Disjoint Cycles and Equitable Colourings in Graphs

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University of British Columbia

Corrádi-Hajnal Theorem

Corrádi-Hajnal, 1963

If G is a graph on n vertices with $n \geq 3k$ and $\delta(G) \geq 2k$, then G contains k disjoint cycles.

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- $k = 1$

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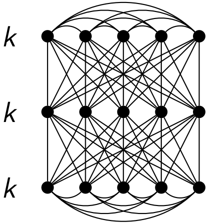

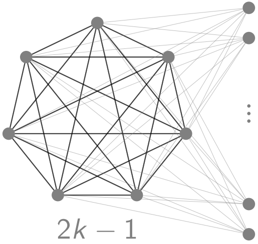

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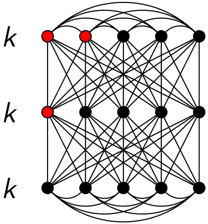

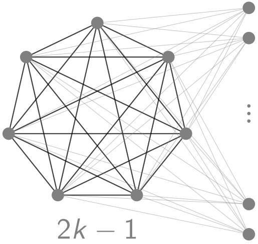

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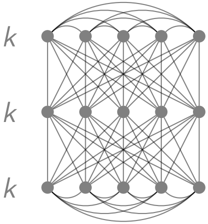

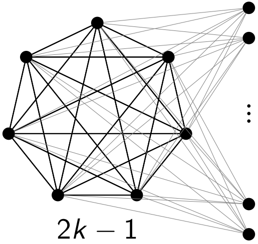

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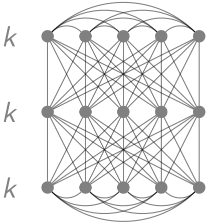

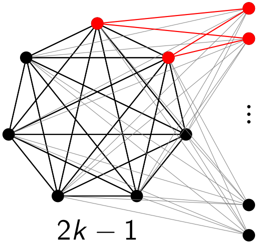

Section 1

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Observation: $k = 1$

If G is a graph where all but one vertex has degree at least 2, then G contains a cycle.

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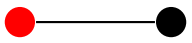

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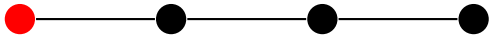

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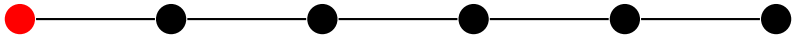

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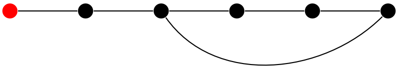
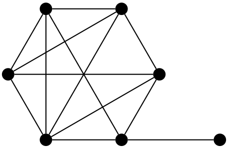


Dirac-Erdős, 1963



Let $V_{\geq c}$ be the number of vertices with degree at least c, etc.

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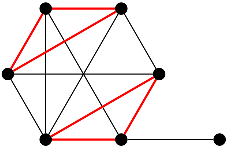
If $V_{\geq 2k} - V_{\leq 2k-2} \geq k^2 + 2k - 4$, $k \geq 3$, then G contains k disjoint cycles.



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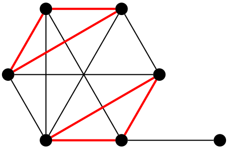

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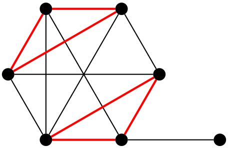

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For, $k \geq 2$, let G be a graph with $|G| \geq 3k$, and no more than t disjoint triangles.
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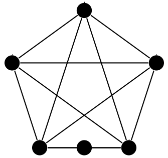

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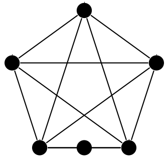

Open:

Is it necessary to consider triangles when $|G|$ is large?

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Implies Corrádi-Hajnal

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Proof (Enomoto)

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Proof (Enomoto)

- Edge-maximal counterexample
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- Minimize number of vertices in cycles
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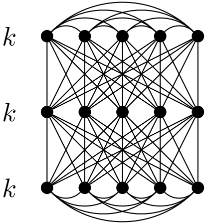
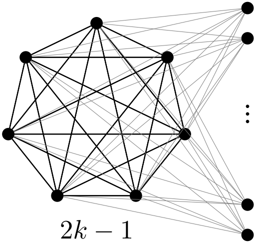
Sharpness:





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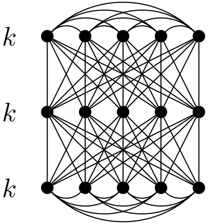

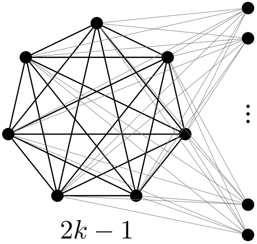

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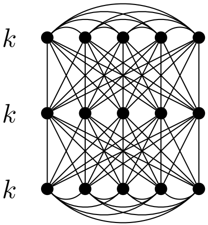

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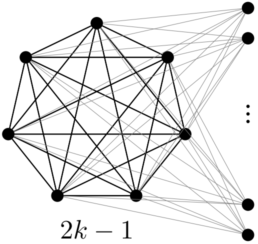


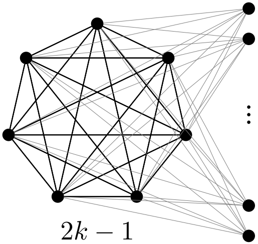
$\alpha(G)$ large

Kierstead-Kostochka-Y., 2017 (link)

Independence Number:

Observation:

$\alpha(G) \geq n - 2k + 1 \Rightarrow$ no k cycles



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For $k \geq 4$, if G is a graph on n vertices with $n \geq 3k + 1$ and $\sigma_2(G) \geq 4k - 3$, then G contains k disjoint cycles if and only if $\alpha(G) \leq n - 2k$.

