A Ramsey Version of Graph Saturation

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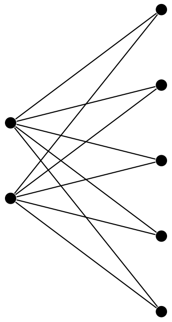

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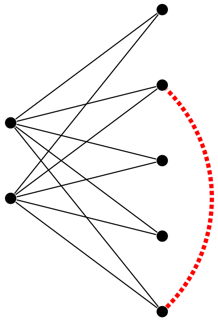

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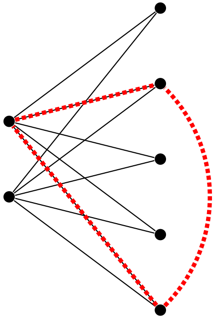

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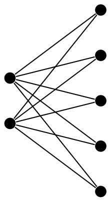

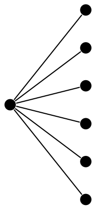

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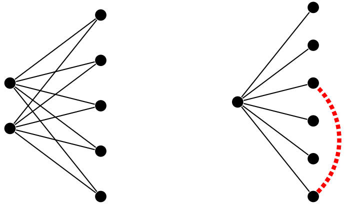

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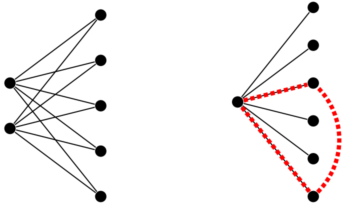

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Given a forbidden family of graphs $\mathcal{F}$, a graph G is $\mathcal{F}$-**saturated** if no member of $\mathcal{F}$ is a subgraph of G, but for every $e \in \overline{G}$, some member of $\mathcal{F}$ is a subgraph of $G + e$.
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Ramsey-Minimal Families

Definitions

Given "forbidden" graphs $H_1, \ldots, H_k$, and any graph G, we write $\mathbf{G} \rightarrow (\mathbf{H_1}, \ldots, \mathbf{H_k})$ if any k coloring of $E(G)$ contains a monochromatic copy of H_i in color i, for some i.

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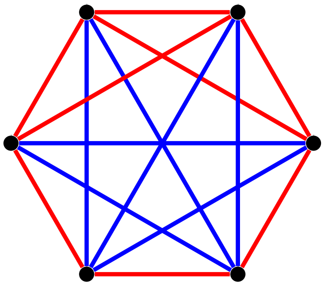

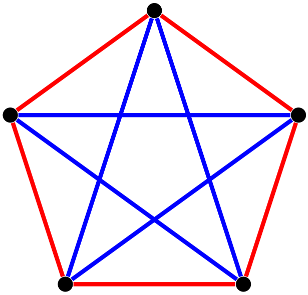

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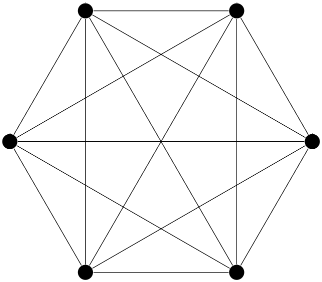

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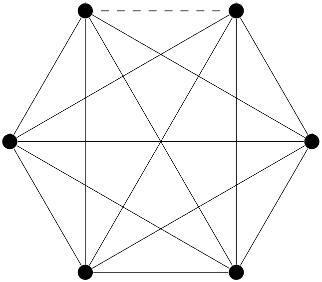

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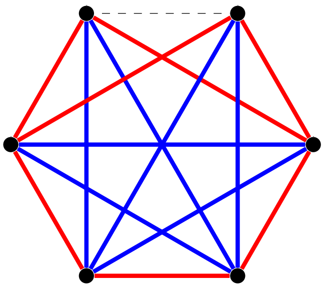

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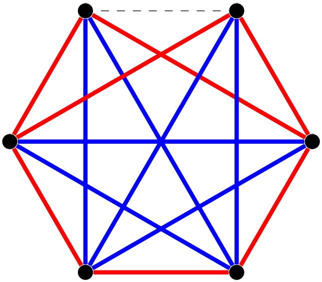

