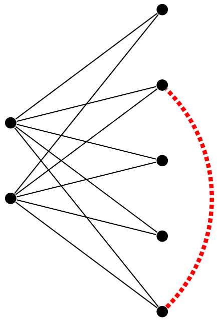

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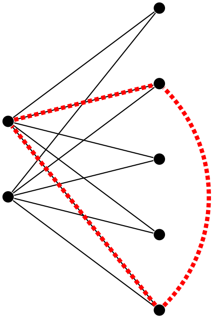

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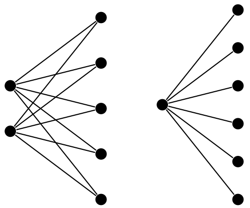

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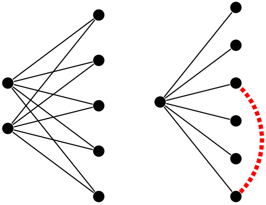

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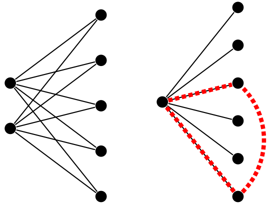

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Given a forbidden family of graphs $\mathcal{F}$, a graph G is $\mathcal{F}$-**saturated** if no member of $\mathcal{F}$ is a subgraph of G, but for every $e \in \overline{G}$, some member of $\mathcal{F}$ is a subgraph of $G + e$.

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Ramsey-Minimal Families

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Given "forbidden" graphs $H_1, \ldots, H_k$, and any graph G, we write $\mathbf{G} \rightarrow (\mathbf{H_1}, \ldots, \mathbf{H_k})$ if any k coloring of $E(G)$ contains a monochromatic copy of H_i in color i, for some i.

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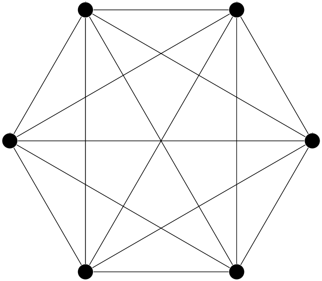

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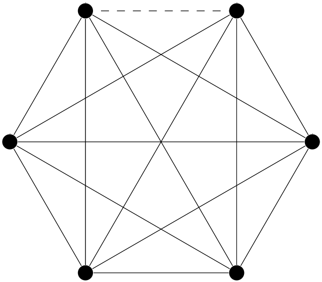

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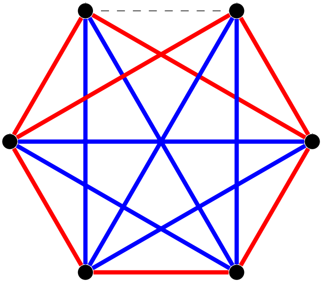

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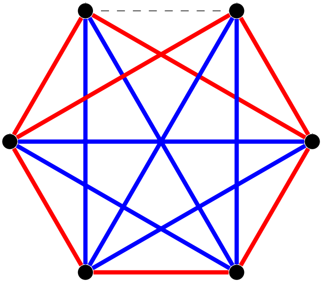

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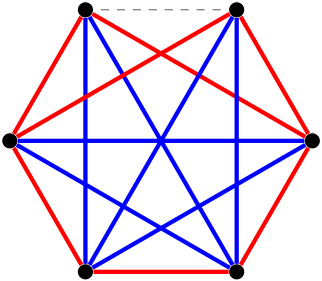

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Saturation of $\mathcal{R}_{min}(K_{k_1}, \ldots, K_{k_t})$

Example

Let $r := r(k_1, \ldots, k_t)$ be the Ramsey number of $(K_{k_1}, \ldots, K_{k_t})$. Then

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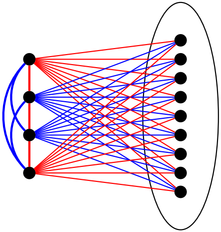

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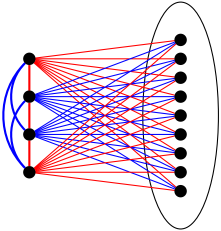

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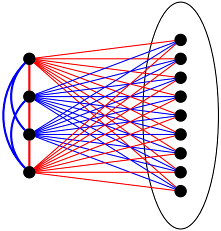