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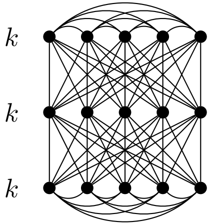

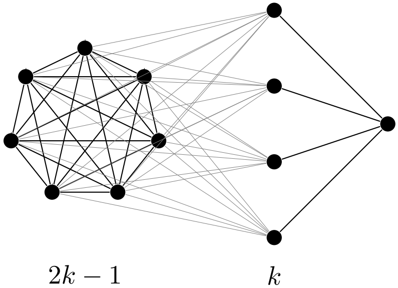

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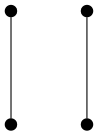

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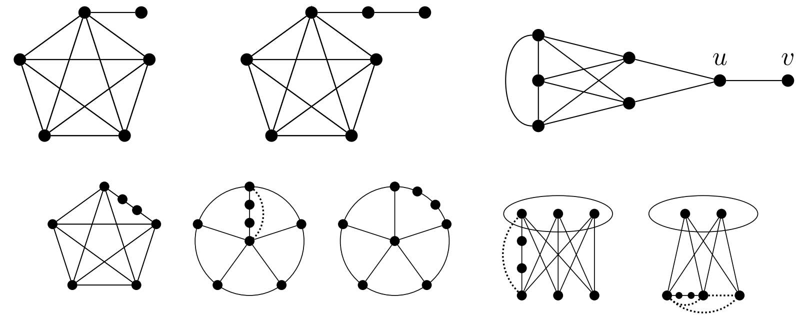

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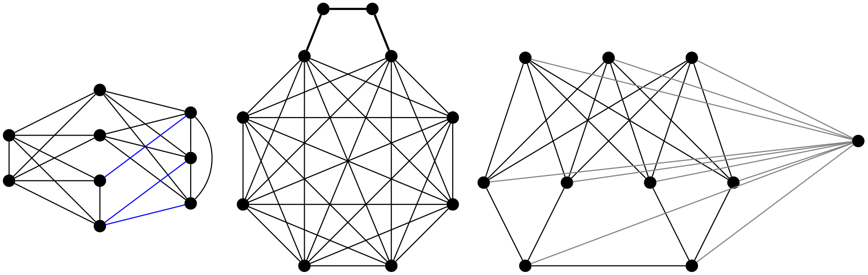

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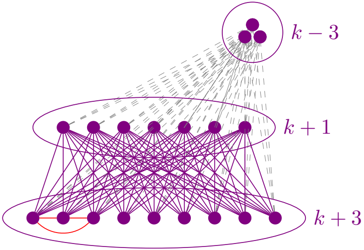
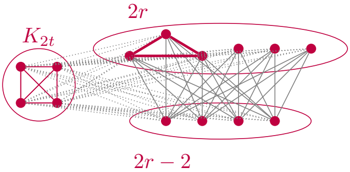
$$\sigma_2 = 4k-4:$$



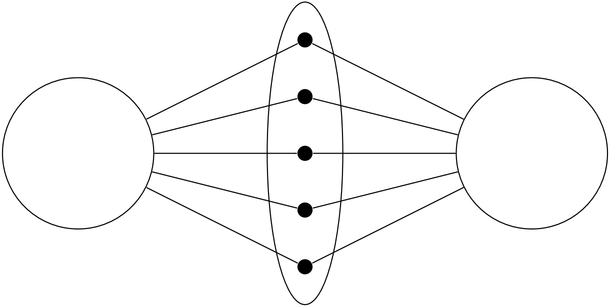


Section 1

Dirac: $(2k-1)$-connected without k disjoint cycles

Dirac, 1963 (link)

What $(2k-1)$-connected graphs do not have k disjoint cycles?



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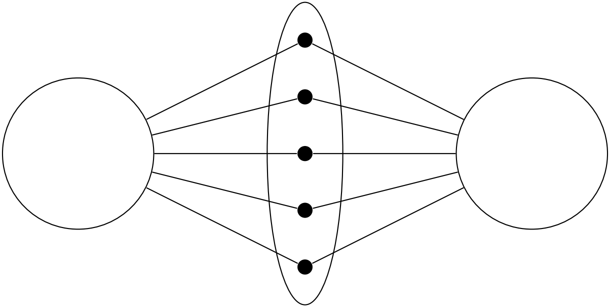


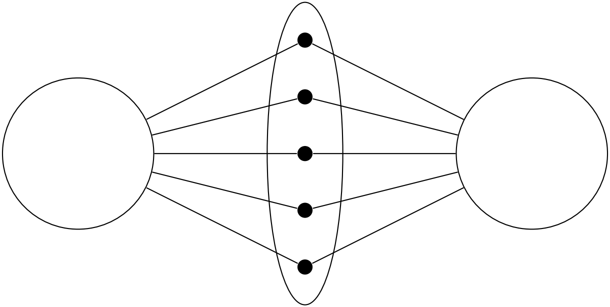
Observation:

G is $(2k-1)$ connected

Dirac: $(2k-1)$-connected without k disjoint cycles

Dirac, 1963 (link)

What $(2k-1)$-connected graphs do not have k disjoint cycles?