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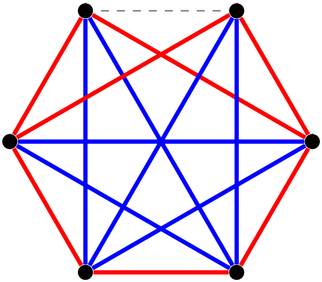

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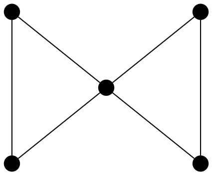

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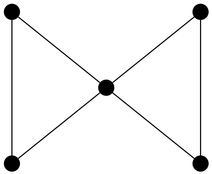

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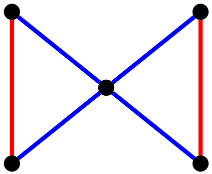

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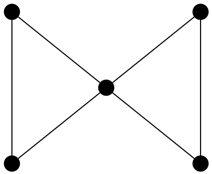

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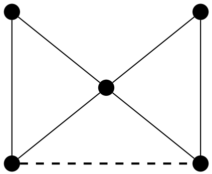

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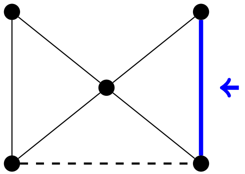

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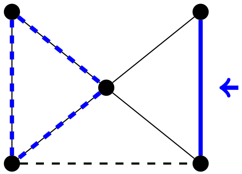

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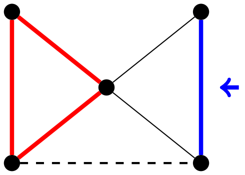

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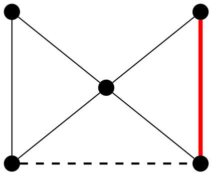

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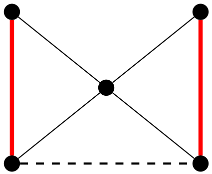

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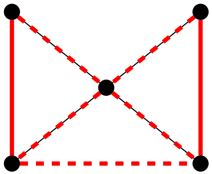

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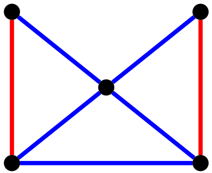

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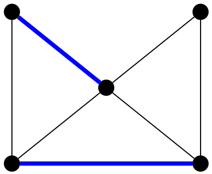

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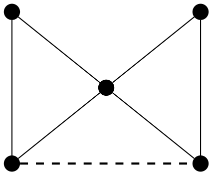

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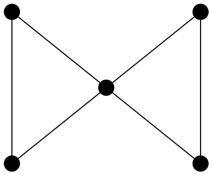

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Saturation of $\mathcal{R}_{min}(K_{k_1}, \ldots, K_{k_t})$

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$$K_{r-2} \vee \overline{K_s}$$

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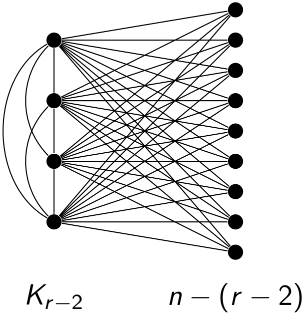

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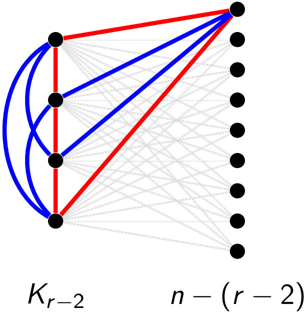

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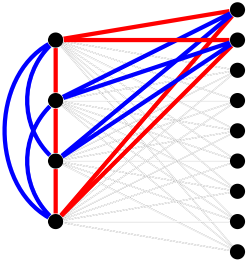