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Hanson-Toft Conjecture, 1987

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Chen, Ferrara, Gould, Magnant, Schmitt; 2011

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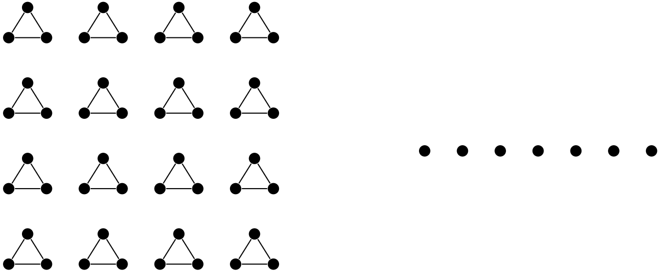

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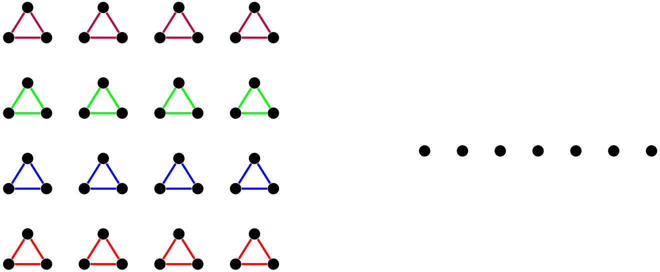

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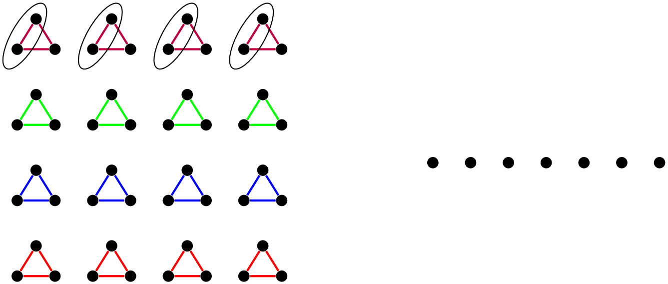

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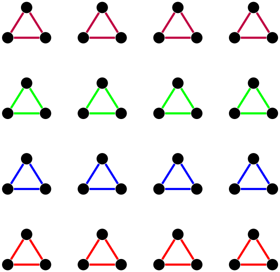

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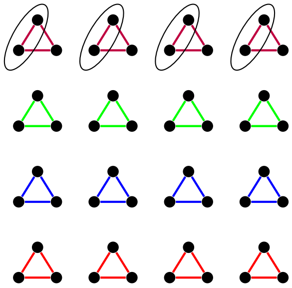

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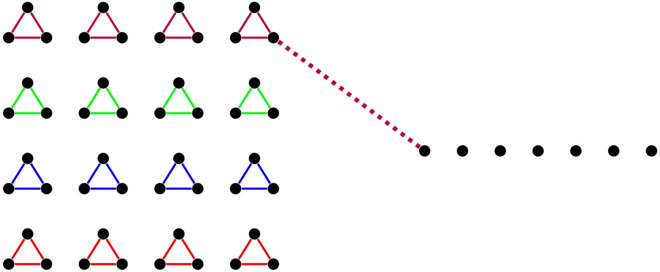

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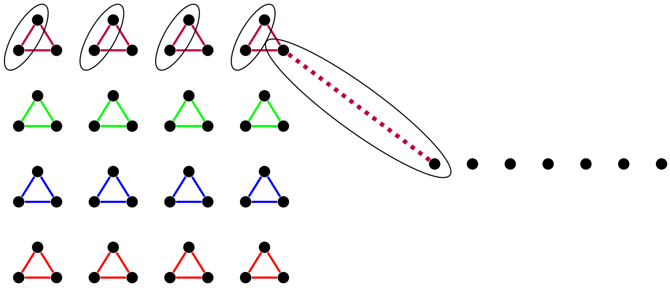