Observation:

G is $(2k-1)$ connected $\Rightarrow \delta(G) \geq 2k-1$

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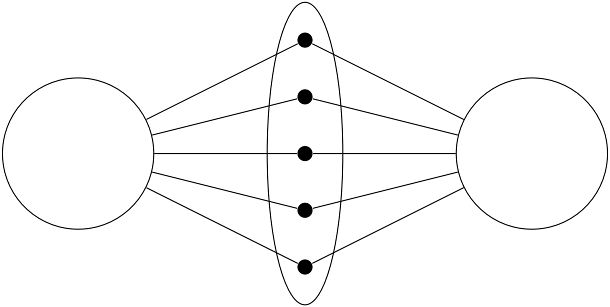

Observation:

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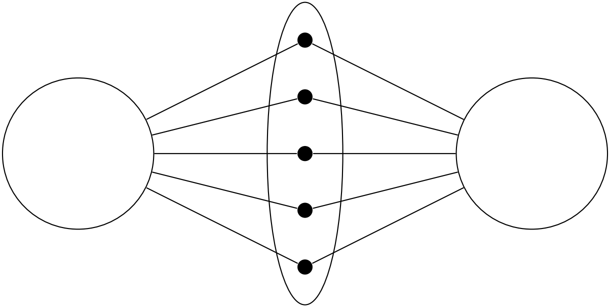

Observation:

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KKY: Holds for $\sigma_2(G) \geq 4k-3$

Dirac: $(2k-1)$-connected without k disjoint cycles

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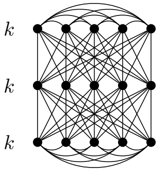
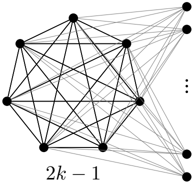
What $(2k-1)$-connected graphs do not have k disjoint cycles?

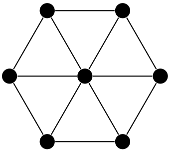
Answer to Dirac's Question for Simple Graphs (KKY 2017)

Let $k \geq 2$. Every graph G with (i) $|G| \geq 3k$ and (ii) $\delta(G) \geq 2k-1$ contains k disjoint cycles if and only if

- if k is odd and $|G| = 3k$, then $G \neq 2K_k \vee \overline{K_k}$, and
- $\alpha(G) \leq |G| - 2k$, and
- if $k = 2$ then G is not a wheel.







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Further:

characterization for multigraphs

Simple Graphs $\rightarrow$ Multigraphs

Idea:

Simple Graphs $\rightarrow$ Multigraphs

Idea:

- Take all 1-vertex cycles

Simple Graphs $\rightarrow$ Multigraphs

Idea:

- Take all 1-vertex cycles
- Take as many 2-vertex cycles as possible (maximum matching)

Simple Graphs $\rightarrow$ Multigraphs

Idea:

- Take all 1-vertex cycles
- Take as many 2-vertex cycles as possible (maximum matching)
- What's left is a simple graph

$(2k-1)$-connected multigraphs with no k disjoint cycles

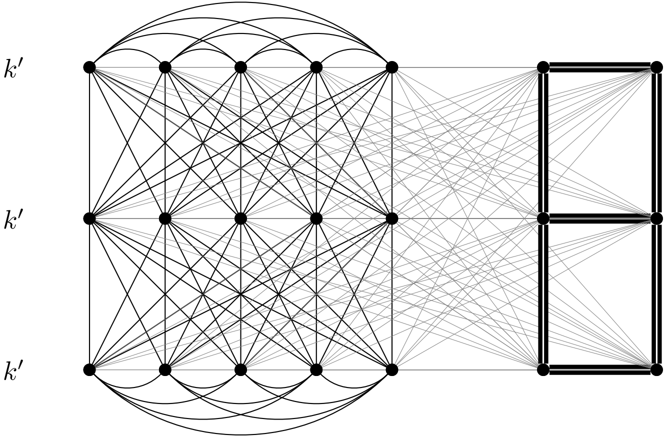
Answer to Dirac's Question for multigraphs: Kierstead-Kostochka-Y. 2015 (link)

Let $k \geq 2$ and $n \geq k$. Let G be an n-vertex graph with simple degree at least $2k-1$ and no loops. Let F be the simple graph induced by the strong edgs of G, $\alpha' = \alpha'(F)$, and $k' = k - \alpha'$. Then G does not contain k disjoint cycles if and only if one of the following holds:

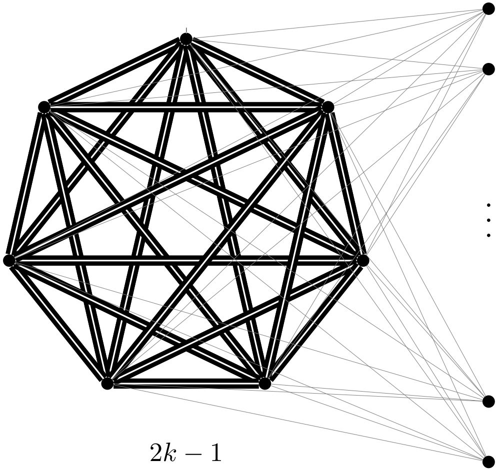
- $n + \alpha' < 3k$;
- $|F| = 2\alpha'$ (i.e., F has a perfect matching) and either (i) k' is odd and $G - F = Y_{k',k'}$, or (ii) $k' = 2 < k$ and $G - F$ is a wheel with 5 spokes;
- G is extremal and either (i) some big set is not incident to any strong edge, or (ii) for some two distinct big sets I_j and $I_{j'}$, all strong edges intersecting $I_j \cup I_{j'}$ have a common vertex outside of $I_j \cup I_{j'}$;
- $n = 2\alpha' + 3k'$, k' is odd, and F has a superstar $S = \{v_0, \ldots, v_s\}$ with center v_0 such that either (i) $G - (F - S + v_0) = Y_{k'+1,k'}$, or (ii) $s = 2$, $v_1 v_2 \in E(G)$, $G - F = Y_{k'-1,k'}$ and G has no edges between $\{v_1, v_2\}$ and the set X_0 in $G - F$;
- $k = 2$ and G is a wheel, where some spokes could be strong edges;
- $k' = 2$, $|F| = 2\alpha' + 1 = n - 5$, and $G - F = C_5$.

k' odd, F has a perfect matching

Example: $k = 8$, $\alpha' = 3$, $k' = 5$.

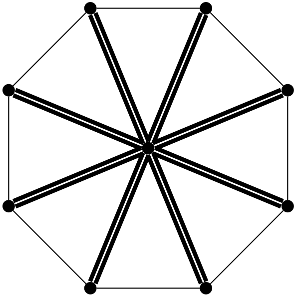


Big independent set, incident to no multiple edges



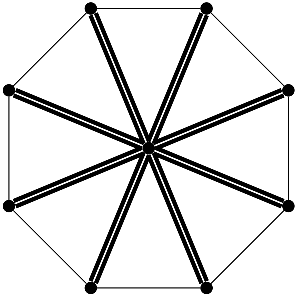
Wheel, with possibly some spokes multiple

Example: $k = 2$



Wheel, with possibly some spokes multiple

Example: $k = 2$



Open

Do the other results in this talk generalize nicely to multigraphs?

Neighborhood Union

Faudree-Gould, 2005

If G has $n \geq 3k$ vertices and $|N(x) \cup N(y)| \geq 3k$ for all nonadjacent pairs of vertices x, y, then G contains k disjoint cycles.

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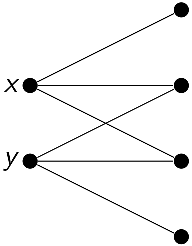

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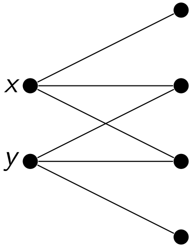

$$d(x) + d(y) = 6$$

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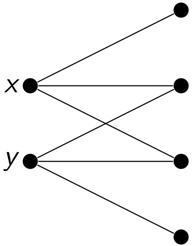

$$d(x) + d(y) = 6$$
$$|N(x) \cup N(y)| = 4$$

Neighborhood Union

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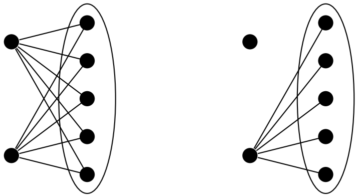
If G has $n \geq 3k$ vertices and $|N(x) \cup N(y)| \geq 3k$ for all nonadjacent pairs of vertices x, y, then G contains k disjoint cycles.

Neither stronger nor weaker than Corrádi-Hajnal.

- If $\delta(G) \geq 2k$, then $\min_{xy \notin E(G)} \{|N(x) \cup N(y)|\} \geq 2k$.
- If $|N(x) + N(y)| \geq 3k$, then $\delta(G) \geq 0$.



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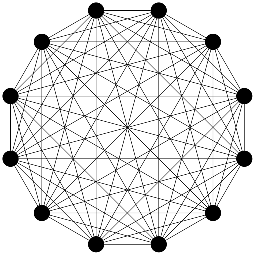
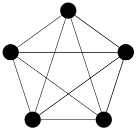
Sharpness:



K_{3k-4}



K_5

Gould-Hirohata-Horn, 2013

Faudree-Gould, 2005

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Gould-Hirohata-Horn, 2013 (link) (conjecture from FG'05)

Let G be a graph on $n > 30k$ vertices such that for any nonadjacent $x, y \in V(G)$, $|N(x) \cup N(y)| \geq 2k + 1$. Then G contains k disjoint cycles.

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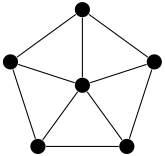
Sharpness of $|N(x) \cup N(y)| \geq 2k+1$:



$k = 2$

Gould-Hirohata-Horn, 2013

Faudree-Gould, 2005

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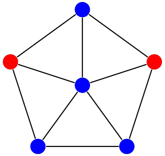

$k = 2$
$|N(x) \cup N(y)| \geq 4$

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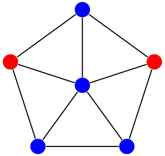

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$|N(x) \cup N(y)| \geq 4$

No two disjoint cycles

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Open:

Perhaps $n > 30k$ is not best possible–can be reduced to $4k$?

Section 2

Finkel, 2008

Posed by Pósa, 1961

Finkel, 2008 (link)

If G is a graph on $n \geq 4k$ vertices with $\delta(G) \geq 3k$, then G contains k disjoint chorded cycles.