K_{r-2} $\quad$ $n-(r-2)$

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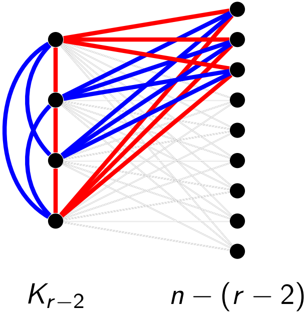

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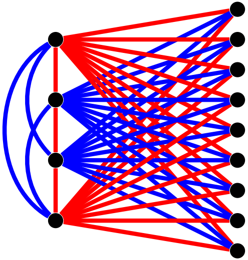

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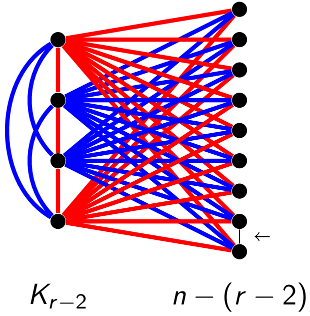

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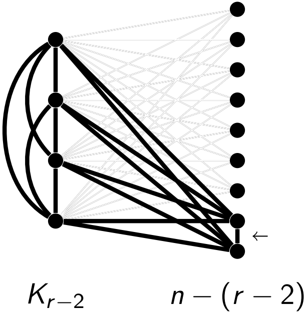

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Hanson-Toft Conjecture, 1987

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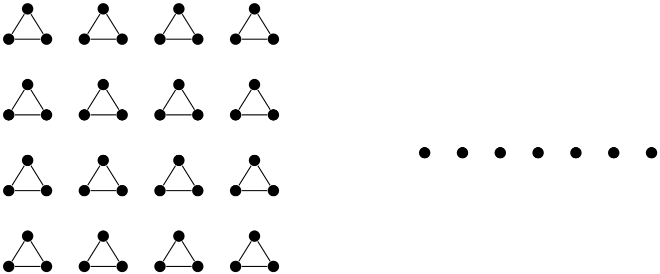

$(5K_2, 5K_2, 5K_2, 5K_2)$

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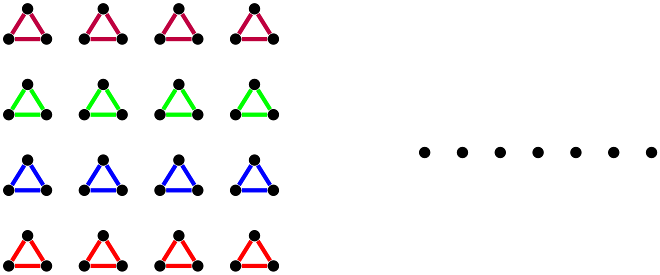

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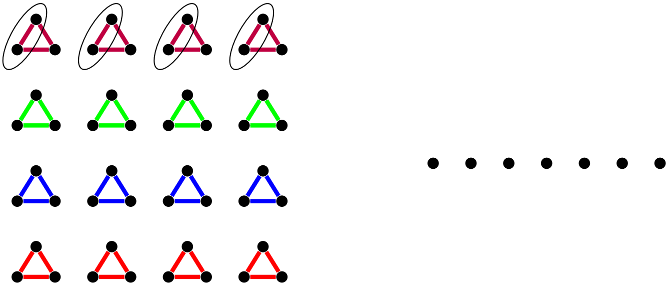

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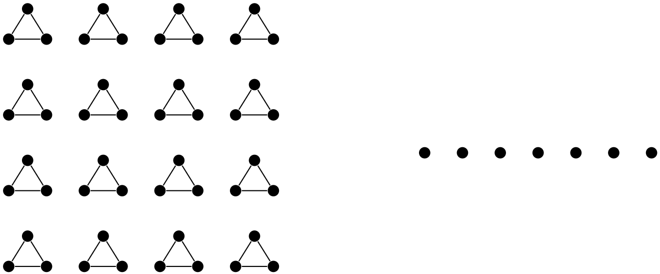

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Useful Observation: “Iterated Recoloring”

Ferrara, Kim, Y.; 2014

Looking (cleverly) at color i allows us to use results from graph saturation of the forbidden subgraph H_i.