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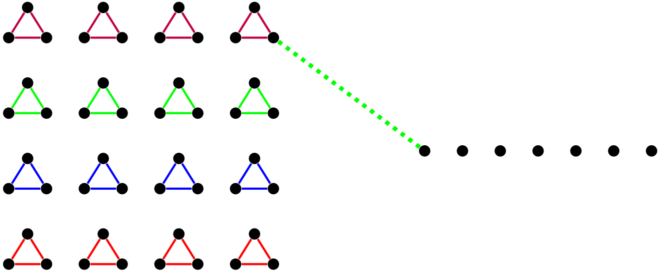

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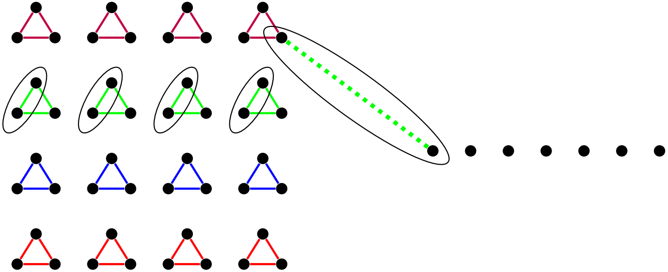

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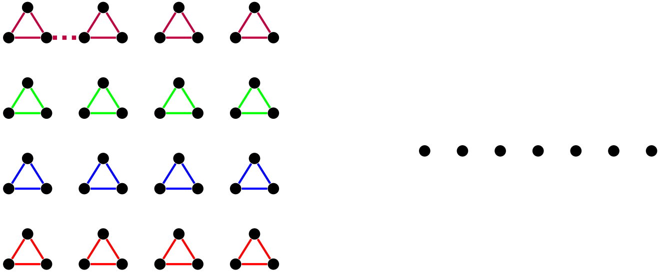

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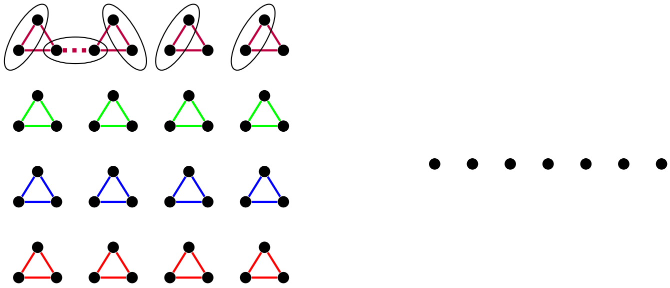

$(5K_2, 5K_2, 5K_2, 5K_2)$

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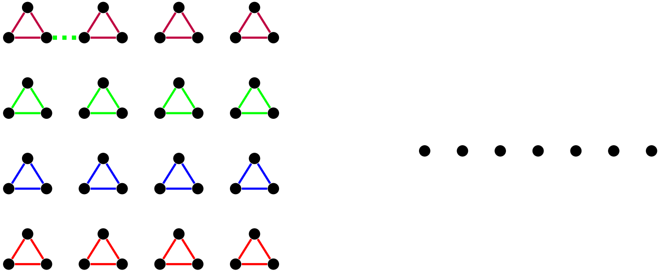

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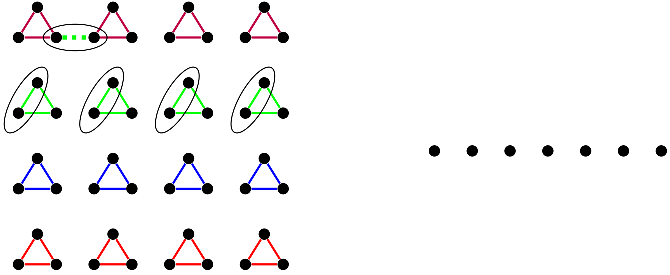

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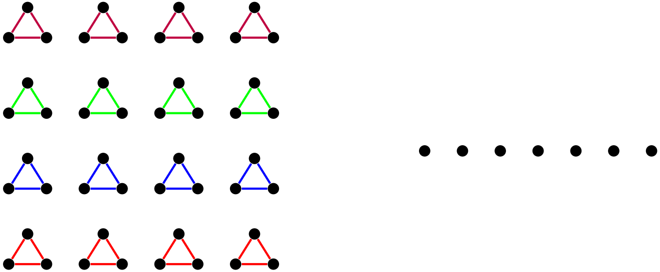