Posed by Pósa, 1961

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$k = 1$:

Finkel, 2008

Posed by Pósa, 1961

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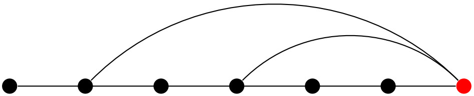

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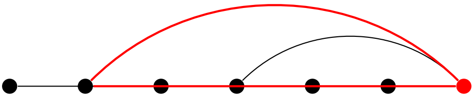

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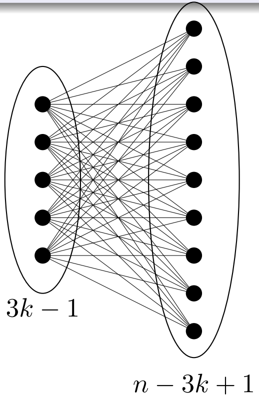

Chorded + Unchorded Cycles

Conjecture: Bialostocki-Finkel-Gyárfás, 2008 (link)

If G is a graph on $n \geq 3r + 4s$ vertices with $\delta(G) \geq 2r + 3s$, then G contains $r + s$ cycles, s of them chorded.

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Chiba-Fujita-Gao-Li, 2010 (link)

Let r and s be integers with $r + s \geq 1$, and let G be a graph on $n \geq 3r + 4s$ vertices. If $\boxed{\sigma_2(G) \geq 4r + 6s - 1}$, then G contains $r + s$ disjoint cycles, s of them chorded cycles.

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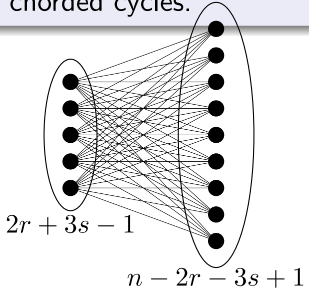

Chorded + Unchorded Cycles: How Sharp Is It?

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Corollary

Let G be a graph on $n \geq 4s$ vertices. If $\sigma_2(G) \geq 6s - 1$, then G contains s disjoint chorded cycles.

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Corollary

Let G be a graph on $n \geq 4s$ vertices. If $\sigma_2(G) \geq 6s - 1$, then G contains s disjoint chorded cycles.

Molla-Santana-Y., 2017 (link)

For $s \geq 2$, let G be a graph $n \geq 4s$ vertices. If $\sigma_2(G) \geq 6s - 2$, then G does not contain s disjoint chorded cycles if and only if $G \in \{K_{3k-1,n-3k+1}, K_{3k-2,3k-2,1}\}$.

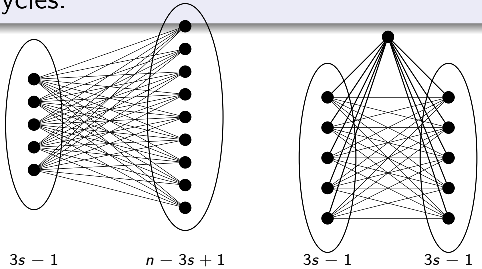
Chorded + Unchorded Cycles: How Sharp Is It?

Chiba-Fujita-Gao-Li, 2010 (link)

Let r and s be integers with $r+s \geq 1$, and let G be a graph on $n \geq 3r + 4s$ vertices. If $\sigma_2(G) \geq 4r + 6s - 1$, then G contains $r + s$ disjoint cycles, s of them chorded cycles.

Corollary

Let G be a graph on $n \geq 4s$ vertices. If $\sigma_2(G) \geq 6s - 1$, then G contains s disjoint chorded cycles.



Chorded + Unchorded Cycles: How Sharp Is It?

Chiba-Fujita-Gao-Li, 2010 (link)

Corollary: If G is a graph on $n \geq 3r + 4s$ vertices with $\delta(G) \geq 2r + 3s$, then G contains $r + s$ cycles, s of them chorded.

Chorded + Unchorded Cycles: How Sharp Is It?

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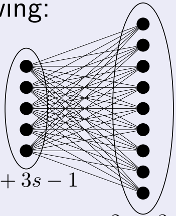
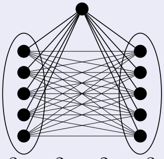
Corollary: If G is a graph on $n \geq 3r + 4s$ vertices with $\delta(G) \geq 2r + 3s$, then G contains $r + s$ cycles, s of them chorded.

Molla-Santana-Y., 2017+

Let r and s be integers with $r + s \geq 1$, and let G be a graph on $n \geq 3r + 4s$ vertices. If $\delta(G) \geq 2r + 3s - 1$, then G fails to contain a collection of $r + s$ disjoint cycles, s of them chorded, if and only if G is one of the following:





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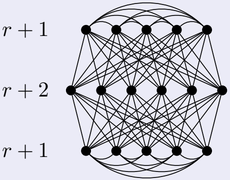
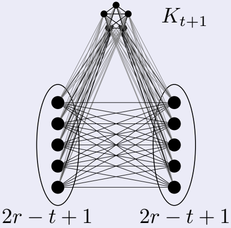
$s = 1$:





Chorded + Unchorded Cycles: Open

Chiba-Fujita-Gao-Li, 2010 (link)

Let r and s be integers with $r + s \geq 1$, and let G be a graph on $n \geq 3r + 4s$ vertices. If $\sigma_2(G) \geq 4r + 6s - 1$, then G contains $r + s$ disjoint cycles, s of them chorded cycles.

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Open:

- In the chorded-cycles-only case: We know what happens if $\sigma_2(G) \geq 6s - 2$; what if $\sigma_2(G) \geq 6s - 3$?