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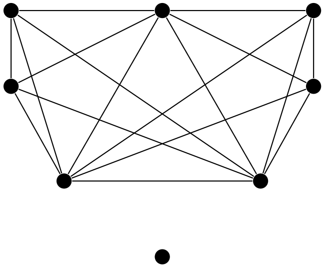

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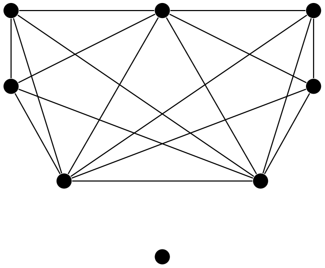

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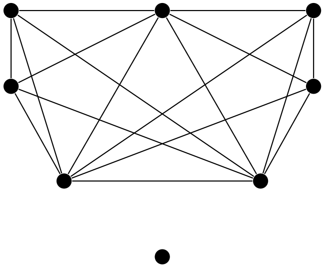

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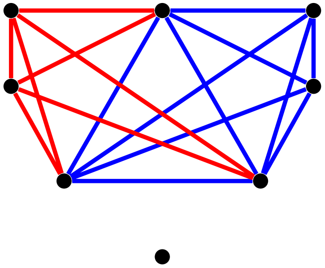

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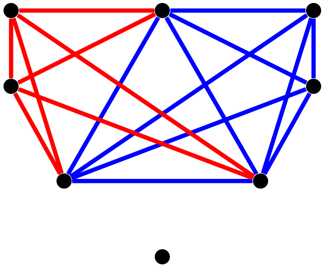

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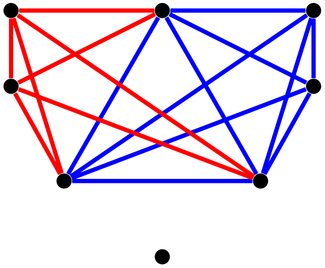

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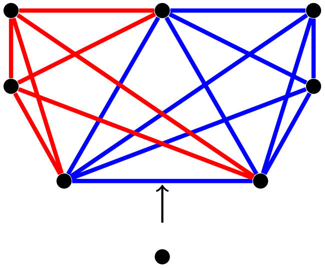

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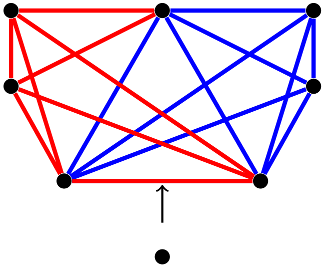

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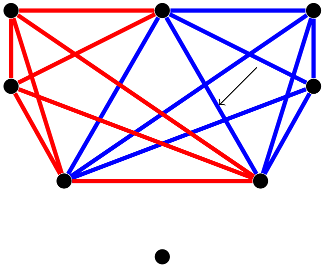

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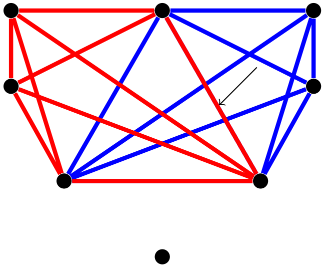

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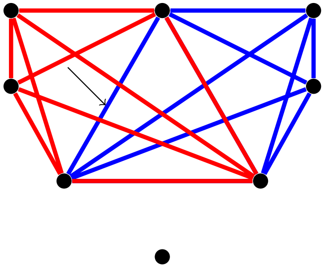

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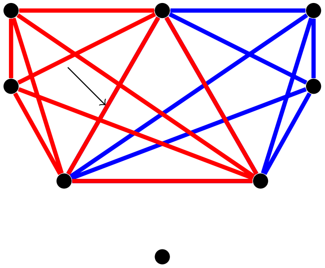