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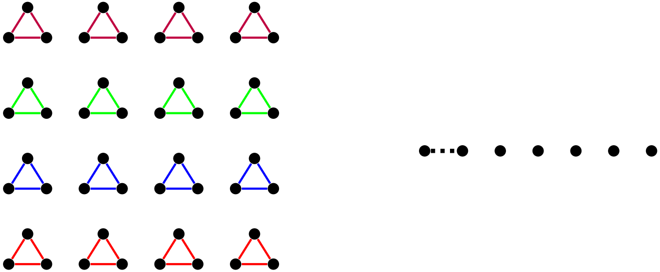

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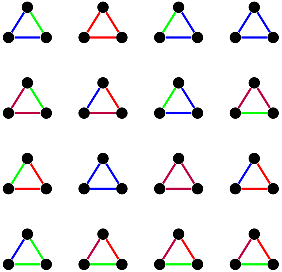

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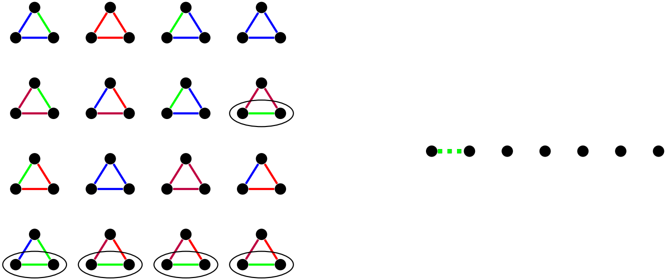

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Sat Number of Ramsey-Minimal Families of Matchings

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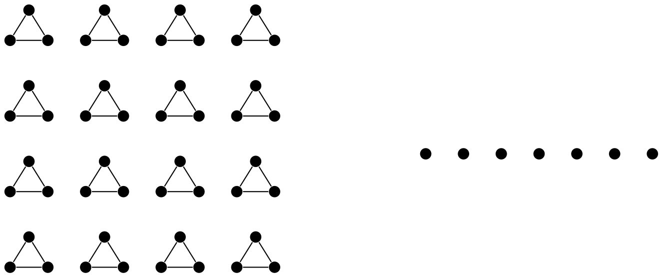

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$\text{sat}(n; \mathcal{R}_{min}(k_1 K_2 + \cdots + k_t K_2)) \leq 3(k_1 + \cdots + k_t - t)$
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Ferrara, Kim, Y.; 2014

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Construction is generally unique: vertex-disjoint triangles with isolates.

Useful Observation

Ferrara, Kim, Y.; 2014

Looking (cleverly) at color i allows us to use results from graph saturation of the forbidden subgraph H_i.

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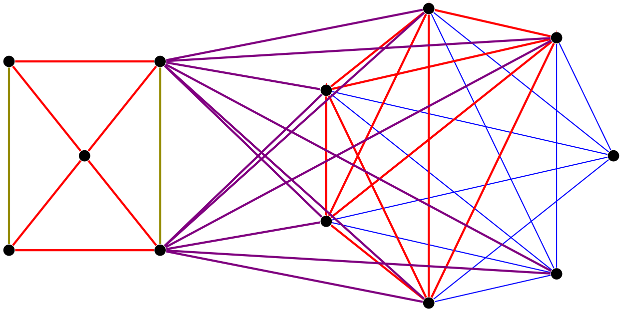

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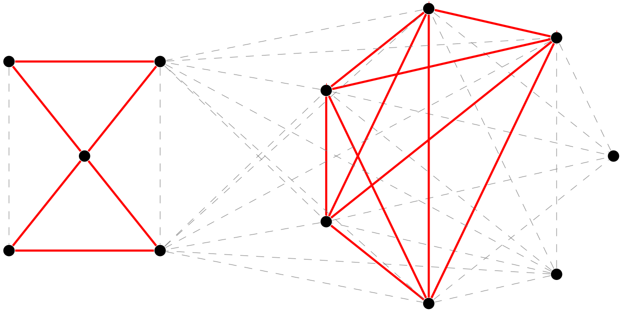

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Corollary

If G is $\mathcal{R}_{min}(H_1, \ldots, H_k)$ saturated, then $G = G_1 \cup \cdots \cup G_k$, where G_i is H_i saturated and all G_i share the same vertex set.

Thanks for Listening!

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