Chorded + Unchorded Cycles: Open

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Let r and s be integers with $r + s \geq 1$, and let G be a graph on $n \geq 3r + 4s$ vertices. If $\sigma_2(G) \geq 4r + 6s - 1$, then G contains $r + s$ disjoint cycles, s of them chorded cycles.

Open:

- In the chorded-cycles-only case: We know what happens if $\sigma_2(G) \geq 6s - 2$; what if $\sigma_2(G) \geq 6s - 3$?
- In the mixed-cycles case: We know what happens if $\delta(G) \geq 2r + 3s - 1$; what about an Ore version?

Neighborhood-Union Conditions

Qiao, 2012 (link)

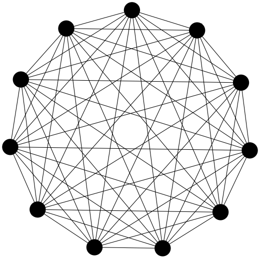
Let r, s be nonnegative integers, and let G be a graph on at least $3r + 4s$ vertices such that for any nonadjacent $x, y \in V(G)$, $|N(x) \cup N(y)| \geq 3r + 4s + 1$. Then G contains $r + s$ disjoint cycles, s of them chorded.

Neighborhood-Union Conditions

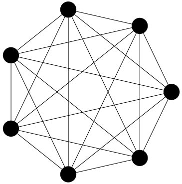
Qiao, 2012 (link)

Let r, s be nonnegative integers, and let G be a graph on at least $3r + 4s$ vertices such that for any nonadjacent $x, y \in V(G)$, $|N(x) \cup N(y)| \geq 3r + 4s + 1$. Then G contains $r + s$ disjoint cycles, s of them chorded.

Sharpness ($r = 0$):



K_{2s+3}



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Open:

- What happens if $|N(x) + N(y)| \geq 3r + 4s$?

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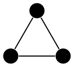
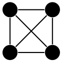
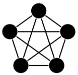
Open:

- What happens if $|N(x) + N(y)| \geq 3r + 4s$?
- Can this be improved for large n, or large k?

Section 2

Multiply Chorded Cycles

We define $f(k)$ to be the number of chords in K_{k+1}, viewed as a cycle. That is, $f(k) = \frac{(k+1)(k-2)}{2}$.

$f(2) = 0$ $\quad f(3) = 2$ $\quad f(4) = 5$

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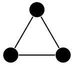

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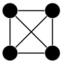

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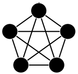

$f(4) = 5$

Gould-Horn-Magnant, 2014 (link)

There exist s_0 and k_0 so that if $s \geq s_0$ and $k \geq k_0$, then there exists an $n_0 = n_0(s,k)$ so that if G has minimum degree at least sk and $|G| > n_0$, then G contains s disjoint cycles with at least $f(k)$ chords.

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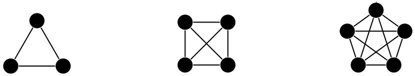

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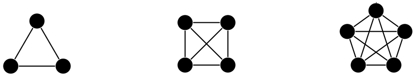

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If $|G| \geq s(k+1)$ and $\delta(G) \geq sk$, then G contains s disjoint cycles, each with at least $f(k)$ chords.

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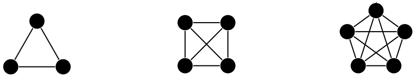

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Hajnal-Szemerédi, 1970

If $|G| = s(k+1)$ and $\delta(G) \geq sk$, then G contains s disjoint copies of K_{k+1}.

Multiply Chorded Cycles

Qiao-Zhang, 2010 (link)

Let G be a graph on $n \geq 4k$ vertices with $\delta(G) \geq \lfloor 7k/2 \rfloor$. Then G contains k disjoint, doubly chorded cycles.

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Sharp for small k:

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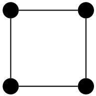
$$\lfloor 7k/2 \rfloor = 3\text{; use } C_4$$



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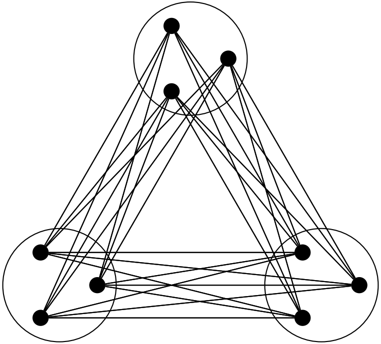
$\lfloor 7k/2 \rfloor = 7$; use $K_{3,3,3}$



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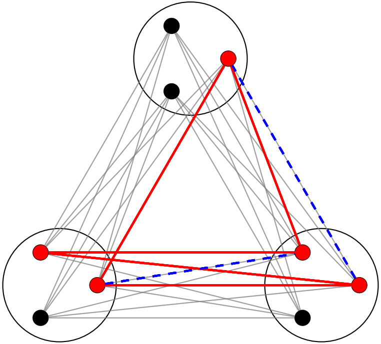

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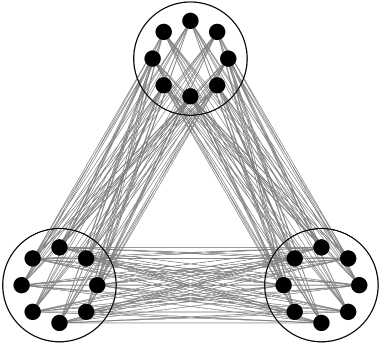
Sharp for small k:

$k = 5$

$\lfloor 7k/2 \rfloor = 17$; use $K_{8,8,8}$



Multiply Chorded Cycles

Gould-Hirohata-Horn, 2015 (link)

If G is a graph on $n \geq 6k$ vertices with $\sigma_2(G) \geq 6k - 1$, then G contains k vertex-disjoint doubly chorded cycles.