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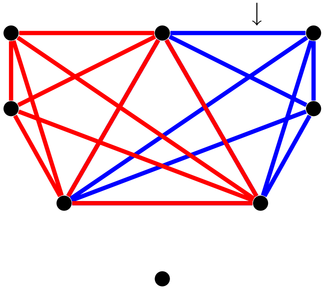

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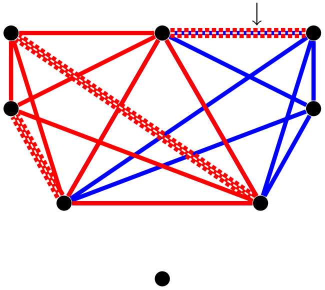

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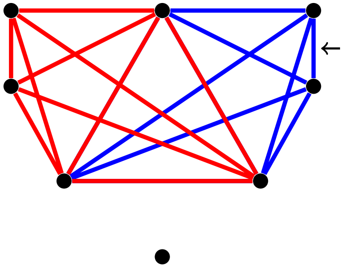

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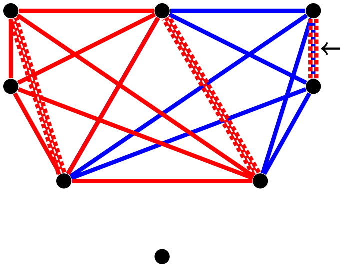

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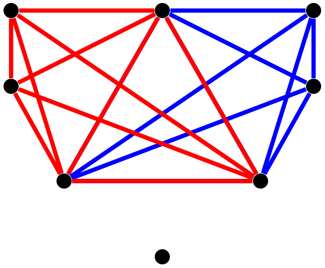

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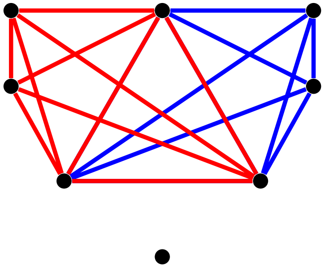

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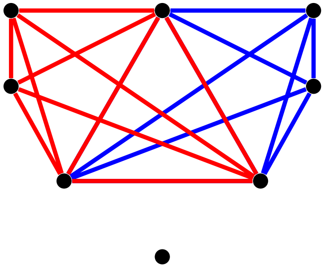

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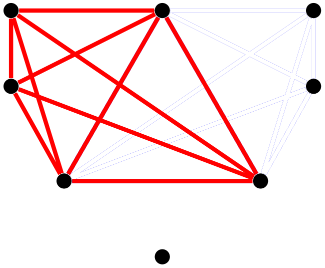

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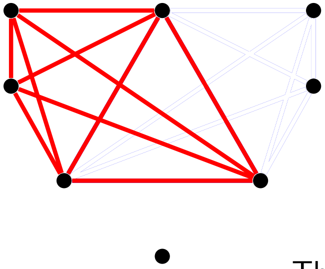

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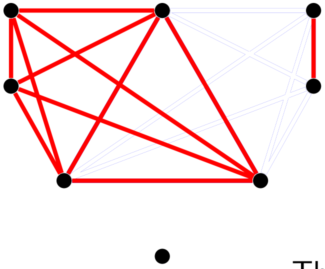

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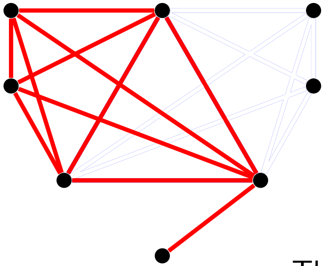

good coloring

↓

make red-heavy

↓

take red subgraph

This (uncolored) subgraph is $3K_2$-saturated.

Useful Observation: "Iterated Recoloring"

Ferrara, Kim, Y.; 2014

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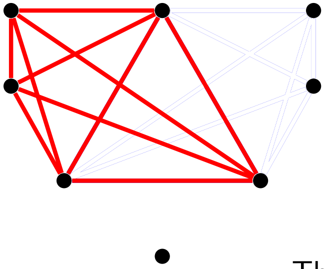

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Thanks for Listening!

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