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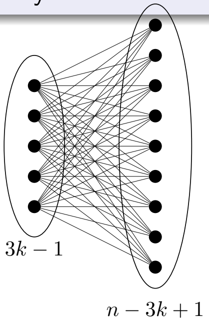

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If G is a graph on $n \geq 6k$ vertices with $\delta(G) \geq 3k$, then G contains k vertex-disjoint doubly chorded cycles.

Conjecture (GHM'14):

If $|G| \geq s(k+1)$ and $\delta(G) \geq sk$, then G contains s disjoint cycles, each with at least $f(k)$ chords.

$\mathbf{k=3}$: If $|G| \geq 4s$ and $\delta(G) \geq 3s$, then G contains s disjoint doubly chorded cycles.

Multiply Chorded Cycles

Conjecture (GHM'14):

If $|G| \geq s(k+1)$ and $\delta(G) \geq sk$, then G contains s disjoint cycles, each with at least $f(k)$ chords.

Chiba-Lichiardopol, 2015 (link)

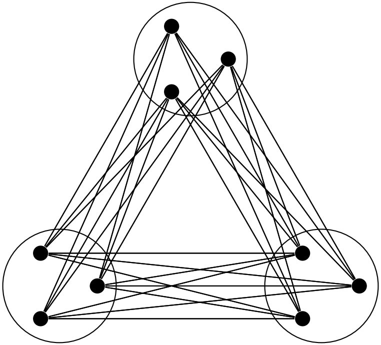
Let s and k be integers, $k \geq 2$, $s \geq 1$.
If G is a graph with $\delta(G) \geq s(k+1) - 1$, then G contains s disjoint cycles, each with at least $f(k)$ chords.

Section 3

Equitable Coloring

Definition

An *equitable k-coloring* of a graph G is a proper coloring of $V(G)$ such that any two color classes differ in size by at most one.



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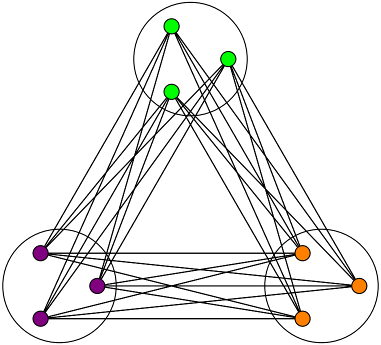

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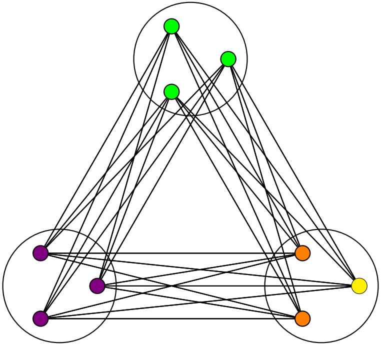

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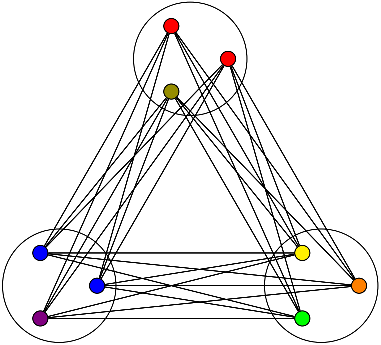

Equitable Coloring and Cycles

$n = 3k$

If G has $n = 3k$ vertices and an equitable k-coloring, then $\overline{G}$ has k disjoint cycles (all triangles).

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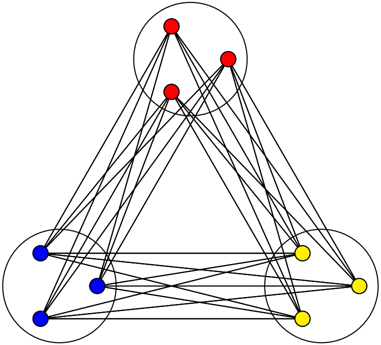

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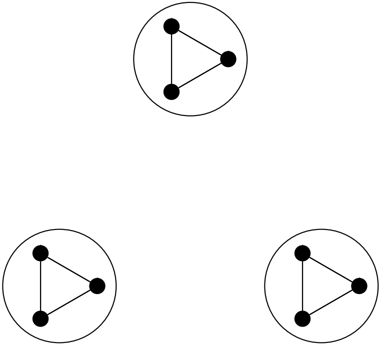

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independent sets $\leftrightarrow$ cliques

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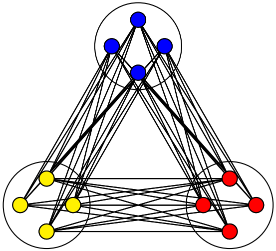

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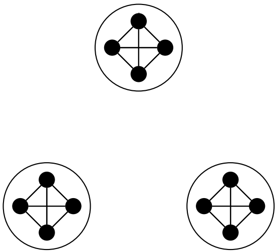

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Cycles, chorded cycles, cycles with $f(k)$ chords, etc: generalizations of cliques.

Equitable Coloring and Cycles

Kierstead-Kostochka, 2008 (link)

If G is a graph such that $d(x) + d(y) \leq 2k - 1$ for every edge xy, then G has an equitable k-coloring.

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Enomoto 1998, Wang 1999

If G is a graph on n vertices with $n \geq 3k$ and $\sigma_2(G) \geq 4k - 1$, then G contains k disjoint cycles.

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$n = 3k$

Equivalent when $n = 3k$: 2(3k-1)-(2k-1)=4k-1

Equitable Coloring and Cycles

Qiao-Zhang, 2010

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$n = 4k$

$\delta(G) \geq \lfloor 7k/2 \rfloor \Leftrightarrow \Delta(\overline{G}) \leq (4k-1) - (\lfloor 7k/2 \rfloor) = \lfloor k/2 \rfloor - 1$

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Equivalent Statement for $n = 4k$

Let G be a graph on $4k$ vertices with $\Delta(G) \leq \lfloor k/2 \rfloor - 1$. Then G is equitably k-colorable.

Chen-Lih-Wu

Hajnal-Szemerédi, 1970

If $k \geq \Delta(G) + 1$, then G is equitably k-colorable.

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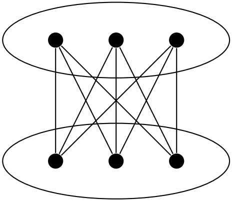

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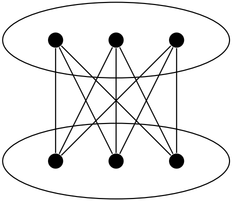

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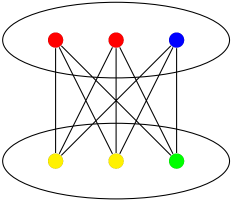

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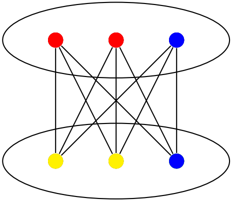

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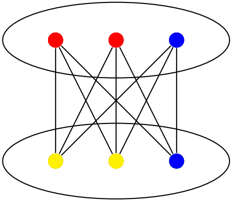

$\Delta(G) = 3$

Chen-Lih-Wu Conjecture, 1994 (link)

A connected graph G is equitably $\Delta(G)$ colorable if G is different from K_m, C_{2m+1} and $K_{2m+1,2m+1}$ for every $m \geq 1$.

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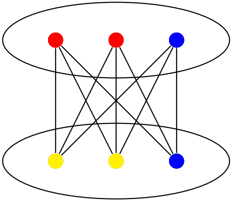

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Chen-Lih-Wu Conjecture Re-stated

If $\chi(G), \Delta(G) \leq k$ and $K_{k,k} \not\subseteq G$, then G is equitably k-colorable.

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CLW true if:

$\delta(G) \geq |G|/2$; $\Delta(G) \leq 4$; G planar with $\Delta(G) \geq 13$; G outerplanar, etc.
Still open in general

Ore Conditions

Kierstead-Kostochka, 2008 (link)

If G is a graph such that for each edge xy, $d(x) + d(y) \leq 2k - 1$, then G is equitably k-colorable.

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If G is a $3k$-vertex graph such that for each edge xy, $d(x) + d(y) \leq 2k + 1$, then G is equitably k-colorable, or is one of several exceptions.

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Equivalent

If G is a graph on $3k$ vertices with $\sigma_2(G) \geq 4k - 3$, then G contains k disjoint cycles, or is one of several exceptions.

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If G is a graph on $3k$ vertices with $\sigma_2(G) \geq 4k - 3$, then G contains k disjoint cycles, or is one of several exceptions.

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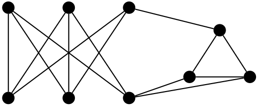
For $k \geq 4$, if G is a graph on n vertices with $n \geq 3k + 1$ and $\sigma_2(G) \geq 4k - 3$, then G contains k disjoint cycles if and only if $\alpha(G) \leq n - 2k$.

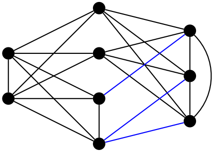
Exceptions

- $k = 3$

Equitable coloring:

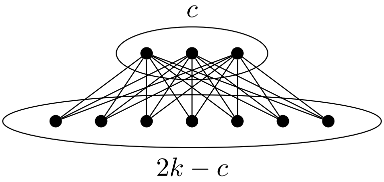


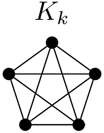
Cycles:



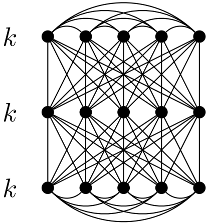
Exceptions

- *Equitable coloring:*





Cycles:



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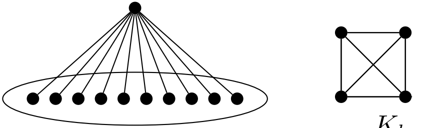
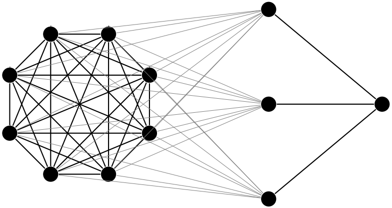


$2k$ K_{k-1}

Cycles:



K_{2k} $k-1$

Thanks!

Slides (with links to references) at:
http://www.math.ubc.ca/~elyse/Talks.html